PHYSICS

Volume II Part I

A Textbook for Class XII

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T. V RAMAKRISHNAN
Indian Institute of Science, Bangalore

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Bangalore
C V. VISHVESHWARA
Indian Institute of Astrophysics
Bangalore

PHYSICS CURRICULUM GROUP

Department of Education in Science and Mathematics, NCERT

R N. MATHUR
B K. SHARMA
R. JOSHI
V. P. SRIVASTAVA

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ARVIND KUMAR
A. N. MAHESHWARI
N. MUKUNDA
RAJARAM NITYANANDA
J. RAMAKRISHNA
T. V RAMAKRISHNAN
S RAI CHOUDHURY
D. P. SENGUPTA
S. V SUBRAMANYAM
C V VISHVESHWARA

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Foreword

In keeping with the National Policy on Education (NPE) 1986, a new generation of physics textbooks for senior secondary classes was published by the National Council of Educational Research and Training (NCERT) during 1988-89. These textbooks were developed by a team of eminent research physicists in active interaction with the physics faculty in the NCERT and some practising teachers. Although these books have been trend-setters for physics education in the country, the feedback on the use of these books received from teachers and students gave a clear message that the Class XII physics textbook needed major revision as the level of mathematics employed in it at many places was far above the competencies envisaged at the senior secondary stage

Prof. T. V Ramakrishnan of the Indian Institute of Science, Bangalore, kindly agreed to take up the responsibility of the revision of the book to make it more effective within the style and approach of the existing book. He has carried out this task with a team consisting of some of the authors from the original team and some new members.

As Chairman of the writing team for this edition, Professor Ramakrishnan has done a commendable job and has produced a high quality user-friendly physics textbook of world class and standard. It is a pleasure to record my sincere thanks to him and to all the members of his writing team for their vision and hard work in writing this book and in preparing the camera-ready manuscript using the latest computer technology. The quality of the computer generated illustrations and text lay-out of this book have set it apart as a pace-setter for NCERT's textbook production.

The development of the manuscript for this edition was carried out with the cooperation, assistance and pedagogical support of a team of faculty members from the Department of Education in Science and Mathematics (DESM), NCERT, under the guidance of Prof. A. N. Maheshwari, Joint Director, NCERT. My special thanks go to Prof. A. N. Maheshwari, Prof. R. N. Mathur, Dr. B. K. Sharma, Shri R. Joshi and Dr. V. P Srivastava for their support to the authors of this book.

I also extend my thanks to Prof. K. V. Rao, Head, DESM, NCERT, for his keen interest and continuous support in the development of this book. My thanks are also due to Mr. C. N. Rao, Head, Publication Department, NCERT, and his Publication Team for ensuring the quality of production of this textbook.

Comments and suggestions for further improvement of the book will be most welcome.

A. K. SHARMA
Director
National Council of Educational
Research and Training

February 1995
New Delhi

Preface to the Revised Edition

An almost completely rewritten version of the Physics textbook for Class XII is before you The general objectives and approach of this edition continue to be the same. Briefly, the main objective is to present Physics as it grows out of phenomena observed in nature as well as in experiments, formulation of basic physical ideas and their precise expression, developing the implications and testing them experimentally, and evolution of new concepts. A related objective is to equip students with the ways of thinking and experimenting, the methods necessary, and the background knowledge of physical phenomena so that these are of qualitative and quantitative use in a wide variety of circumstances. Our approach in trying to realize this objective has been to adopt a somewhat informal direct style, and to start with a description of phenomena and experiments. The basic physical concepts and principles implied are then drawn out and clearly expressed Their consequences are discussed. A number of solved examples in the text as well as a large number of exercises ard additional exercises illustrate how the principles are applied in a variety of physical situations. In trying to realize the rather general objectives mentioned above, we have benefited in many ways from the use of the first edition and from the nearly six years that have passed since it came out.

We have had the advantage of extensive feedback from classroom use of the first edition of the text In July 1990, a meeting sponsored by the NCERT was held in the Regional College of Education, Mysore, at which teachers who used the book, subject experts, and authors discussed the text extensively. A number of concrete suggestions regarding the style of presentation, level and use of mathematics, reduction in the amount of subject matter, etc. were made, and they have been largely incorporated. The authors have also individually benefited from taking part in a number of summer courses organized for physics teachers at the +2 level. A preliminary draft of the present version was discussed at a review workshop (organized by the NCERT) conducted in Bangalore, in April 1993. This, and continuous interaction with colleagues in the Physics Curriculum Group of the DESM as the manuscript evolved, had an effect on the book.

In the following pages, we have attempted to meet our general objectives with the advantage of the inputs mentioned above. We have tried to make the presentation as simple, concrete and direct as possible while retaining accuracy. Simple examples are emphasized. Mathematical formalism is kept at the minimum necessary level (Vectors and differentiation are used; the integral symbol is occasionally employed, but a knowledge of integration is neither assumed nor used.) A large number of figures are used to assist in visualization. Both the amount and the level of material in several chapters have been reduced. Important ideas, definitions, etc. are often highlighted by bold letters. Each chapter ends with an extensive summary Each chapter has a number of solved examples integrated with the text. The unsolved problems have been divided into two categories, namely exercises and additional exercises. Exercises often involve substitution or relatively simple analysis, calculation and application. They are confidence building steps towards additional exercises, which may have harder questions. The specific changes in different chapters include the following:

- a simpler approach to Gauss's theorem in electrostatics based on symmetry and with many applications,
- a simpler discussion of the electrostatic potential,
- reduction of material on thermoelectricity,
- a more direct and brief discussion of electromagnetic induction as well as of alternating currents,
- a simpler, descriptive approach to electromagnetic waves, and
- a discussion of semiconductor rectifiers and transistors basically as circuit elements with given behaviour.

Supplementary themes, somewhat advanced discussions and historical material are all in starred sections, which are to be out of the scope of examinations.

The computer revolution has finally caught up with this edition. The text was typeset using a public domain distribution of TEX with a large number of macros specially developed for this book by Dr. Biswadeb Dutta (of Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore). A two-column text such as this, with its variety of symbols, typefaces, insertions, etc is a formidable task. Dr. Dutta's ingenuity, unusual ability and selfless commitment have made this book physically realizable. I would also like to thank the typists, namely Ms. G. Sukanya, Ms. A. M. Shanta Kumari, Mr. N. Gopalakrishnan, Mr. G. Manjunatha and Mr. M. Renugopal for doing such a good job and for putting up with several rounds of corrections, additions, etc. and generally helping well beyond the call of duty. The drawings were generated on the computer by Mr. A. S. Prabhakar and Mrs. Geeta Prabhakar, who in spite of difficult circumstances cheerfully carried out many rounds of changes and improvements in the original drawings. I would also like to thank Mr. G. Chandramohan of the Indian Academy of Sciences, Bangalore, for careful reading of proofs and general advice regarding the production of this book and Mr. M. K. Dasgupta for helping to put some figures on to the text. All these professionals as well as the authors gave of their time and expertise unstintingly because of their commitment to the cause of good education for the young of our country.

For everyone connected with this book, including the authors, the work was something beyond their normal activity This has led to delays, and to haphazard progress. In spite of this, the commitment of everyone involved has seen the book through. The encouragement, patience and active support of colleagues at the NCERT has made the crucial difference to the coming out of this revised edition, in its form as well as in content. We would particularly like to thank Professor A. K. Sharma, Director, NCERT; Professor A N. Maheshwari who first as Head, DESM and now as Joint Director, NCERT strongly supported us; Professor K. V. Rao, Head, DESM who gave us all encouragement and support in carrying out this task We would also like to thank our colleagues in the Physics Group at the NCERT for going through this version line by line, checking the text, figures and exercises. Special mention must be made of Dr. V. P. Srivastava, who worked for several weeks in Bangalore in the final stages, assisting in the birth of the book.

We have tried to make the book interesting and thought-provoking while presenting the essentials of physics appropriate at this stage of education In spite of our efforts, many errors might still remain, and changes in presentation might be desirable in many places. We request readers of the book to bring these to our attention so that corrections and improvements can be made as necessary.

T V. RAMAKRISHNAN
Chairman of the Author Team
Indian Institute of Science
Bangalore

Preface

The National Policy on Education adopted, in 1986, the Plan of Action evolved to implement the accepted National Policy, and the countrywide debate thereon led to a decision that the teaching of science should be introduced in an integrated way and pursued up to class X as a part of the study of Nature without compartmentalising it into narrow disciplines such as Physics, Chemistry, Botany, Zoology, etc. With this background of science, it was further decided to teach Physics, Chemistry, etc. as separate subjects in classes XI and XII In response to these decisions the Minister for Human Resource Development, Government of India, appealed to men of science in the universities and research institutions to get together to develop curricular material for teaching of science and of separate scientific disciplines at the school level. The response to this appeal was immediate and enthusiastic. Working Groups consisting of university professors, school teachers and research scientists were constituted to develop curricular material. The present book is the result of the efforts of the Working Group in Physics In this endeavour, the Working Group gratefully acknowledges the unstinted help and cooperation as also valuable pedagogical input received from Dr. P. L Malhotra, Director, NCERT, and his colleagues in the Department of Education in Science and Mathematics (DES&M).

The physics Working Group, after long deliberation, considered it necessary to initiate a new approach to the study of Physics at the school level. This approach is essentially based on the active participation of the students in the learning process through experimentation, supplemented by demonstration by teachers, oriented activity by the students and discussion leading to the understanding of the basic concepts in Physics, without loss of mathematical rigour. The efforts of the Group have been to relate, as far as possible, the teaching of Physics to what a student sees and does in everyday life. In addition, it is intended to transmit in some measure the thrill and excitement in the study of Physics which would help the students to understand Physics better and find something new for themselves. The Working Group is only too conscious that there is no unique way of teaching Physics and indeed one can conceive of very many ways of doing so. This book is only an humble effort of the Working Group in pursuance of their perception.

The Working Group strongly feels that classroom teaching, experimentation by the students, demonstration by the teachers, outdoor oriented activity, the process of evaluation, etc all contribute to the teaching of Physics and it has been our effort to produce a total package. Although initially we thought of integrating all these into one book, however, for the sake of convenience, the textbook and the laboratory manual are separated. This textbook contains a large number of questions at the end of each chapter which are indeed to supplement the textbook. The group is also conscious that every student has his own pace of learning and some students can progress at a rapid pace and assimilate more difficult concepts as compared to the average student In order to sustain the interest of such students, the textbook contains portions which are starred (*) and are printed in small type. The omission of these portions will not affect the learning process of the average student.

In order to achieve the objective of presenting Physics as it grew arising out of keen observations of nature and natural phenomena, formulation of basic concepts, conceptualising a model, and predictions made, based on the models that have been built, the authors have adopted a somewhat informal direct style, appealing directly to the reader's experience and curiosity, while describing a large number of phenomena. In order to facilitate easy comprehension, a number of illustrations and diagrams have been given. It has been our attempt to present basic ideas with care, simultaneously bringing out their subtleties. Physics has grown and is still growing. This process of growth implies existence of areas of ignorance, inadequate understanding etc. We have attempted to bring the attention of the students to some of these areas. Interesting additional material, which is connected with the rest of the text but is not in the prescribed syllabus is included in smaller print as starred (*) sections.

We believe that one of the important ways in which physics is learnt is by solving problems requiring application and critical appreciation of the principles dealt within the text. In pursuance of this conviction, the book includes a large collection of solved examples and exercises–about 500 in all.

Because of the above reasons, this textbook is somewhat big compared to the earlier ones (though not in comparison to textbooks for students of this age in other countries). Briefly, we believe that the kind and level of detailed treatment given here will help in comprehension, enjoyment and application. Most emphatically, the material is not to be memorized, but to be read, thought about, appreciated, and applied (now and later). As a matter of fact, the number of topics covered (syllabus) is *less* than that in earlier syllabi. The idea is to build up the ability of the student to think about phenomena scientifically and to stimulate his interest in them. It is not to burden him with facts, lists and formulae.

The text falls into three natural parts. The major portion of the book, comprising chapters I to IX is concerned with electricity and magnetism. Chapter X and XI are concerned with optics The remainder of the book (chapters XII to XV) has to do with modern physics.

The laboratory manual contains three components, viz., (i) experiments to be conducted by the students themselves which are designed to clarify or bring out the application of the concepts dealt within the textbook (ii) demonstration by the teacher in the class intended to exemplify the operation and working of physical principles and concepts and (iii) oriented activity by the students designed to convey the ideal that Physics is not only to be studied in the class or a laboratory but should become a part of life.

The Working Group is only too conscious of the shortcomings and limitations of the material. The practical difficulties in implementing the course will become clear after full scale trial. Teachers in urban and rural schools are our primary concern and we look forward to a meaningful appraisal of the material by them. We also look forward to the reaction of the young students to whom it is addressed. We also look up to senior physicists in the universities and research institutions for their mature criticism of the material presented here from the standpoint of the contents as well as the way of presentation For these reasons the present edition is being brought out as an experimental edition which will undergo early revision after we receive the feedback from various sources.

Because of constraints of time, this edition has had to be brought out in some haste. As a consequence, we are conscious that a number of errors have crept in, some of which will be indicated in the corrigendum attached to part II of the book. In spite of this, a number of mistakes might have gone in unnoticed. We shall indeed be grateful if these are brought to our attention so that the final version of the text can be brought out without errors and mistakes.

All my colleagues join me in offering our grateful thanks to Prof. C. N. R. Rao, Chairman, Science Advisory Committee to the Prime Minister, who conceived the idea of Working Groups, for his guidance and stimulation; Dr. P. L. Malhotra, Director, NCERT; Dr. B Ganguly, Head, DES&M, and his colleagues for their help in this endeavour.

This book has been made possible by the dedication and help of many. Mr. G. Chandramohan of the Indian Academy of Sciences, Bangalore, made the book press-worthy, read and corrected the proofs, and generally lent his time and professional expertise unstintingly. Mr. Y. Hatwalne and Mr. B. Dutta, Ph.D. research scholars of the Department of Physics, Indian Institute of Science have given their time and knowledge of physics in checking the manuscripts, helping to get them typed, getting the figures drawn and generally making the manuscripts suitable for the audience. Mr. G. Fernandes and Ms. P. Lalantika of the Centre for Theoretical Studies, Indian Institute of Science, Bangalore, did an extremely good job of typing several revisions of the book. The draughtsmen, Mr. C. M. Aswatha and Mr. R. Babu were very constructive and helpful. We are also thankful to the teachers who participated in the New Delhi workshop for their careful reading of the book, discussions, and suggestions for change.

V G. BHIDE
Poona University, Pune

A Note for Teachers

Most of the exercises in this book are designed to create learning situations for students to help them comprehend the contents of the textbook thoroughly. The collection of exercises here is, however, quite large and attempting to solve all of them in the limited time available in a year may be burdensome for many students. Items which are not completely straightforward applications of material in the text have, therefore, been put in the category of additional exercises. All students should be encouraged to attempt to solve as many of the exercises and additional exercises as possible.

A Note for Students

A. Regarding the Book

This book is meant to increase your understanding of the world around you, by describing phenomena carefully, and by providing you with the basic ideas and methods for their scientific explanation. Our hope is that after reading the book, thinking about what it says, going through solved examples and working through the exercises, you will be able to figure out things by yourself In order to achieve this aim, we have tried to give many physical examples, describe carefully how theoretical models are made, emphasize basic principles and to relate the wide variety of phenomena to these We have also given a large number of solved examples and unsolved exercises with answers

For the above reasons, the book is somewhat big (though no bigger than what students of the same age use in other countries). We strongly believe that this will actually help you in understanding, enjoying and applying physics. The book is *not* a list of facts and equations to be painfully memorized. It will, however, have to be read and read again, and worked through We believe that you are at an age where you are not only full of questions, but also, with a little help (this book) can yourself find answers (at least for questions concerned with physical science!). If you find that things are not clear you are most welcome to write to us at the address indicated.

B. Regarding Examples and Exercises

This book contains a large collection of examples and exercises – about 500 in all. Answers to nearly all the exercises are given at the end of the book and hints or partial solutions have been provided to many exercises

The exercises have been divided into two broad categories; exercises and additional exercises The former should be attempted first. They often require direct use of what you have learnt, and will help you to be sure of the basic results and their application The additional exercises may need more work, but are based only on what is developed in the text In some cases, hints are given for solution. You can, if you like, first solve only the exercises, and after this preparation, depending on interest as well as available time, attempt to solve as many of the additional exercises as you like.

It is most important that you try to work out the exercises regularly. Try at least 15 to 20 exercises every week, and we believe you will notice after a few months of this regular practice an improvement in your comprehension of the subject. We wish you happy problem-solving in physics.

A Note on Notation

In this book, all symbols are given in italics, e g. a mass m, a charge q or potential V The exceptions are vectors, which are given in boldface, e.g. electric field **E**. Coordinates and units are given in roman, e.g a point P, or 3V (for volts) or 7 3 A (for amperes). We have followed SI units throughout the book.

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SOME SYMBOLS FOR UNITS OF PHYSICAL QUANTITIES

SI Symbols		Symbols other than SI that are Commonly Used	
Name	Abbreviation	Name	Abbreviation
ampere	A	angstrom	Å
candela	cd	atmosphere	atm
coulomb	C	British thermal unit	Btu
farad	F	calorie	cal
henry	H	degree	°
hertz	Hz	degree celsius	°C
joule	J	centimetre	cm
kelvin	K	Curie	Ci
kilogram	kg	degree Fahrenheit	°F
meter	m	dyne	dyn
mole	mol	electron volt	eV
newton	N	foot	ft
ohm	Ω	gauss	G
pascal	Pa	gram	g
radian	rad	horsepower	hp
second	s	hour	h
siemens	S	minute (of arc)	′
steradian	sr	minute (of time)	min
tesla	T	pound	lb
volt	V	revolution	rev
watt	W	second (of arc)	″
weber	Wb	unified atomic mass unit	u
		year	y

THE GREEK ALPHABET

Alpha	Α	α	Iota	Ι	ι	Rho	Ρ	ρ
Beta	Β	β	Kappa	Κ	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	Τ	τ
Delta	Δ	δ	Mu	Μ	μ	Upsilon	Υ	υ
Epsilon	Ε	ϵ	Nu	Ν	ν	Phi	Φ	ϕ,φ
Zeta	Ζ	ζ	Xi	Ξ	ξ	Chi	Χ	χ
Eta	Η	η	Omicron	Ο	ο	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational Value	BEST (1986) VALUE	
			Value[a]	Uncertainty[b]
Speed of light in a vacuum	c	3.00×10^{8} m/s	2.99792458.	exact
Elementary charge	e	1.60×10^{-19} C	1.60217738	0.30
electron mass	m_e	9.11×10^{-31} kg	9.1093897	0.59
Proton mass	m_p	1.67×10^{-27} kg	1.6726230	0.59
Neutron mass	m_n	1.68×10^{-27} kg	1.6749286	0.59
Electron-charge-to-mass ratio	e/m_e	1.76×10^{11} C/kg	1.75881961	0.30
Permittivity constant	ϵ_0	8.85×10^{-12} F/m	8.85418781762	exact
Permeability constant	μ_0	1.26×10^{-6} H/m	1.25663706143	exact
Planck constant	h	6.63×10^{-34} J.s	6.6260754	0.60
Universal gas constant	R	8.31 J/mol K	8.314510	8.4
Avogadro constant	N_A	6.02×10^{23} mol^{-1}	6.0221367	0.59
Boltzmann constant	k	1.38×10^{-23} J/K	1.380657	11
Faraday constant	F	9.65×10^{4} C/mol	9.6485309	0.30
Rydberg constant	R	1.10×10^{7} m^{-1}	1.0973731534	0.0012
Gravitational constant	G	6.67×10^{-11} m^3/s^2 kg	6.67260	100
Bohr radius	a_0	5.29×10^{-11} m	5.29177249	0.045
Electron magnetic moment	μ_e	9.28×10^{-24} J/T	9.2847700	0.34

[a] Values given in this column should be given the same unit and power of 10 as the computational value. [b] Parts per million.

SI PREFIXES*

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

⋆ In all cases the first syllable accented, as in ná-no-mé-ter

SCIENCE RELATED VALUES

Curiosity, quest for knowledge, objectivity, honesty and truthfulness, courage to question, systematic reasoning, acceptance after proof/verification, open-mindedness, search for perfection and team spirit are some of the basic values related to science. The processes of science, which help in searching the truth about nature and its phenomena are characterised by these values. Science aims at explaining things and events. Therefore to learn and practise science :

- **Be inquisitive about things and events around you.**
- **Have the courage to question beliefs and practices.**
- **Ask 'what', 'how' and 'why' and find your answers by critically observing, experimenting, consulting, discussing and reasoning.**
- **Record honestly your observations and experimental results in your laboratory or outside it.**
- **Repeat experiments carefully and systematically if required, but do not manipulate your results under any circumstance.**
- **Be guided by facts, reasons and logic. Do not be biased in one way or the other.**
- **Aspire to make new discoveries and inventions by sustained and dedicated work.**

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational Value	BEST (1986) VALUE	
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Speed of light in a vacuum	c	3.00×10^{8} m/s	2.99792458.	exact
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Boltzmann constant	k	1.38×10^{-23} J/K	1.380657	11
Faraday constant	F	9.65×10^{4} C/mol	9.6485309	0.30
Rydberg constant	R	1.10×10^{7} m^{-1}	1.0973731534	0.0012
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg	6.67260	100
Bohr radius	a_0	5.29×10^{-11} m	5.29177249	0.045
Electron magnetic moment	μ_e	9.28×10^{-24} J/T	9.2847700	0.34

[a] Values given in this column should be given the same unit and power of 10 as the computational value. [b] Parts per million

SI PREFIXES*

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

⋆ In all cases the first syllable accented, as in ná-no-mé-ter.

SCIENCE RELATED VALUES

Curiosity, quest for knowledge, objectivity, honesty and truthfulness, courage to question, systematic reasoning, acceptance after proof/verification, open-mindedness, search for perfection and team spirit are some of the basic values related to science. The processes of science, which help in searching the truth about nature and its phenomena are characterised by these values. Science aims at explaining things and events. Therefore to learn and practise science :

- **Be inquisitive about things and events around you.**
- **Have the courage to question beliefs and practices.**
- **Ask 'what', 'how' and 'why' and find your answers by critically observing, experimenting, consulting, discussing and reasoning.**
- **Record honestly your observations and experimental results in your laboratory or outside it.**
- **Repeat experiments carefully and systematically if required, but do not manipulate your results under any circumstance.**
- **Be guided by facts, reasons and logic. Do not be biased in one way or the other.**
- **Aspire to make new discoveries and inventions by sustained and dedicated work.**

CHAPTER 1

Electric Forces, Charges and Fields

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1.1 Introduction

Many practical conveniences and appliances we use in daily life are based on the principles of electricity and magnetism. Lights, fans and motors, radio and television are familiar examples. So are electric power generation, wireless and telephone communication, batteries and so on. Indeed, modern civilisation depends heavily on the science of electricity and magnetism. Many of these applications also require understanding the properties of different kinds of materials.

For daily purposes we depend mainly on current electricity for the working of appliances. Historically, however, it was frictional electricity that was understood first, while current electricity came to be understood later. As we shall see, there is no basic difference between the two. So we begin our study with frictional electricity and electric charge; in later Chapters we turn to current electricity, which is flow of charge.

In the study of the laws of projectiles and falling bodies, we learn about the force of *gravity* This is also the force that explains the motion of the moon around the earth, and of the planets and comets around the sun. Apart from gravity, in all processes of normal daily experience only two other forces are involved. These are *electric* and *magnetic* forces. Often these are referred to jointly as the *electromagnetic* force It is this force that leads to all the laws of chemistry, the properties of matter from atoms to bulk materials, and even the processes that occur in the cells of living organisms

In the monsoon season, and also at other times, you may have seen the very impressive sight of flashes of lightning during thunderstorms. This is a large scale electrical phenomenon which occurs in nature. We will study a little bit about it in the next Chapter. Apart from this natural example, it usually requires special effort to show the existence of electromagnetic forces and phenomena in nature, and to see the electrical basis of chemical and biological processes.

The major part of this text book will deal with the basic features of electricity and magnetism. At many places in our study, some understanding of the properties of matter will be important, so this will be included. In this Chapter, we introduce the ideas of electric charge, electric forces between charges, and the electric field We study these further in the next Chapter as well. In later Chapters we will learn about electric currents, their thermal and chemical effects, and the close connections between electricity and magnetism. This leads us on to electromagnetic waves, optics and optical instruments, and some parts of modern physics. One Chapter is devoted to an account of solids and semiconductor devices, which are so commonly used today

1.2 Frictional electricity – two kinds of charge – likes repel and unlikes attract – the electroscope

A glass rod rubbed with a silk cloth, or a hard rubber rod with a woollen cloth, is able to attract small pieces of paper, straw, lint, light feathers, hair, etc. If a rubber or plastic comb is passed through dry hair on a dry day, it can also attract such small objects. In each of these cases, we say that the rod after rubbing, or the comb after running through dry hair, is *electrified*, or has become *electrically charged*.

Any solid material rubbed with any other material under suitable conditions gets charged to some extent in this way In all cases, it is the friction during rubbing that makes the object, whether glass or rubber rod or comb, get charged So these are examples of *frictional electricity*. Here are more examples: a piece of ebonite rubbed with wool; a woollen sweater or coat sliding against a plastic seat, rubber shoes brushing over a woollen carpet.

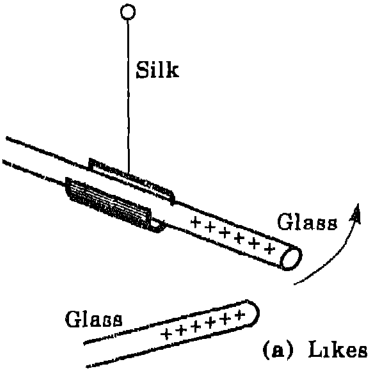
Effects of this kind have been known for a long time. Simple experiments can be done to show that there are two kinds of electric charge. They also show that in some cases there are repulsive forces, unlike the attractive forces described above. Let us describe such an experiment.

Take a glass rod and rub one end of it vigorously with a silk cloth. Then suspend it by a long silk thread, so that it can swing horizontally. Now take another glass rod, rub one end with silk, and bring it near the hanging rod. It is found that the hanging rod swings *away*, that is, the two charged ends *repel* one another. This is shown in Fig 1.1a.

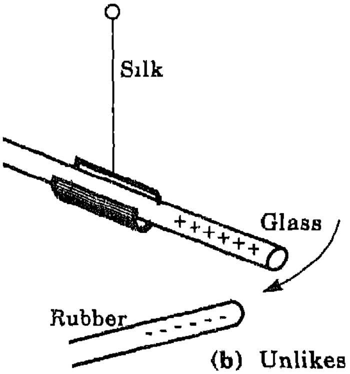
Next, suppose we take a hard rubber rod and rub one end of it with a woollen cloth. If this is brought near the hanging glass rod, we see something different. The glass rod swings *towards* the rubber rod, that is, the two charged ends *attract* one another. We show this in Fig 1.1b.

A third experiment, Fig. 1 1c, completes the picture. If we hang a hard rubber rod, charged by rubbing with wool, from a long thread, and bring another charged rubber rod near it. we find again that they repel one another, as was the case with two glass rods.

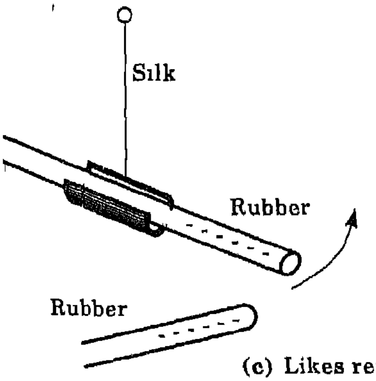
From many such experiments, it becomes clear that two kinds of "electrification" can be produced on bodies by friction, that is, there are two kinds of electric charge Two bodies carrying the *same* kind of electric charge *repel* one another – in the above experiment, two charged glass rods or two charged rubber rods. On the other hand, two bodies carrying *different* or *opposite* kinds of electric



(a) Likes repel



(b) Unlikes attract



(c) Likes repel

Figure 1.1: Attraction and repulsion between charged objects

charge *attract* one another - in the above experiment, one charged glass rod and another charged rubber rod. We can say in brief. *likes repel and unlikes attract.*

The fact that there are two kinds of electric charge can be conveyed in another way. Consider the glass rod which has been electrically charged by rubbing with a silk cloth. If the rod and cloth are again brought into contact with one another, the charge disappears, the rod is now no longer able to attract or repel another charged object! It is as though the cloth and rod have neutralised one another, something like positive and negative numbers being able to cancel one another when we add them.

The two kinds of electric charge are called *positive* and *negative*. This is a matter of convenience, or a convention. According to this convention, the charge on a glass rod rubbed with silk is said to be positive. The charge on a rubber rod rubbed with wool is then said to be negative In Fig 1.1 these conventions have already been used to indicate the kind of charge on each rod. We can now say: two positively charged bodies, or two negatively charged bodies, repel one another. But a positively charged body, and another negatively charged body, attract one another.

★ The beginnings of an understanding of electric charges and their properties, and also of magnetism, go back to the 6th century B C , that is, 2500 years ago Thales of Milotus, one of the founders of Greek science, knew that if a piece of amber is rubbed with a woollen cloth, it would then attract light feathers, dust, lint, pieces of leaves etc. Amber is a yellow resinous (that is, gumlike) substance found on the shores of the Baltic sea The Greek name for amber was "electrum", and this is the origin of all the familiar words. electricity, electric force, electric charge, and finally, the electron

Similarly the early knowledge about magnetism goes back to the Greeks and, independently, the Chinese The Greeks knew that certain rocks from the island of Magnesia in Asia minor (west of present day Turkey) had the strange capacity to attract pieces of iron These are rocks of lodestone or magnetite, an iron ore. The Chinese were aware that thin long pieces of certain natural iron ores, if suspended from a string and free to swing horizontally, would naturally orient themselves roughly in the north-south direction. By about 1000 A D, that is, 1000 years ago, the Chinese were using magnetic compasses for navigation.

The first person to write a scientific account of the early experiences of electricity and magnetism was William Gilbert He lived in the 16th century, from 1540 to 1603, and was personal doctor to Queen Elizabeth I of England He had done experiments like the ones we described earlier, rubbing glass rods with silk, rubber shoes against a woollen carpet, etc, which produced electrically charged bodies In his book "De Magnete" ("On the Magnet", published in 1600), he gave the name "electrica" for substances like amber which became electrically charged by rubbing The observation that there are two kinds of electric charge, and that likes repel while unlikes attract, was known to Gilbert

In these early days the two kinds of charge were called "resinous" and "vitreous" The charge produced on a rubber rod or on a piece of amber when rubbed with wool was called resinous (Remember, amber is like resin!) On the other hand, the charge produced on a glass rod when rubbed with silk was called vitreous.

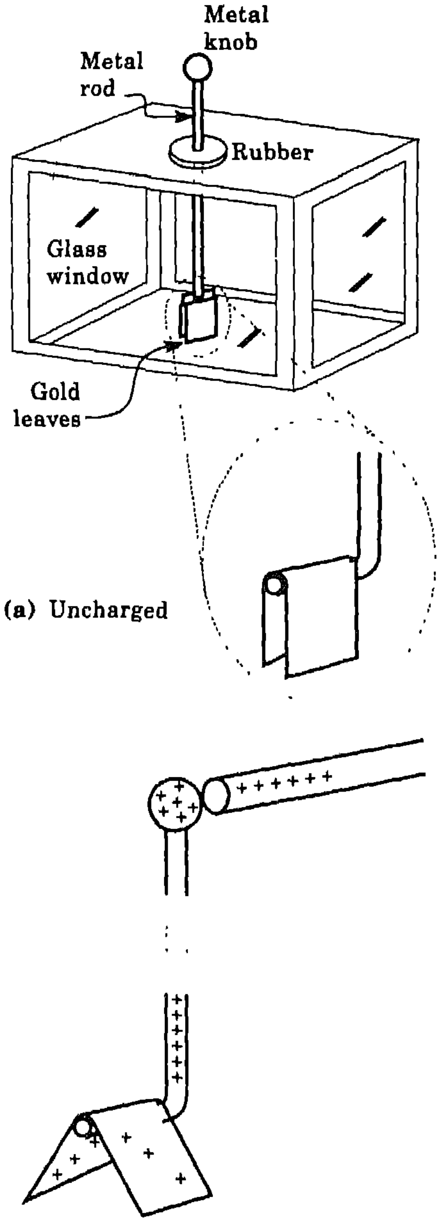
It was the American scientist-statesman Benjamin Franklin (1706-1790) who later introduced the words *positive* in place of vitreous, and *negative* in place of resinous This convention has been followed ever since But it is in fact an unfortunate choice in one respect The electron, so important in all of chemistry and the study of the properties of matter, turns out by this convention to have a negative charge! ★

The Electroscope: A simple apparatus with which we can detect the presence of electric charge on a charged body is the *leaf electroscope*. It can also give us a rough idea of the amount of charge on the body This instrument is now a museum piece since many, much more sensitive, electronic instruments have been developed. But just because it is so simple, it is worthwhile describing it briefly at this point Actually there are several kinds of electroscopes, we describe one typical kind to understand the main features.

Here is how an electroscope is constructed. It contains a vertical metal rod, with a round metal ball or knob on top, housed in a box as shown in Fig. 1.2a. The part of the top lid which holds the rod in place, and which is in contact with the rod, is made of material like hard rubber or amber Two very thin gold leaves (or two thin strips of aluminium) are attached to the bottom end of the rod, inside the box, as shown. (Alternatively, one may have a right angle bend at the bottom end of the rod. Then a longer strip of thin gold leaf is draped over, and fastened to, the horizontal portion of the rod (see inset in Fig. 1.2). The sides of the box contain glass portions like windows Through them the positions of the leaves can be seen. The purpose of the box is to protect the rod and the thin leaves from air currents, and from dust.

When the electroscope is uncharged, the two leaves hang parallel and vertically downwards, because of their own weight, as in Fig 1.2a Suppose now we bring a charged object, say a glass (or rubber) rod which has been rubbed with silk (or wool), and touch the metal knob at the top of the electroscope with it. Then the charge flows from the charged object through the electroscope metal rod onto the gold leaves. The two leaves now repel one another – likes repel (This will happen whether we had brought in a charged glass rod or a charged rubber rod, that is, a positively or a negatively charged object) So they no longer hang vertically down but diverge from one another as in Fig. 1.2b. This can be seen through the glass windows. The amount by which the leaves have diverged gives us an idea how much electric charge has been put on the leaves

Another way to charge an electroscope is to use a battery, rather than touch the knob with a charged object. Thus if we connect one terminal of a battery to the knob and the other to the electroscope body (or "casing" as it is called), again the leaves become charged and hence will diverge.



(a) Uncharged

(b) Charged

Figure 1.2: The leaf electroscope

In Section 1.5 we will study the quantitative aspects of electrical forces between charged bodies. Then it will be easy to understand why the degree of divergence of the leaves is an indication of the amount of charge on them.

1.3 Properties of electric charge

As in the case of other physical quantities such as mass, length and time, the measurement of the quantity of charge on a charged body needs a unit. The SI unit of electric charge is the *coulomb*, written as C. We will introduce it in Section 1.5. But even at this point, we can understand some of the basic properties of electric charge quite easily.

Quantization. Many experiments have shown that in nature there is a certain *smallest amount* of electric charge, and all other electric charges are *integral multiples* of this basic amount. This property is called the *quantization of electric charge.*

We have learnt that electric charge can be positive or negative. The smallest amounts of positive and negative charge are the charges of a proton and an electron respectively. These have exactly the same magnitude but opposite signs. Two protons (or two electrons) repel one another, while a proton and an electron attract one another.

We use the symbol e for the magnitude of the smallest amount of charge. So the charge of a proton is e, and that of an electron is $-e$. (Remember that it is because of Benjamin Franklin's convention that we say the electron has a negative charge). The numerical value of e will be given after the coulomb has been defined.

Quantization of electric charge means, then, that any charged body, large or small, has a total charge which is an integral multiple of e, namely, $\pm e$, $\pm 2e$, $\pm 3e$, ... No fractions of $\pm e$ have been seen.

When we deal with most practical situations involving electrically charged bodies and large scale electrical phenomena, however, the magnitude of the proton and electron charges is *so small* that we can essentially ignore the quantization of charge. As an example, when we switch on a 60-watt electric bulb, the number of electrons passing through the filament in each second is a really enormous number, approximately 2×10^{18}. (You will be able to calculate this later on, but you can already appreciate how large this number is). So on this scale the charge on a single electron is very small indeed. This is why we can suppose that the charge on a charged body of reasonable size, something we can directly see and handle, can have any continuous value It is similar to supposing that water is a continuous fluid - for most practical purposes we can ignore the fact that actually it consists of individual molecules.

Classical concepts of electricity and magnetism cannot explain why electric charge is quantized. (So far modern physics has also not been able to explain this satisfactorily!). In many situations, as we said, we may and do ignore this property of electric charge. However when we study the physical and chemical properties of individual atoms and molecules, and properties of matter where its atomic structure is important, then we cannot ignore the quantization of charge.

Additivity: There is a property of electric charge which we used implicitly in the previous discussion Let us suppose we have a charged body, with various amounts of positive and negative charge distributed over different parts of it. Then the total charge of the body is the *sum* of all these individual contributions, each being taken with its proper sign. So this is similar to the process of adding real numbers, some positive and some negative. We say electric charge is *additive*.

For example we may have a body which has positive and negative charges distributed in such a way that they add up to zero. Then the total charge is zero and the body is overall *neutral*. On the other hand suppose for example we have a glass rod which has been charged by rubbing with a silk cloth Then positive charges would be distributed all over the rubbed portion, and the total charge on the rod would be nonzero and positive.

This additive property of electric charge is similar to such a property for mass. the total mass of an extended body or a system of bodies is the sum of the masses of its parts. The difference however is that all masses are positive, while charges can be positive or negative.

Conservation Another important property of electric charge is that it is *conserved*. From the study of elastic collisions, for example, we are familiar with the conservation of energy and momentum. The total energy is the same before and after such a collision, and similarly the total momentum. They do not change in the course of time. Electric charge obeys a similar law. If some amount of charged matter is contained in a certain volume of space, and no charged matter is allowed to enter or leave this volume, then the total charge within the volume will never change. We can say: ***the total charge of an isolated system does not change.***

★ In the early days of the study of electricity, for example at the time of Benjamin Franklin, it was thought that electric charge was a continuous fluid Somewhat later, by about 1870 and the time of James Clerk Maxwell, (1831-1879) the laws of electrolysis had shown that there probably was a smallest amount of electric charge in nature We will come to this in Chapter 4 in connection with the thermal and chemical effects of electric currents

The charge of the electron was first measured in a series of experiments performed at the University of Chicago during 1909-1913 by the American experimental physicist Robert Andrews Millikan (1868-1953). A brief description of this famous experiment will be given in Chapter 12 on Electrons and Photons

The fact that the charges of the proton and the electron have the same magnitude, though opposite in sign, has been experimentally verified to a very great accuracy Thus it is now known that the equality of the magnitudes of these two charges holds to an accuracy of one part in 10^{20}

★

We can illustrate this with the discussion of frictional electricity in the previous Section When a glass rod gets positively charged by rubbing with a silk cloth, in the process the silk gets negatively charged (and to the same extent!) This can be checked quite simply, if the cloth is taken near the charged glass rod, hanging from a thread, the rod is attracted by, and swings towards, the cloth Therefore the rubbing of the rod by the cloth results in both becoming charged, one positively and the other negatively. The total charge remains zero, as it was to begin with. This suggests that what happens in frictional electricity is a *transfer* of electric charge from one body to the other, and not *creation* of charge We will soon see that it is electrons that are transferred.

Experiments have shown that the law of conservation of electric charge is obeyed in all processes, both large scale and microscopic ones. An interesting example is in the process of electrolysis, which we will study in Chapter 4

★ There are numerous examples of processes involving microscopic elementary particles, in which one sees the conservation of electric charge Here is a very interesting and striking one Related to the electron is another particle called the positron It has the same mass as the electron, but exactly the opposite charge When an electron and a positron collide, they sometimes annihilate each other, that is, they both disappear What remains is electromagnetic radiation, which has no charge One sees that the total electric charge was zero in the beginning, and remains zero in the end The reverse process also occurs· under suitable conditions, electromagnetic radiation (in the form of a gamma ray) can create an electron-positron pair, and itself disappear! Again, total charge is conserved

A few more comments about the relationships among the properties of electric charge may be helpful. Conservation of electric charge makes sense whether or not we take account of quantization, that is, even in a classical description when charge is imagined to be a continuous quantity. The conservation of electric charge is so basic that without it we cannot have any description of electricity and magnetism at all!

We compared above some properties of electric charge with those of mass. As pointed out, a major difference is that while charge can be of either sign, mass is always positive Another difference is this, according to special relativity, the mass of a body increases as its speed increases However, the charge on a body remains constant and does not change as the speed of the body changes This is experimentally very well verified, for example in the operation of giant accelerating machines which speed up charged particles to great velocities Lastly, while charge is fundamentally quantized, mass does not appear to follow a similar rule of quantization ★

1.4 Atomic structure – conductors and insulators

Atomic Structure: From chemistry we know that there are about one hundred different elements in nature, ninety-two of which are naturally occurring and the rest are artificially produced. Atoms of various elements in combinations make up molecules. For example, a molecule of water consists of two hydrogen atoms and one oxygen atom. The lightest atom is hydrogen. Then come helium, lithium, beryllium, and so on up the periodic table Along the way we have elements like iron, cobalt, nickel and copper. Towards the end of the periodic table are heavy elements like lead and uranium.

To appreciate better the electrical properties of matter, let us very briefly recall here some aspects of atomic structure At the centre of every atom of any element is a tiny nucleus, with a diameter of about 10^{-15}m. It contains protons and neutrons. (The nucleus of hydrogen is a single proton). Comparatively far away from the nucleus, at a distance of about 10^{-10}m, are the electrons The nucleus together with these electrons is the atom. Since protons and neutrons have masses about 1840 times the electron mass, the nucleus accounts for practically the entire mass of the atom.

The electrons may be crudely pictured as travelling along various orbits around the nucleus, so that the atom as a whole appears like a tiny solar system. More correctly, one must picture the electrons as forming a smeared out cloud of electric charge about 10^{-10}m away from the nucleus. We will study these matters in more detail in Chapter 13.

The neutron has zero electric charge In an atom in its normal condition, there are equal numbers of protons in the nucleus and electrons outside. Therefore the total electric charge of the atom is zero If in some way one or more electrons are *removed* from a neutral atom, what remains is a *positively charged ion*. On the other hand, *addition* of electrons to a neutral atom produces a *negatively charged ion*. Both processes are called *ionization*

Matter in bulk is ordinarily electrically neutral - in any piece of material the positive proton charges and the negative electron charges cancel each other perfectly. Therefore, because there are both attractive and repulsive electric forces, electrical effects can cancel out to a very great degree. (In contrast, as all masses are positive and gravitational forces always attractive, gravitational effects do not cancel out in this way). Normally it takes special effort to produce a large scale separation of positive and negative charges in matter.

Conductors and Insulators We are familiar with the distinction between *conductors* and *insulators* In a conductor we can have a flow of electric charge, that is, an electric current This is not possible in an insulator. We can understand these contrasting properties on the basis of the atomic nature of matter.

In all atoms, the innermost electrons are more tightly held by the nucleus than the outer ones. Depending on the particular atom (or in some cases molecule), the latter can be removed with greater or less effort. It is these outer electrons, by rearrangement and transfer from one atom or molecule to another, that are involved in all chemical reactions.

Materials are classified into conductors and insulators according to their ability to hold on to, or their affinity for, the outer electrons. The basic distinction is this: conductors allow large scale movement of electric charges through them, insulators do not Examples of conductors are metals like copper, gold, aluminium etc; the earth, our bodies, damp wood; and electrolytes, which are solutions of salts or acids or alkalis in water.

In a piece of metal, the outermost electrons do not stay attached to their atoms. Each atom donates one or more electrons to the material as a whole These electrons are then able to move throughout the material, with greater or lesser ease depending on the particular metal. In 1 cc of copper, for instance, there are approximately 10^{23} such mobile electrons! In most conductors the mobile charges are electrons, but in electrolytes both ions and electrons are able to move.

Insulators are also called *dielectrics*. In them, each electron remains attached to its own atom, or molecule, and is not able to move away freely. Examples are nonmetallic solids, oils, glass, rubber, plastic, dry wood, paper, silk and wool. Electric charges placed upon insulators are called *static* charges since they cannot move or drain away.

The classification of materials into conductors and insulators is not a very sharp one. Each conductor shows some resistance to the flow of charge, and each insulator permits some flow of charge But it is a useful and broad classification. Fused quartz, for example, has 10^{25} times as much resistance to the flow of charge as copper, so it is a very good insulator indeed!

Semiconductors are materials whose properties lie in between those of conductors and insulators. An example is silicon. Moreover the electrical properties of semiconductors depend on the temperature. We study these in some detail in Chapters 3 and 14.

We can now understand better what happens in frictional electricity. Whether we rub a glass rod with silk, or a rubber rod with wool, *both materials must be insulators*. The rubbing causes a transfer of electrons from one to the other. Compared to glass, silk has a greater affinity for electrons. So in this case electrons are transferred from glass to silk. As a result the former gets positively charged, the latter negatively. We also see that to give a body a positive charge means to remove electrons from it, and to give it a negative charge is to add electrons to it. In the case of rubber and wool, the former has a greater affinity for electrons than the latter.

We can also see now the need for certain precautions in creating static electric charge by friction. Since our hands and bodies are conductors, we must not touch a charged body, or the charged portion of an insulating body, with our hands. If we did, we would drain away the accumulated charge, whether it is positive or negative. For example, if an electroscope has been charged and the leaves have diverged, and we then touch the metal knob, the leaves will collapse! If we want to charge a copper or other metal rod, we must hold it by an insulating wooden or rubber handle, and not touch the metal directly.

Returning to the construction of the electroscope outlined in Section 1.2, it becomes clear why the rod connecting the knob on top to the leaves at the bottom has to be metallic. It must conduct electric charge from the top to the bottom. To avoid any leakage of charge away from the metal parts, the rod must be held in place in the electroscope by a support of nonconducting material like rubber or amber. Both these are indicated in Fig.1.2.

Charging by induction. In Section 1.2 we saw how an electroscope can be charged *by contact with* a charged object. For instance, to give it a negative charge we touch the knob with a negatively charged rubber rod. Then electrons flow from the rod into the electroscope, so it also becomes negatively charged. If we touch the knob with a positively charged glass rod, electrons would flow from the electroscope into the rod, leaving the electroscope positively charged.

There is another indirect method of charging an electroscope, called *charging by induction.* Here direct contact with the charged object is not needed. And, as we will now see, the charge on the electroscope turns out to be *opposite in sign* to that on the charged object!

As illustrated schematically in Fig 1.3, the steps are as follows:

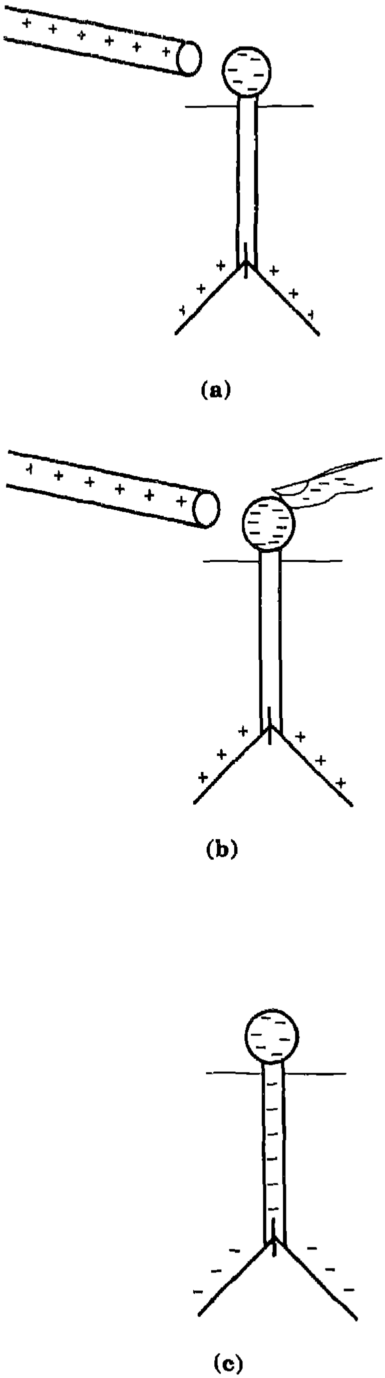
(a) We bring, say, a positively charged glass rod *near* the knob of an uncharged electroscope, but not in contact with it (Fig.1.3a). Electrons are attracted from the leaves of the electroscope to the knob. This makes the leaves positively charged, and they diverge, though as a whole the electroscope is neutral. If we were to remove the glass rod now, the leaves would collapse as the electroscope would return to its original condition. However we now do something else.

(b) Keeping the glass rod nearby, we touch the knob (Fig. 1.3b). Now, as we have seen, our bodies are conductors. So the positively charged glass rod will attract electrons in our fingers towards itself. Since we are touching the knob, these electrons will flow onto the knob and join the electrons that had already collected there in the previous step (a). At this stage the electroscope has a net negative charge on it.

(c) Now *first* remove the finger, *then* take away the glass rod. The net negative charge remains on the electroscope, and distributes itself all over, that is, on the knob and metal rod and leaves (Fig. 1.3c). The leaves will still diverge, but now carrying negative charge, unlike as in Fig. 1.3a.

We have ended up with a net negative charge on the electroscope, *opposite* in sign to the charge on the glass rod. This negative charge was of course supplied during step (b), as described above, when we were touching the knob.

You may wonder why, in step (b) above, we said there would be a flow of electrons from the finger to the electroscope knob, and not the other way around. After all, if more electrons have to reach the knob, would they not be repelled by the electrons already there? We have assumed however that there is enough



(a)

(b)

(c)

Figure 1.3: Charging by induction

positive charge on the glass rod nearby to overcome this repulsion, and to cause additional electrons to move from finger to knob.

This indirect method of charging by induction is often more convenient than charging by direct contact with a charged body. You can try to reconstruct the steps by which, starting with a negatively charged rubber rod, at the end the electroscope would have a net positive charge.

1.5 Coulomb's law and the coulomb

Now we come to the detailed law of force between electric charges. This is called *Coulomb's Law*. It is the electrical analogue of Newton's Law of Universal Gravitation.

Let two bodies have charges q_1 and q_2 (in units which will be described soon). Let them be at rest, in vacuum, and separated by a distance of r metres. We assume that the sizes of both bodies (their linear dimensions) are small compared to r. Then we may call them *point charges* How do the electric forces exerted by them on one another depend on their charges and their separation? These dependences can be described thus:

Coulomb's Law: The electric forces exerted on each other by two point charges q_1 and q_2, separated by a distance r, are proportional to the product $q_1 q_2$, and inversely proportional to the square of the distance of separation. They act along the line joining the two charges. These forces are attractive if $q_1 q_2$ is negative (unlike charges), repulsive if $q_1 q_2$ is positive (like charges). We express these dependences by writing

$$F \propto \frac{q_1 q_2}{r^2}. \tag{1.1}$$

We go into more detail below.

★ The earliest experiments to find the law of electric force were performed by Henry Cavendish (1731-1810) in England in 1772, but his results remained unknown till about sixty five years after his death The experiments whose results were first publicly announced were performed in France by Charles Augustin de Coulomb (1736-1806) around

1784-85 Even before these experiments, both Benjamin Franklin and Joseph Priestley had suspected that, like the force of gravity, electric forces too obeyed an inverse square law

Coulomb's experiments used a torsion balance and small charged metal spheres The idea of the torsion balance was used later also by Cavendish in 1797 in his experiments to measure Newton's gravitational constant G.

In Coulomb's time, there was yet no unit for measuring electric charges However he was able to obtain the q_1q_2 dependence in the following clever way By touching a charged metallic sphere to another identical but uncharged sphere, he could divide the charge on the former into two equal parts Then by a repetition he could divide it into four equal parts, and so on With such changes in the quantity of charge on each body he could measure the resulting changes in the electric forces, and so arrive at the q_1q_2 dependence

We said above that the charged bodies are assumed to be at rest If they are moving, then other (magnetic) forces are also present These forces are proportional to the velocities of the charges We study such forces in Chapter 5 and later Chapters For small velocities, compared to the speed of light which is approximately $3\times10^8\mathrm{ms}^{-1}$, these magnetic forces can be neglected, and the electric Coulomb force is the main one. ★

Let us now express Coulomb's law in a more detailed mathematical form Then the magnitudes and directions of the forces become clear Let the two point charges q_1 and q_2, numbered 1 and 2 for convenience, be at points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively (with respect to some origin O). The vector leading *from* 1 *to* 2 is the difference between $\mathbf{r}_2$ and $\mathbf{r}_1$, and will be written as $\mathbf{r}_{21}$. The distance of separation r is the magnitude of $\mathbf{r}_{21}$. In detail:

Position vector of charge q_1 is $\mathbf{r}_1$,
Position vector of charge q_2 is $\mathbf{r}_2$;
Vector *from* 1 *to* 2 $= \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{r}_{21}$,
Vector *from* 2 *to* 1 $= \mathbf{r}_1 - \mathbf{r}_2 = \mathbf{r}_{12} = -\mathbf{r}_{21}$;
Distance between 1 and 2 $= r = |\mathbf{r}_{12}| = |\mathbf{r}_{21}|$. (1 2)

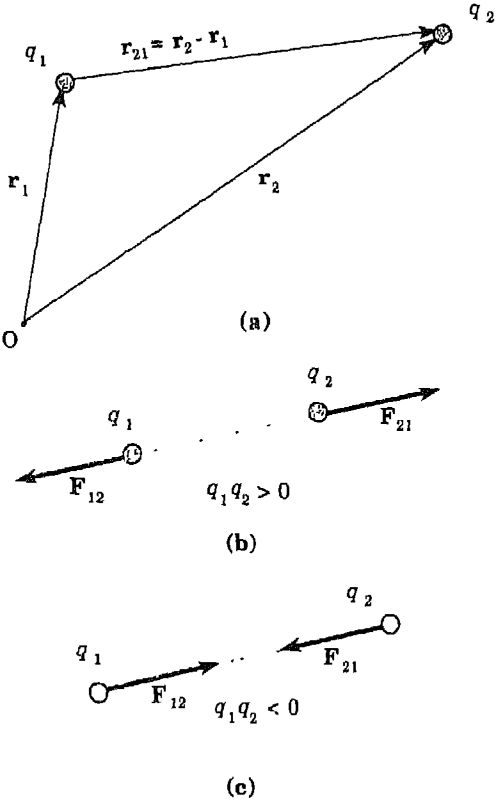


Figure 1.4: Illustrating Coulomb's Law

These are shown in Fig. 1.4a drawn in a two-dimensional plane. Then the expression of Coulomb's Law is this:

$$\begin{aligned}\mathbf{F}_{21} &= \text{force } on \text{ 2 } due\ to \text{ 1}\\ &= k\,\frac{q_1q_2}{r^2}\,\frac{\mathbf{r}_{21}}{r},\\ \mathbf{F}_{12} &= \text{force } on \text{ 1 } due\ to \text{ 2}\\ &= -\mathbf{F}_{21} = k\,\frac{q_1\,q_2}{r^2}\,\frac{\mathbf{r}_{12}}{r}.\end{aligned} \tag{1.3}$$

Here k is a positive constant. We see that Coulomb's Law is in agreement with Newton's Third Law of Motion: **The electric forces exerted by two electric charges on each other are equal in magnitude and opposite in direction.**

(If, for simplicity, we consider the case where the charge q_1 is located at the origin O, then the vector $\mathbf{r}_1$ is zero. If we then write $\mathbf{r}$ for the position vector of the other charge q_2, the expressions for the forces become slightly simpler

$$\begin{aligned}\mathbf{F}_{21} &= \text{force } \textit{on } 2 \textit{ due to } 1 \\ &= k\, q_1\, q_2\, \frac{\mathbf{r}}{r^3}, \\ \mathbf{F}_{12} &= \text{force } \textit{on } 1 \textit{ due to } 2 \\ &= -k\, q_1\, q_2\, \frac{\mathbf{r}}{r^3}\ )\end{aligned}$$

The vectors $\mathbf{r}_{21}/r$ and $\mathbf{r}_{12}/r$ used in Eq. (1.3) are of unit length, that is, they are *unit vectors*. We write them for convenience as $\hat{\mathbf{r}}_{21}$ and $\hat{\mathbf{r}}_{12}$:

$$\begin{aligned}\hat{\mathbf{r}}_{21} &= \frac{\mathbf{r}_{21}}{r} \\ &= \left\{\begin{array}{l}\text{unit vector pointing in} \\ \text{the direction } \textit{from } 1 \textit{ to } 2,\end{array}\right. \\ \hat{\mathbf{r}}_{12} &= \frac{\mathbf{r}_{12}}{r} = -\hat{\mathbf{r}}_{21} \\ &= \left\{\begin{array}{l}\text{unit vector pointing in} \\ \text{the direction } \textit{from } 2 \textit{ to } 1.\end{array}\right.\end{aligned} \tag{1.4}$$

These unit vectors, along with the sign of $q_1 q_2$, fix the directions of $\mathbf{F}_{21}$ and $\mathbf{F}_{12}$ respectively. The forces can be neatly written as

$$\begin{aligned}\mathbf{F}_{21} &= k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21}, \\ \mathbf{F}_{12} &= k\,\frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}}_{12}\end{aligned} \tag{1.5}$$

and the factors giving the magnitudes and the directions are immediately clear. As shown in Figs 1.4 b,c and since k is positive, if $q_1 q_2 > 0$ (like charges) the force on 2 is *away from* 1, that on 1 is *away from* 2. They *repel* one another. On the other hand, if $q_1 q_2 < 0$ (unlike charges) the force on 2 is *towards* 1, that on 1 is *towards* 2 They *attract* one another.

Example 1.1: A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B carried by an insulating handle is brought close to A to a distance of 9.0 cm between their centres The resulting repulsion of A is noted (for example by shining a beam of light and measuring the deflection of its shadow on a calibrated screen) Spheres A and B are touched by uncharged spheres C and D respectively. C and D are then removed, and B is brought closer to A to a distance of 4.5 cm between their centres. What is the expected repulsion of A on the basis of Coulomb's law? Spheres A and C and spheres B and D have identical sizes Ignore the sizes of A and B in comparison to the separation between their centres.

Answer: Let the original charge on sphere A be q and that on B be q'. At a distance r between their centres, the magnitude of the electrostatic force on each is given by

$$F = k\frac{qq'}{r^2}$$

neglecting the sizes of spheres A and B in comparison to r. When an identical but uncharged sphere C touches A, the charges redistribute on A and C and, by symmetry, each sphere carries a charge $(q/2)$ Similarly, after B touches D, the redistributed charge on each is $(q'/2)$ If now the separation between A and B is halved, the magnitude of the electrostatic force on each is

$$F' = k\frac{(q/2)(q'/2)}{(r/2)^2} = k\frac{qq'}{r^2} = F.$$

Thus the electrostatic force of repulsion on A is unaltered.

The coulomb: In what units are the charges q_1 and q_2 to be measured? What is the value of the constant k? Since we are using SI units for mechanical quantities, the natural procedure would seem to be the following: The unit of charge should be defined so that two unit positive point charges exactly one metre apart repel each other with a force of one newton. If this were the definition of unit charge, we could conveniently set $k = 1$.

This is, however, *not* the way in which the SI unit of charge is defined! The SI unit of electric charge is the *coulomb*, denoted by C. It is chosen to get a simple form for the magnetic effects of moving charges, that is, for the magnetic forces among charges in motion. (We will see this in Section 5.3 in Chapter 5) This is a convention we have to get used to.

We can now say: if in Eq. (1.3) the charges are measured in coulombs and the distance in metres, then the forces $\mathbf{F}_{21}$ and $\mathbf{F}_{12}$ will come out in newtons if for k we put in the experimentally determined value

$$k = 8.98755 \times 10^9 \ \ \mathrm{Nm}^2/\mathrm{C}^2. \qquad (1.6)$$

For most practical calculations the approximate value $k \simeq 9 \times 10^9$ Nm^2/C^2 is good enough. So for two point charges q_1 and q_2 a distance r apart we have:

$$\text{Magnitude of force} \simeq 9 \times 10^9 \times \frac{q_1(\text{coulombs}) \times q_2(\text{coulombs})}{[r(\text{metres})]^2} \text{ newtons} \qquad (1.7)$$

Sometimes the constant k is written as $1/4\pi\epsilon_0$, and ϵ_0 is called "the permittivity of the vacuum". (This phrase will be explained in the next Chapter) For practical purposes we can take, to reasonable accuracy,

$$k = 1/4\pi\epsilon_0 : \ \epsilon_0 \simeq 9 \times 10^{-12} \ \ \mathrm{C}^2/\mathrm{Nm}^2 (1.8)$$

★ We may mention that, according to the most recent conventions for time and length measurements, in SI units the combination $4\pi\ \epsilon_0$ has the *exact* value $(10^7/c^2)$ C^2/Nm^2, where c the speed of light in vacuum has the *exact* value 2.99792458 x 10^8 m/s ★

The charge of an electron can now be expressed in terms of the coulomb. We can state it in two equivalent ways

Charge of an electron = $e = 1.6 \times 10^{-19}$ C,

Charge of 6.2×10^{18} electrons = -1C.

(1.9)

If we write "proton" in place of "electron" here, all the negative signs go away!

How large (or small) by "everyday" standards is one coulomb of charge? Even on the macroscopic scale, the coulomb is an *immense* amount of charge. If we had two small charged bodies one metre apart, each ca
ing one coulomb of charge, we see from
(1.7) that they would repel one another w
a force of about 9×10^9 N. This is an *en*
mous force in terms of daily experience.
roughly equal to the weight of 10^9 kg of m
ter on the earth, which is about the weigh
thirteen million people!

Why do we then still say that the coul
is a "reasonable" amount of charge, and
it as the SI unit? It is because the magn
effects of moving charges happen to be *m*
weaker than electric effects for moderat
locities of the charges, say a few m/s.
will see this when we come to Chapter 5
order that these magnetic effects may ap
to have a reasonable magnitude in our e
day system of mechanical units, we hav
use such a large amount of charge as th
unit of charge.

Comparing electrical and gravitat
forces: We have mentioned already
while gravitational forces are always a
tive, electrical forces can be attractive
pulsive since electric charges can be of
sign. Both forces have an inverse squar
tance dependence - this is common to
ton's and Coulomb's Laws.

How strong are electric forces in co
to gravitational ones? To compare th
can find the ratio of these forces betwee
an electron and a proton. In this case
forces are attractive, and due to the co
distance dependence the ratio is indep
of distance. We find, using $m_e \simeq 9 \times 1$
$m_p \simeq 1.6 \times 10^{-27}$kg, G = Newton's g
tional constant $\simeq 6.7 \times 10^{-11}$ Nm^2/kg
the values of k and e given in Eqs (1
that:

Ratio of electric force to gravitation
between proton and electron

$$= ke^2/Gm_em_p \simeq 2.4 \times 10^{39}.$$

This is dimensionless and gives us so
how much stronger electric forces ar
hearted practical demonstration is this
plastic comb run through your hair o

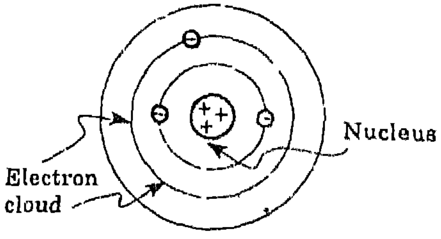
day can lift a small piece of paper upwards, and thus overcome the force of gravity exerted by the entire earth on the paper!

Notice that this fact - the far greater strength of electric forces compared to gravity - has nothing to do with the enormity of one coulomb of charge we described earlier. Here the ratio defined in Eq (1.10) is independent of our choice of unit of charge (or mass, for that matter). It just compares two distinct forces present in nature between electrons and protons which occur in nature.

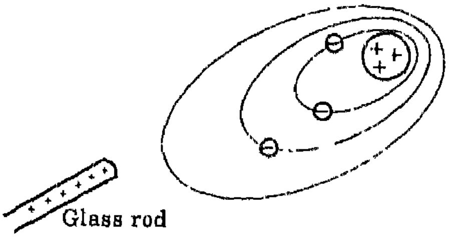
This enormity of electric forces as compared to gravity can be brought out in yet another illuminating way, due to Feynman. We have mentioned that bulk matter is practically always electrically neutral. If now you stand at arm's length from your friend, and instead of being *precisely* electrically neutral each of you had an excess of electrons over protons of just *one per cent*, the force of repulsion between you would, in Feynman's words, "be enough to lift a "weight" equal to that of the entire earth!".

★ *Verification and range of validity of Coulomb's Law – Polarization* Direct verification of Coulomb's Law to great accuracy is not easy since it is awkward to deal with point charges. It is more efficient to derive various consequences of Coulomb's Law, and then test them to great accuracy. Thus our belief in the exact validity of this law is in this sense somewhat indirect. In this way it has been experimentally checked that in the expression (1.7) for the magnitude of the electric force, the power of the distance in the denominator is within the limits $2 \times (1 \pm 10^{-9})$. So we do have an inverse *square* law to extremely high accuracy.

From other evidence we also know that the law works all the way from separations of the order of 10^{-15} m (within the nuclei of atoms) to macroscopic distances of several kilometres. Thus the dependence on distance has been checked over some 18 orders of magnitude!



(a) Normal atom



(b) Polarized atom

Figure 1.5: Illustrating polarization of atoms in an insulator. The figure is not to scale

This inverse square distance dependence means that electric forces are stronger at closer distances between charges. This qualitative feature helps us understand a feature of frictional electricity not mentioned earlier. Suppose you bring a positively charged glass rod near a small piece of paper. The latter is attracted and flies towards the rod. But once it *touches* the rod, it gets *repelled* and flies away! Why does this happen?

The reason is as follows. Paper being an insulator, there are no free mobile electrons in it. Normally within each atom there is the cloud of negative electrons on the outside, and the positive nucleus at the centre. This is shown in Fig. 1.5a where for simplicity we have pictured the electrons as travelling in orbits even though, as we said earlier, they are actually smeared out into a cloud. But when the positively charged glass rod is brought nearby, in each atom the electrons are attracted towards the rod while the nucleus is pushed away – see Fig. 1.5b. This distortion of each atom in the presence of the glass rod is called *polarization* – we study it in more detail in the next Chapter. While each atom as a whole remains neutral, the attractive force on the electrons is stronger than the repulsive force on the nucleus, since the former are on the average closer to the glass. Each atom in the paper thus feels a net tiny attraction towards the rod. The cumulative effect on all the atoms is a sizeable force, making the paper fly towards the rod.

However, once the paper touches the rod, some of the charge on the latter passes into the former as well. In this case actually some electrons are transferred from paper to glass. Then the paper gets a net positive charge, just like the glass. While the attraction due to polarization continues, normal coulomb repulsion of like charges is now also present. The latter overcomes the former, so the paper flies away. If instead of the glass rod a negatively charged rubber rod had been used, you can convince yourself that once again polarization causes initial attraction of paper to rod, then there is repulsion.

Polarization of a dielectric is involved in another context too. In stating Coulomb's Law it was assumed that the point charges are in vacuum. What happens if instead they are in air or some other dielectric substance, say oil? Then the two given point charges cause a polarization of the atoms and molecules in the dielectric around them. The net effect is that the force between the point charges is *reduced* by a certain factor compared to what it would be in vacuum. This factor depends on the dielectric, and for air it is very nearly unity to within one part in two thousand. Practically, then, Coulomb's Law may be used as it stands for point charges in air rather than in vacuum. We study these aspects somewhat further in the next Chapter. ★

1.6 Superposition principle - electric field

The superposition principle: Coulomb's Law gives the magnitude and directions of the forces that two point charges (situated in vacuum) exert on one another. What if we have several charges, say $q_1, q_2, q_3 \ldots$ at points P_1, P_2, P_3 . in space? What is the total force on the charge q_1, say, due to all the other charges $q_2, q_3, \ldots$? The answer is very simple. We first use Coulomb's Law for the force $\mathbf{F}_{12}$ on q_1 due to q_2 alone, ignoring all the other charges; then for the force $\mathbf{F}_{13}$ on q_1 due to q_3 alone; and so on. Finally we add these individual force vectors to obtain the total force on q_1 as shown in Fig. 1.6

$$\begin{aligned}\mathbf{F}_1 &= \left\{\begin{array}{l}\text{total electric force on } q_1\\ \text{due to } q_2, q_3, \ldots\end{array}\right.\\ &= \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots,\\ \mathbf{F}_{12} &= \text{force on } q_1 \text{ due to } q_2 \text{ alone},\\ \mathbf{F}_{13} &= \text{force on } q_1 \text{ due to } q_3 \text{ alone}, \ldots\end{aligned} \tag{1.11}$$

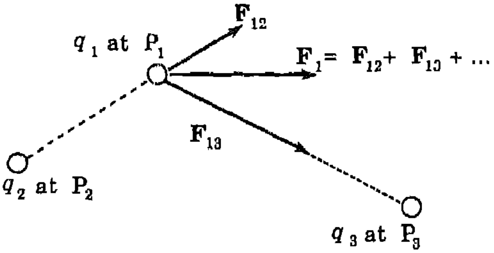


Figure 1.6: Force on one charge due to many others

This rule is called the *superposition principle.* It is just like the parallelogram law of forces in mechanics: to find the resultant or total force on a body on which many forces act, we have to add the individual forces vectorially. In the present case, all the forces considered are electric in origin. And the important point is that *the forces with which two charges attract or repel one another are not affected by the presence of other charges.*

To find the total force on charge q_2 at position P_2 (or q_3 at P_3, etc.), we follow an exactly similar procedure

$$\begin{aligned}\mathbf{F}_2 &= \text{total force on } q_2 \text{ due to } q_1, q_3, \ldots\\ &= \mathbf{F}_{21} + \mathbf{F}_{23} + \ldots,\\ \mathbf{F}_{21} &= \text{force on } q_2 \text{ due to } q_1,\\ \mathbf{F}_{23} &= \text{force on } q_2 \text{ due to } q_3, \ldots\end{aligned} \tag{1.12}$$

In both Eqs (1.11,1.12), for each individual contribution we must use Coulomb's Law.

Let us illustrate this with a very simple example. Suppose charges $q_1 = q_2 = q$, $q_3 = -q$ are at the corners of an equilateral triangle of side l, as in Fig. 1.7. We have chosen a sim-

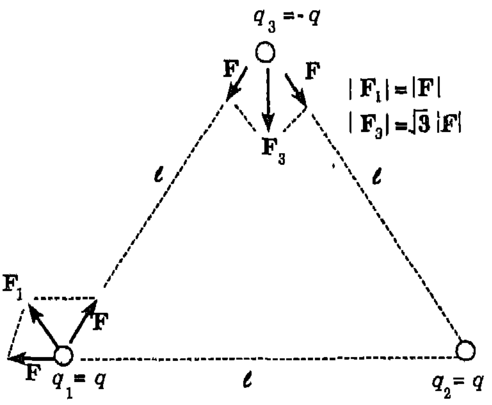

$q_3 = -q$
F
F
$|\mathbf{F}_1| = |\mathbf{F}|$
$|\mathbf{F}_3| = \sqrt{3}\,|\mathbf{F}|$
$\mathbf{F}_3$
l
l
$\mathbf{F}_1$
F
F
$q_1 = q$
l
$q_2 = q$

Figure 1.7: Illustrating the Superposition Principle

ple geometrical arrangement, and equal magnitudes for the charges, to simplify the situation as much as possible. For each pair the forces of attraction or repulsion have the same magnitude, say $F = q^2/4\pi\epsilon_0 l^2$; only the directions change. The charge $q_1 = q$ is repelled by $q_2 = q$ and attracted by $q_3 = -q$, each with magnitude F. We can then see by vector addition that the total force $\mathbf{F}_1$ on q_1 is as shown, and it also has the magnitude F. On the other hand, $q_3 = -q$ is attracted towards q_1 as well as q_2; so the total force $\mathbf{F}_3$ on it is vertically downwards as shown. Using the method of components for finding the resultant, its magnitude is $|\mathbf{F}_3| = 2\mathrm{F}\cos\pi/6 = \sqrt{3}F$.

★ We can express Eq (1 11) for the total force on charge q_1 in terms of the position vectors of the various charges Let q_1 be at $\mathbf{r}_1$ (the position vector of the point P_1), q_2 at $\mathbf{r}_2$, q_3 at $\mathbf{r}_3$ etc. Then using Eq. (1 4) we can write:

$$\begin{aligned}
\mathbf{F}_{12} &= \text{force on } q_1 \text{ due to } q_2 \\
&= \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12}, \\
\mathbf{F}_{13} &= \text{force on } q_1 \text{ due to } q_3 \\
&= \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_3}{r_{13}^2}\,\hat{\mathbf{r}}_{1}\ ,. \\
\mathbf{F}_1 &= \text{total force on } . \\
&= \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots \\
&= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1\, q_3}{r_{13}^2}\,\hat{\mathbf{r}}_{13} + \ldots\right).
\end{aligned} \tag{1.13}$$

Here r_{12} is the distance between q_1 and q_2, while $\hat{\mathbf{r}}_{12}$ is the unit vector pointing from q_2 towards q_1; and so on Fig 1 8 will help you understand the situation ★

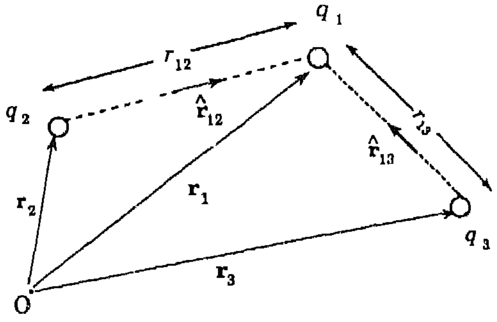


Figure 1.8: Directions and distances to understand Eq (1.13)

We can say, by way of recapitulation, that there are three important physical facts conveyed by Coulomb's Law of electric forces:

(i) For each individual pair of charges, there is a $1/r^2$ dependence on the separation.

(ii) For each pair there is a dependence on the product of charges, with repulsion for like charges and attraction for unlike ones.

(iii) For many charges, the force on any one is found, as a vector sum, by the superposition principle.

Properties (i) and (iii) hold for gravity as well, with charge replaced by mass.

The electric field. Coulomb's Law describes the forces between charges in the spirit of "action at a distance". However far apart they may be, charges affect one another across the distance separating them. This is similar to gravity acting between point masses.

We can however view the situation slightly differently by introducing the concept of the *electric field*. Instead of saying that the effect of a charge q_1 on another charge q_2 is given in a *single step* by Coulomb's Law, we break it up into *two steps*: First, the charge q_1 creates a field at all points of space around it; second, this field acts on the charge q_2, whereever it may be located, and gives the force on it due to q_1. Reversing the roles of the charges we also say: q_2 creates an electric field around itself; this then acts on q_1 and gives the force on q_1 due to q_2.

Let us now define the electric field in a general situation Suppose some arrangement of charges is given, say $q_1, q_2, q_3, \ldots$ at positions $P_1, P_2, P_3, \ldots$ in space. These are called the *sources* of the electric field. At a general point P in space, the electric field produced by them is defined to be the force experienced by a unit positive "test" charge placed at P: Electric field at point P in space = **E** at P

$$\begin{aligned} &= \text{force on } +1\text{C of charge placed at P,} \\ &\quad \text{due to } q_1 \text{ at } P_1, q_2 \text{ at } P_2, q_3 \text{ at } P_3, \ldots \end{aligned} \tag{1.14}$$

Of course we know that 1C of charge is an enormous amount. What we actually want in defining the electric field is the *force per unit charge*. So we may place a small test charge q_0, say, at the point P and measure the force **F** on it. Then we get the electric field at P as the ratio $\mathbf{F}/q_0$ (See Fig. 1.9):

$$\begin{aligned} \mathbf{F} &= \text{total force exerted by} \\ &\quad q_1, q_2, q_3 \ldots \text{(sources) on test} \\ &\quad \text{charge } q_0 \text{ at point P} \quad \Longrightarrow \\ \mathbf{E} &= \text{electric field at P} \\ &= \mathbf{F}/q_0. \end{aligned} \tag{1.15}$$

It is now seen that the unit for electric field is newton per coulomb. For convenience we assume q_0 to be positive, so **F** and **E** have the same direction.

The electric field has two important properties : (i) it is independent of the amount of charge q_0 used as test charge; (ii) it also obeys a superposition principle. Let us see why. According to the superposition principle for electric forces, the total force **F** on the test charge q_0 is the vector sum of many contributions: one due to q_1, another due to q_2, and so on for each source charge. Each contribution is calculated using Coulomb's Law and ignoring other source charges. But in each contribution q_0 appears as a multiplying factor. Therefore in the ratio $\mathbf{F}/q_0$ there is no dependence left on q_0 at all! We can turn Eq (1.15) the other way around and also say:

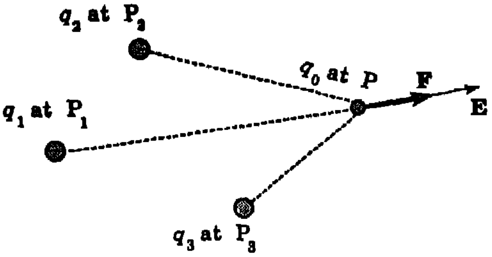


Figure 1.9: The electric field at a point due to given sources

$$\begin{aligned} \mathbf{E} &= \text{electric field at point P} \\ &\quad \text{in space} \quad \Longrightarrow \\ \mathbf{F} &= \text{force on any charge } q_0 \\ &\quad \text{placed at P} = q_0\mathbf{E}. \end{aligned} \tag{1.16}$$

As for the second property, it also follows from the force **F** being a sum of terms, one due to each source charge. So after dividing by q_0 the same must be true of **E** as well:

$$\begin{aligned} \mathbf{E} &= \text{electric field at point P due to} \\ &\quad q_1 \text{ at } P_1, q_2 \text{ at } P_2, \ldots \\ &= \text{electric field at P due to } q_1 \text{ alone,} \\ &\quad \text{plus electric field at P due to } q_2 \\ &\quad \text{alone, plus} \ldots \end{aligned} \tag{1.17}$$

This is what we mean by saying that the electric field also obeys a superposition principle. The sum here is of course a sum of vectors.

We may combine the ideas leading to Eqs (1.16,1.17) and say in one sentence: the electric field at a point depends only on the sources, and not on the test charge; it is a (vectorial) sum of contributions, one from each source charge.

We will soon look at some simple examples of the electric field produced by various sources. But the idea is so important that it may be helpful to make a few more descriptive remarks. Given the source charges and their locations, we can imagine placing our test charge q_0 at various points P in space, one after another. Using the ratio definition (1.15) we then get both the direction and the magnitude of the electric field at all points of space (avoiding the locations of the sources!). To say that there is a nonvanishing electric field at a certain point simply means that if any charge were placed there, it would experience a force, and would accelerate. This is all that the electric field represents. It is just a convenient way of knowing in advance what electric force would act on a charge q_0 if it were placed in various locations - no more and no less.

An electric field is said to be *uniform* in a certain region of space if both the direction and the magnitude of **E** are the same at all points there. Otherwise, it is *nonuniform*. In a nonuniform field, either the direction or the magnitude or both vary from point to point.

★ The idea of the electric field is similar in spirit to that of the gravitational field. There too the field is a replacement for an action at a distance description. When we study the motion of bodies near the earth, say, the gravitational effect of the earth is represented by its gravitational field. This is just the gravitational attraction exerted by the earth on a unit mass. Therefore the units for gravitational field are force per unit mass, or acceleration, ms^{-2}. As we know, close to the earth's surface, over a sufficiently small portion of it, this is a constant field pointing vertically downwards, with magnitude $g \simeq 9.80\ \mathrm{ms}^{-2}$.

It is to Michael Faraday (1791-1867), and later to James Clerk Maxwell (1831-1879), that we owe the development of the ideas of electric and magnetic fields. These are indispensable for the understanding of electromagnetism. In this Chapter, we have introduced the electric field imagining a static situation. This means the sources do not move, and at each point in space the electric field does not change in the course of time. In later Chapters we will have to consider more general situations, where the field does vary with time.

The electric field has been defined in Eq. (1.15) as the ratio of force **F** to test charge q_0. In some situations it becomes necessary to view this as a limiting process. Suppose, for example, that the sources consist partly of charges on conductors. If we now bring a test charge q_0 to a point P, it will exert a force on the source charges, if located on conductors, they may then move! Thus the process of bringing a test charge to point P to measure the field there changes the very thing we wish to measure! One way out is to imagine using smaller and smaller test charges, then we define the field as a limit as q_0 tends to zero

$$\mathbf{E} \text{ at P} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0},$$

$$\mathbf{F} = \text{force on charge } q_0 \text{ at P.} \qquad (1.18)$$

The idea is that by reducing q_0 gradually to zero, we continually diminish the disturbance it causes on the sources.

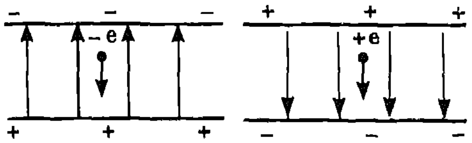
There is however a catch in this argument! We know that in nature charge is quantized. The smallest nonzero amount of charge is $e \simeq 1.6 \times 10^{-19}\mathrm{C}$. So in fact we are not able to let the test charge q_0 of Eq. (1.18) approach zero continuously. Only in large scale or classical situations, where the quantization of charge may be ignored, can we adopt the limiting procedure (1.18) to define the electric field. ★

Example 1.2 : **An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0 \times 10^4\ \mathrm{NC}^{-1}$ [Fig. a]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. b]. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.**

Answer : In (a) the field is upwards, so the negatively charged electron experiences a downward force of magnitude eE where E is the magnitude of the electric field. The acceleration of the electron is,

$$a_e = eE/m_e$$

where m_e is the mass of the electron.



(a) (b)

Starting from rest, the time required by the electron to fall through a distance h is given by

$$\begin{aligned} t_e &= \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}} \\ \text{for e} &= 1.602 \times 10^{-19}\text{C}, \\ m_e &= 9.110 \times 10^{-31}\text{kg}, \\ E &= 2.0 \times 10^{4}\text{NC}^{-1}, \\ \text{and } h &= 1.5 \times 10^{-2}\text{m}, \\ t_e &= 2.9 \times 10^{-9}\text{s} \end{aligned}$$

In (b), the field is downwards, and the positively charged proton experiences a downward force of magnitude eE. The acceleration of the proton is,

$$a_p = eE/m_p$$

where m_p is the mass of the proton; $m_p = 1.673 \times 10^{-27}$ kg. The time of fall for the proton is

$$t_p = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}} = 1.3 \times 10^{-7}\text{s}$$

Thus the heavier particle (proton) takes a greater time to fall through the same distance. This is in basic contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Note that in this example we have ignored the acceleration due to gravity in calculating the time of fall. To see if this is justified, let us calculate

$$a_p = \frac{eE}{m_p} = \frac{(1.602 \times 10^{-19}\text{C}) \times (2.0 \times 10^{4}\ \text{NC}^{-1})}{1.673 \times 10^{-27}\ \text{kg}},$$
$$= 1.9 \times 10^{12}\text{ms}^{-2}$$

which is enormous compared to the value of g (=9.8 ms^{-2}). The acceleration of the electron is even greater. Thus the effect of acceleration due to gravity can be ignored in this example.

Examples of Electric Fields: The simplest example of an electric field is that produced by a single point charge. Let it be positive, and of amount q. Suppose we bring a test charge q_0 (also positive) to a distance r from q. It will be *repelled* with a force of magnitude $q_0q/4\pi\epsilon_0r^2$, in the direction *away from* the source q. Divide this force by q_0, the amount of test charge. We find that the electric field at a distance r from q has magnitude $q/4\pi\epsilon_0r^2$, and points directly away from q. This is shown in Fig. 1.10a.

The field exists at every point in three-dimensional space, but we can only give a two-dimensional picture of it. This is shown in Fig. 1.10b. The positive source q is at the centre. At all points distance r away, the field has the magnitude $q/4\pi\epsilon_0r^2$. At each point its direction is *radially outwards* from q. As the distance r increases, the magnitude decreases as $1/r^2$. So, twice as far away the field is one-fourth as strong; three times as far away means one-ninth as strong; and so on.

In Fig. 1.10b we have drawn arrows at a few points, to give an approximate idea of the field at each point. These arrows, always pointing away from q, get shorter and shorter as we go farther away from q.

Suppose the source charge q is negative. Then the positive test charge q_0 is *attracted* towards q. So at each point the electric field vector points radially *inwards*. This is shown in Fig. 1.10c.

We can find a simple expression for this electric field of a single charge. Let $\mathbf{r}$ be the vector (in three dimensions) from q to a general point P where we place our test charge q_0. (See Fig. 1.10a). Then Coulomb's Law gives us:

$$\begin{aligned} \mathbf{F} &= \text{force on test charge } q_0 \text{ at point } P \\ &= \frac{1}{4\pi\epsilon_0}\frac{q_0q}{r^2}\frac{\mathbf{r}}{r} \Rightarrow \end{aligned}$$

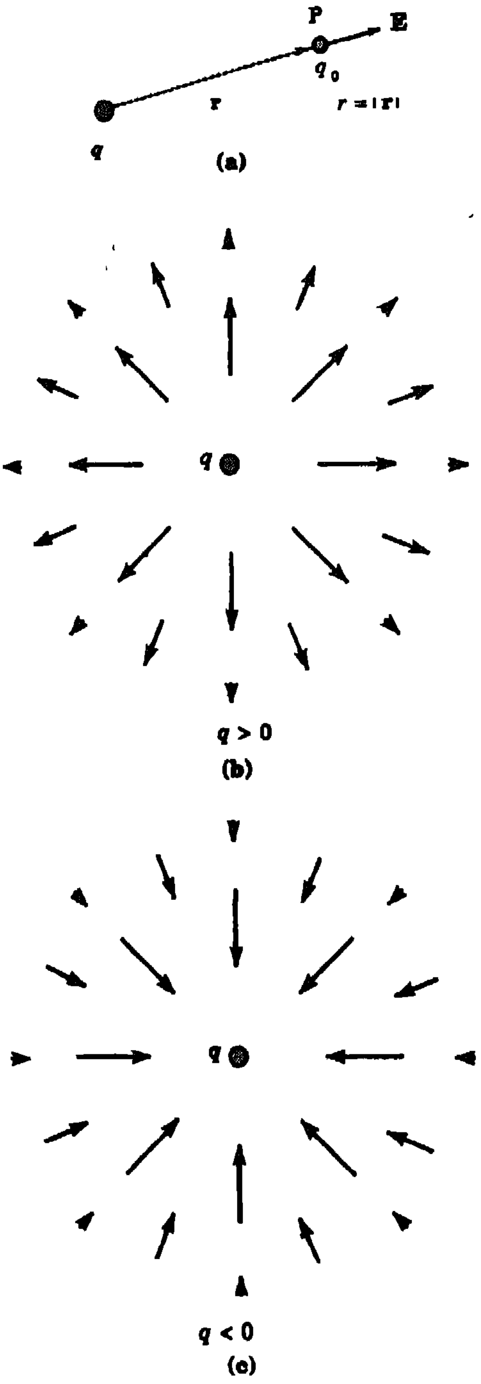


Figure 1.10: Electric field of a point charge

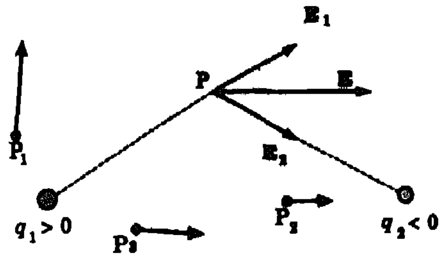


Figure 1.11: Electric field of two charges of opposite signs

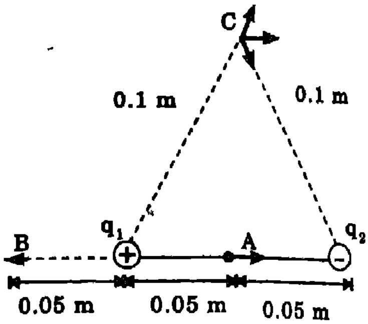
$$\begin{aligned}\mathbf{E} &= \text{electric field vector at } P \text{ due to} \\ &\quad \text{point charge } q \\ &= \frac{\mathbf{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\frac{\mathbf{r}}{r}.\end{aligned} \tag{1.19}$$

The sign of q, positive or negative, determines whether $\mathbf{E}$ points away from, or towards, the source q

As another example, take two point charges q_1 and q_2. Suppose q_1 is positive and q_2 is negative. To find the field at a point P, we use the superposition principle. Draw the electric field vector at P due to q_1 alone, then that due to q_2 alone. Then add these two vectors to get the total field at P. In Fig. 1.11 we have shown a picture of this. At the point marked P, the two individual fields marked $\mathbf{E}_1$ and $\mathbf{E}_2$ add by the parallelogram law to give the total field $\mathbf{E}$. The field $\mathbf{E}_1$ caused by q_1 is in the direction q_1 to P; $\mathbf{E}_2$ caused by q_2 points in the direction P to q_2. At a few other points the field vectors are marked by arrows to give an approximate idea of the nature of the field.

Example 1.3: Two point charges q_1 and q_2 of $+10^{-8}$C and -10^{-8}C respectively are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in the figure

Answer: The electric field vector $\mathbf{E}_1$ at A due to the positive charge q_1 points towards



the right and it has a magnitude

$$E_1 = (9 \times 10^9 \text{Nm}^2\text{C}^{-2}) \frac{(10^{-8}\text{C})}{(0.05\text{m})^2}$$

$$= 3.6 \times 10^4\text{NC}^{-1}.$$

The electric field vector $\mathbf{E}_2$ at A due to the negative charge q_2 points towards the right and it has a magnitude

$$E_2 = (9 \times 10^9\text{Nm}^2\text{C}^{-2}) \frac{(10^{-8}\text{C})}{(0.05\text{m})^2}$$

$$= 3.6 \times 10^4\text{NC}^{-1}.$$

Magnitude of the total electric field at A

$$E_A = E_1 + E_2 = 7.2 \times 10^4\text{NC}^{-1}$$

$\mathbf{E}_A$ is directed towards the right.

The electric field vector $\mathbf{E}_1$ at B due to the positive charge q_1 points towards the left and it has a magnitude

$$E_1 = (9 \times 10^9 \text{ Nm}^2\text{C}^{-2}) \frac{(10^{-8}\text{C})}{(0.05\text{m})^2}$$

$$= 3.6 \times 10^4\text{NC}^{-1}.$$

The electric field vector $\mathbf{E}_2$ at B due to the negative charge q_2 points towards the right and it has a magnitude

$$E_2 = (9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}) \frac{(10^{-8}\text{ C})}{(0.15 \text{ m})^2}$$

$$= 4 \times 10^3 \text{ NC}^{-1}.$$

Magnitude of the total electric field at B

$$E_B = 3.2 \times 10^4 \text{ NC}^{-1}$$

$\mathbf{E}_B$ is directed towards the left.

The magnitude of each electric field vector, at point C, of charges q_1 and q_2 is

$$E_1 = E_2 = (9 \times 10^9 \text{ Nm}^2\text{C}^{-2}) \frac{(10^{-8}\text{ C})}{(0.1 \text{ m})^2}$$

$$= 9 \times 10^3 \text{ NC}^{-1}.$$

The directions in which these two vectors point are indicated in the figure. The resultant of these two vectors is

$$E_C = E_1 \cos\frac{\pi}{3} + E_2 \cos\frac{\pi}{3}$$

$$= 9 \times 10^3 \text{ NC}^{-1}$$

Vector $\mathbf{E}_C$ points towards the right.

★ In Eq (1.13) we have an expression for the total force on a charge q_1 due to other charges $q_2, q_3, \ldots$ Think of q_1 as our test charge, and suppose there are only two other charges, q_2 and q_3. Divide the total force $\mathbf{F}_1$ acting on q_1, by the charge q_1. Then we get the electric field vector at position $\mathbf{r}_1$ due to two source charges, q_2 at $\mathbf{r}_2$ and q_3 at $\mathbf{r}_3$.

Electric field at $\mathbf{r}_1$ =

$$\frac{1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} + \frac{q_3}{r_{13}^2}\,\hat{\mathbf{r}}_{13}\right)$$

r_{12} = distance from $\mathbf{r}_1$ to $\mathbf{r}_2$,
$\hat{\mathbf{r}}_{12}$ = unit vector from $\mathbf{r}_2$ to $\mathbf{r}_1$,
r_{13} = distance from $\mathbf{r}_1$ to $\mathbf{r}_3$,
$\hat{\mathbf{r}}_{13}$ = unit vector from $\mathbf{r}_3$ to $\mathbf{r}_1$. (1.20)

If we have more point charges producing the field, we have more contributions of the above forms ★

Continuous charge distributions: So far we have considered electric fields due to point charges. There are however cases where we have to deal with *continuous distributions of charge*. Here we ignore charge quantization. We assume the total amount of charge involved is very much greater than that on a single electron or proton.

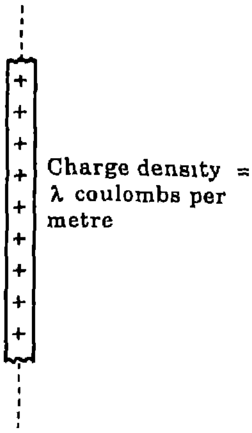
With such charge distributions we generally have to use the methods of integral calculus to find the electric field. We will not study such methods in any detail. Nevertheless it is useful to describe some simple examples of such charge distributions, since the ideas are quite easy to grasp. In a few cases, the electric field that is produced will also be described without derivation.

Let us first look at one-dimensional examples. The simplest is a thin straight line or rod along which charge is uniformly distributed. So we have a constant charge per unit length, or *charge density*, say λ coulombs per metre, σ Cm^{-1}. This is shown in Fig. 1 12a. Another example is a circular ring of radius R metres, say, with a total charge of q coulombs distributed uniformly over it. The charge density then is $q/2\pi R$ Cm^{-1}. These are illustrated in Fig 1 12a,b. Now let us come to two-dimensional cases. Again the simplest is a uniform plane sheet of charge, with a constant charge per unit area, say σ coulombs per square metre, or $\sigma\mathrm{Cm}^{-2}$ This is shown in Fig. 1.12c Another simple example is the surface of a sphere of radius R metres, say, over which a total charge of q coulombs is spread uniformly Then the charge density is $q/4\pi R^2$ Cm^{-2}. This is illustrated in Fig 1.12d.

Our last example is in three dimensions. Again we take a sphere of radius R metres, and spread a total charge of q coulombs uniformly inside it. We now have a constant charge per unit volume. The charge density is $q/(4/3\pi R^3)$ Cm^{-3}. This is depicted in Fig. 1.12e (to be carefully distinguished from Fig. 1.12d!).

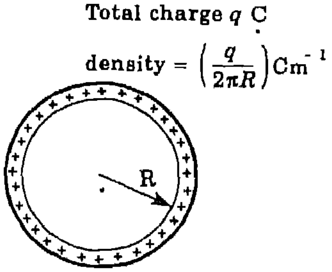
In each example, depending on the dimensions, we see that the units for charge density are different. Cm^{-1} or Cm^{-2} or Cm^{-3} as the case may be. Moreover, in each of the cases illustrated, the charge density could be negative rather than positive.

To obtain the electric field produced by these (or other) continuous charge distributions, the idea we use is the following. We imagine dividing the source into a large number of small elements. Then we use Coulomb's Law to find the field at a given point due to each element. Finally we add up all these contributions vectorially to get the total field. It



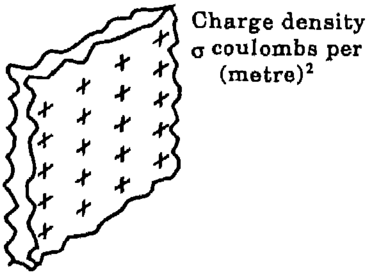
(a) Straight line of charge

Figure 1.12 a



(b) Ring of charge

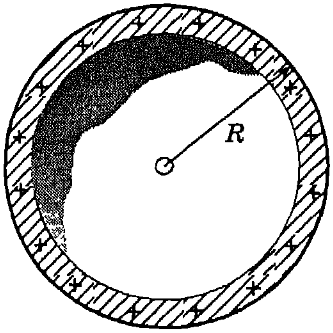
Figure 1.12 b



(c) Plane sheet of charge

Figure 1.12 c

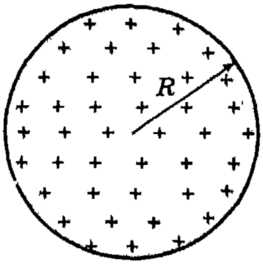
is in this last step that integral calculus is usually needed.



Total charge q C.
surface density $= \frac{q}{4\pi R^2}$ Cm^{-2}

(d) Spherical surface of charge

Figure 1.12 d



Total charge q C.
Volume density $= \left(q\Big/\frac{4}{3}\pi R^3\right)$ Cm^{-3}

(e) Solid sphere of charge

Figure 1.12 e

Figure 1.12: Continuous charge distributions - examples

We will not present these details here. Nevertheless in a few cases it is worth describing the field because it is rather simple. In the next Chapter we will indicate a way by which these results may be found.

Let us first take the cases of the straight line and the plane sheet. In both cases for simplicity we assume the charge distribution is infinite in extent, in length or area as the case may be, and the charge density is positive. (This is unphysical since it means the total charge is infinite, but it leads to simple results for the fields. The alternative is to ask for the fields only at points very close to the line or the plane, in comparison to the linear size of the source). Then the field due to the line of charge is everywhere perpendicular to the line, pointing radially away, and inversely proportional to the distance from the line:
Field due to infinite straight line, of uniform charge density λ, at perpendicular distance r away =

$$\frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}, \text{ radially outwards} \qquad (1.21)$$

This is shown in Fig. 1.13a. For the plane sheet the result, shown in Fig. 1.13b, is even simpler:

Field due to infinite plane sheet,
of uniform charge density σ
= uniform on each side of plane,
perpendicular to and away from
plane, magnitude $\sigma/2\epsilon_0$. (1.22)

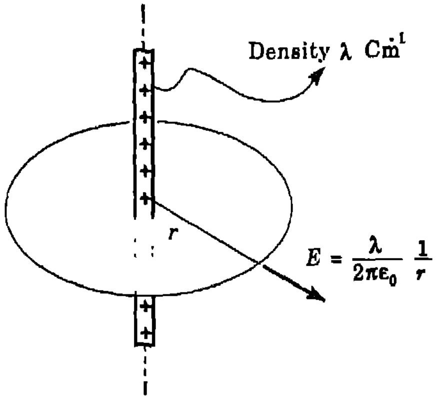
You can see that in these two expressions, the dimensions of the charge densities are just right to give the dimension NC^{-1} to the field. If the density is negative, we simply reverse the direction of the field.

For a uniform spherical surface layer, or a solid sphere, of radius R, the total charge q is finite. In both cases, at all points *outside* the sphere, the field is the same as that produced by a *point* charge q at the centre of the sphere! This important result is indicated in Figs 1.13c.

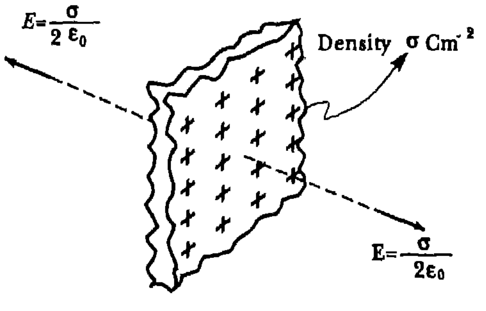
Field due to uniform spherical surface layer, or uniform solid sphere, of radius R and total charge q, at distance r from centre of sphere $(r > R)$ =

$$\frac{q}{4\pi\epsilon_0}\frac{1}{r^2} \text{ radially} \begin{cases} \text{outwards if } q > 0, \\ \text{inwards if } q < 0. \end{cases} \qquad (1.23)$$

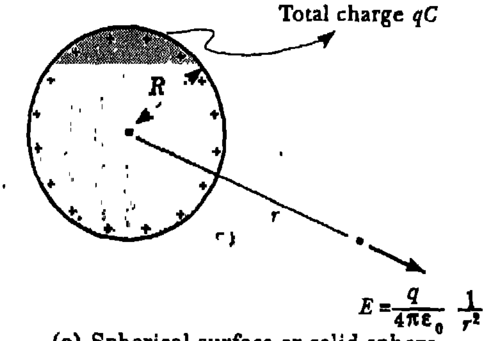
★ The results described above require integral calculus for their derivation There is however



(a) Straight line


$E=\frac{\sigma}{2\varepsilon_0}$
Density σ Cm⁻²
$E=\frac{\sigma}{2\varepsilon_0}$

(b) Plane sheet



(c) Spherical surface or solid sphere

Figure 1.13: Electric fields of some continuous charge distributions

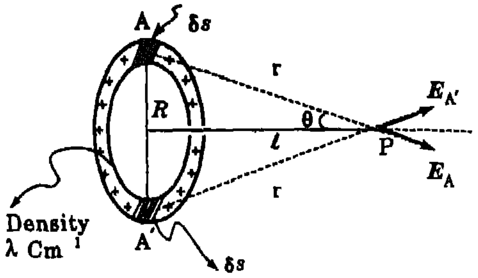


Figure 1.14: Field of ring of charge at points on axis

one case - a uniform circular ring of charge - in which at special points we can find the field using elementary arguments. We present the derivation for this case

The situation is illustrated in Fig. 1.14 Assume the charge density $\lambda = q/2\pi R$ is positive. Take a point P on the axis, distance l from the plane of the ring A small portion of the ring at the point A, of length δs, has charge $\lambda\delta s$. It produces an electric field $\mathbf{E}_A$ at P as shown A similar portion of the ring at the diametrically opposite point A′, of the same length δs produces the field $\mathbf{E}_{A'}$. We can see that the components of $\mathbf{E}_A$ and $\mathbf{E}_{A'}$ along the axis add up, but their perpendicular components exactly cancel. This happens for each pair of diametrically opposite portions of the ring

We see then that the total field at P is along the axis, and is the result of adding such components due to all portions of the ring Further the angle θ as shown is the same all over the ring. So we can easily get the total field.

Component of $\mathbf{E}_A$ parallel to axis

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda\delta s}{r^2}\cos\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{(R^2+l^2)}\frac{l}{(R^2+l^2)^{1/2}}\delta s;$$

Total field at P

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{(R^2+l^2)}\frac{l}{(R^2+l^2)^{1/2}}\,2\pi R$$

$$= \frac{q}{4\pi\epsilon_0}\frac{l}{(R^2+l^2)^{3/2}} \text{ along the axis.} \qquad (1.24)$$

We see that because of the simple geometry, and cancellation of components perpendicular to the axis, we can get this result easily (If the point P were not on the axis, we could not have avoided use of integral calculus!) At the centre of the ring, $l = 0$, the field vanishes. Here the fields due to diametrically opposite portions of the ring cancel completely. ★

1.7 Lines of force

In the examples of electric fields given in the previous Section, we drew pictures with arrows representing the field vectors at a few points. These pictures give us some idea how the field changes from point to point in space.

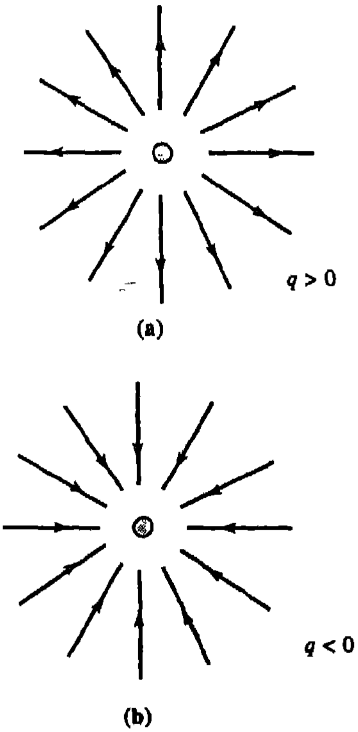


Figure 1.15: Lines of force for single point charge

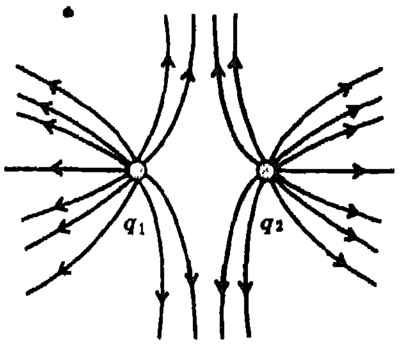


Figure 1.16: Lines of force for two positive charges $q_1 > q_2 > 0$

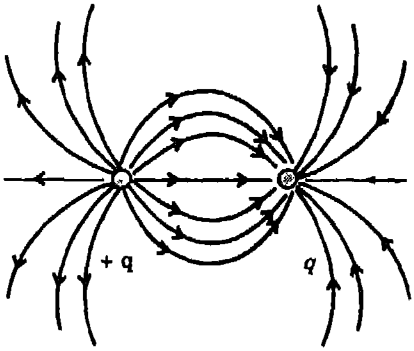


Figure 1.17: Lines of force for two equal and opposite charges $q, -q$

There is another way to convey the general features of the electric field produced by given sources. This is based on the idea of *electric lines of force*. A line of force is a line drawn in such a way that at each point on it the tangent is in the direction of the electric field there. So a line of force carries a definite sense or direction along it.

In general the direction of the electric field vector changes from point to point. Therefore lines of force are, generally, curved lines. Even though lines of force are to be drawn in three dimensions, we are limited to drawing pictures in two dimensions. We will therefore only look at simple enough situations where two-dimensional pictures are adequate. In the special case of a field which is uniform in a region of space, the lines of force become parallel straight lines.

Lines of force are drawn to help "map out

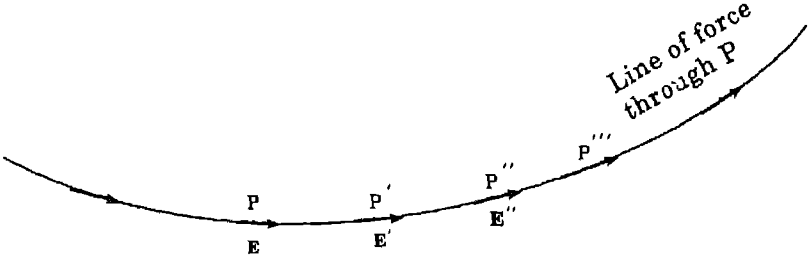


Figure 1.18: Constructing a line of force

the field" and get a feeling for the electric field in any situation. We have to choose how many lines we wish to draw. In that sense, they are imaginary lines. They give us partial qualitative information about the field.

Let us now look at some examples of lines of force for simple cases, and later describe their general properties. For a single positive point charge q, the field vectors point radially outwards everywhere. So the picture of lines of force, drawn in any plane containing the charge, looks as in Fig. 1.15a. In case q is negative, the arrows on these lines of force must point inwards as in Fig. 1.15b. At this point you must compare these figures of lines of force with Figs 1.10b,c which show the electric field vectors in these two cases. Notice also that these lines of force are straight lines, though they are not parallel as they would be for a uniform field.

Next let us look at the example of two positive point charges $q_1 > q_2 > 0$. The lines of force are shown in Fig. 1.16. We do get a strong feeling that the charges are pushing one another apart!

Lastly let us look at two equal and opposite charges, q and $-q$ placed a certain distance apart. Now the lines of force are strikingly different from the previous case of two positive charges, and are shown in Fig. 1.17. This picture conveys strongly the idea of attraction between the two charges!

You may wonder how we might actually draw, or construct, a line of force. We describe it briefly. Take any point P in space, and let the electric field vector there be $\mathbf{E}$. Move from P by a very small distance in the direction of $\mathbf{E}$, reaching a nearby point P$'$. Let the electric field at P$'$ be $\mathbf{E}'$. Then move from P$'$ in the direction of $\mathbf{E}'$, to a nearby point P$''$. And so on from P$''$ to P$'''$, . . . , in the direction of $\mathbf{E}''$, . . . By bringing these points closer and closer, we ultimately get a continuous curve starting out at P in the direction of $\mathbf{E}$. The construction is indicated in Fig. 1.18.

This method ensures the basic requirement: at each point the tangent to the line of force is in the direction of the electric field there. Starting from P, we can also go *backwards* in the direction of $-\mathbf{E}$, and continue that way. So the line of force extends on both sides of the starting point P. By now you see: you could have taken any point on the line of force as starting point, and the above procedure would allow you to build it up completely. ★

We have already said that at each point on a line of force the tangent is in the direction

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★ It is an important historical fact that Michael Faraday, who invented these ideas of lines of force and applied them to both electricity and magnetism, depended a great deal on such pictures to guide him in his discoveries. In fact the vector nature of electric and magnetic fields was not fully appreciated by him, and he relied much more on ideas based on lines of force. He was quite nonmathematical in his approach, and had a very powerful intuitive way of thinking. The complete system of equations of electricity and magnetism, found by Maxwell in 1865, have to be written in terms of the vectors $\mathbf{E}$ and $\mathbf{B}$, not in terms of lines of force. This shows the limitations of the latter.

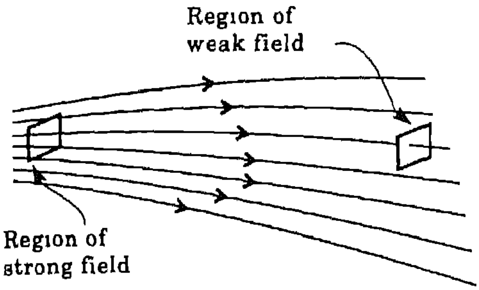

Figure 1.19: Density of lines of force as proportional to strength of field

of the electric vector there Here are some other important properties which are simple and easy to grasp, so we give them here in a descriptive way·

(a) Two lines of force cannot cross at a common point. For if they did, we would have two different directions for the electric field at that point, which is not possible!

(b) Once we have drawn a set of lines of force, their density, or relative closeness or thinness, indicates the strength of the electric field at various places. Lines of force crowd near each other where the field is strong, and move far apart where it is weak. This is seen in Figs 1.15-1.17. We can say: if at some point we imagine a small element of area drawn perpendicular to the lines of force there, the number of lines of force cutting the area element is proportional to the magnitude of E there, as illustrated in Fig. 1.19.

(c) As long as we consider fields which are static, that is, which do not change with time, lines of force start at positive charges and end at negative ones only (in some cases they may start or end at infinity).

(d) Lines of force are continuous and do not have breaks in them.

1.8 Electric dipoles

An *electric dipole* is a pair of equal and opposite charges, q and $-q$, separated by some distance, $2a$. In this Section we study some properties of dipoles

Why are dipoles interesting and important? In Section 1.5 we described the phenomenon of polarization When a dielectric is placed in an electric field, within each atom (or molecule) the positive nuclei and the negative electrons are pulled in opposite directions and get slightly separated Their charges are of course equal and opposite. Thus in effect each atom or molecule becomes a tiny dipole (if it was not already so!). Many electrical properties of dielectrics can be understood on this basis Dipoles are important in our understanding of magnetism too, as we shall see in Chapters 5 and 6. (Those are of course magnetic dipoles).

We must look at two aspects of dipoles, namely : the nature of the electric field created by a dipole; and the forces (and torques) that act on a dipole if it is placed in some electric field.

The field of a dipole: Let us take up the first aspect. Fig. 1 17 already gives us some idea of the field due to a dipole. We can find the magnitude and direction of the electric field

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★ In the examples given, we could draw pictures of lines of force in a plane This is because in these cases each line of force indeed lies in some plane But in more general cases each line of force may wind around in three-dimensional space, and not lie in any plane at all Then we would be unable to draw any two-dimensional pictures!

Here is another interesting point, what happens if there is a point P_0 at which the electric field vanishes? This is so for example in the case of two positive charges illustrated in Fig 1 16 Since we assumed $q_1 > q_2 > 0$, the point P_0 is on the straight line joining q_1 to q_2, nearer to q_2 than to q_1. Whenever the field vanishes at a point P_0, there is no line of force passing through P_0. Lines of force nearby P_0 avoid that point! ★

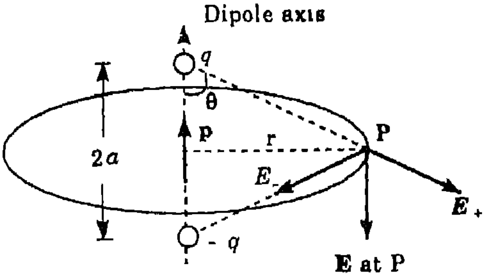


Figure 1.20: Dipole electric field at points in equatorial plane

vector at some points quite easily. As illustrated in Fig. 1.20, let the point P be in the *equatorial plane* of the dipole, at a distance r from the centre of the dipole. (The *axis* of the dipole is the direction from the negative to the positive charge; the equatorial plane is perpendicular to the axis and contains the centre) The electric vectors $\mathbf{E}_+$ and $\mathbf{E}_-$ at P, due to the charges q and $-q$ (always here q is positive), are as shown. They have equal magnitudes, and their vector sum is easily seen to be opposite in direction to the dipole axis.

Dipole electric field at

points in equatorial plane

$$= 2\frac{q}{4\pi\epsilon_0}\frac{1}{a^2 + r^2}\cos\theta$$

$$= \frac{2aq}{4\pi\epsilon_0}\frac{1}{(a^2 + r^2)^{3/2}}$$

direction opposite to dipole axis.

(1.25)

The charge q and the separation $2a$ appear here in the combination $2aq$. This motivates our defining the *dipole moment* of the dipole. We include also the direction of the dipole axis, and so we define:

Dipole moment vector $\mathbf{p}$ =

vector along dipole axis,

with magnitude $2aq$. (1.26)

The dimensions are evident: coulomb meter. In words: the dipole moment vector points from the negative to the positive charge, with magnitude equal to the magnitude of the individual charges times the distance of separation. We can now write Eq. (1.25) in vector

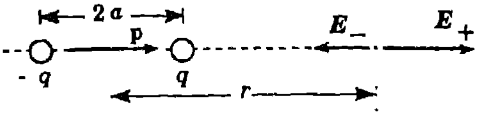


Figure 1.21: Dipole electric field at points on axis

form:

Dipole field at points in equatorial plane, distance r from centre

$$= -\frac{\mathbf{p}}{4\pi\epsilon_0}\frac{1}{(a^2 + r^2)^{3/2}}. \qquad (1.27)$$

If the distance r is much greater than the length a, we get a simpler expression:

Dipole electric field at great distances in equatorial plane

$$\simeq -\frac{\mathbf{p}}{4\pi\epsilon_0}\frac{1}{r^3}, \quad \text{for} \quad r >> a \qquad (1.28)$$

We see that at large distances only the dipole moment vector is needed, not charge q and separation $2a$ individually. Notice also the *inverse cube* dependence on distance. For a single point charge the Coulomb's Law gives an inverse square dependence for the electric field. The difference can be understood easily. With a dipole, at great distances, the fields due to q and $-q$ almost cancel each other, but not quite. (Indeed, if the separation $2a$ were zero, the two charges would have been on top of one another, and there would be no field at all!). What remains, after the near cancellation, is what we find in Eq (1.28), behaving as $1/r^3$ instead of as $1/r^2$.

We can also calculate the strength of the field at points along the axis, far away 'in front' or 'at the back'. Referring to Fig. 1.21, we find:

Magnitude of dipole electric

field at distance r along

the axis, front or back,

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]. \qquad (1.29)$$

In the figure, $E_\pm$ denote the fields due to $\pm q$ respectively. For r much greater than a, using the binomial expansion we can write in an approximate way:

$$\frac{1}{(r-a)^2} \simeq \frac{1}{r^2}\left(1+\frac{2a}{r}\right),$$

$$\frac{1}{(r+a)^2} \simeq \frac{1}{r^2}\left(1-\frac{2a}{r}\right). \qquad (1.30)$$

Therefore, taking into account the direction as well (as seen in Fig. 1.21), we get,

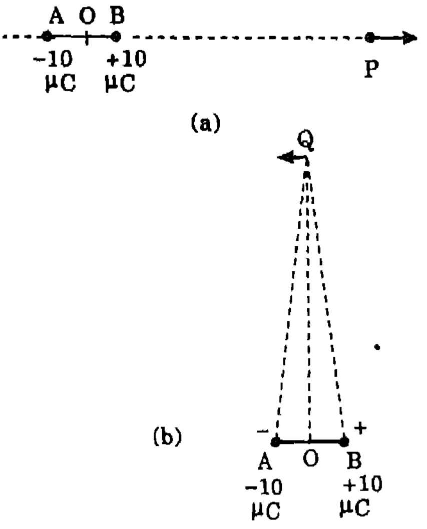
Dipole electric field at great distance r along the axis, front or back,

$$\simeq \frac{p}{4\pi\epsilon_0}\,\frac{2}{r^3}. \qquad (1.31)$$

Compared to the result for points in the equatorial plane, Eq. (1.28), we see the *magnitude is doubled* and the *direction is reversed*.

This illustrates an important fact: in comparing the fields of a point charge and of a dipole, on the one hand the $1/r^2$ dependence is replaced by $1/r^3$ dependence. On the other hand, both the direction and the magnitude of the dipole electric field depend on the angle between the dipole moment vector and the direction to the point where the field is considered. The source of the field in this case has a built-in sense of direction. We have seen this dependence here by comparing equatorial points with points on the axis. This is also brought out by the picture of lines of force in Fig. 1.17. We see that the field produced by a dipole is geometrically more complicated than that of a point charge.

Example 1.4: Two charges ±10 μC are placed 5.00 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole.



Answer: (a) Field at P due to charge +10 μC

$$= \frac{10^{-5}\ \mathrm{C}}{4\pi(8.854\times10^{-12}\ \mathrm{C^2N^{-1}m^{-2}})} \times \frac{1}{(15-0.25)^2\times10^{-4}\ \mathrm{m^2}}$$

$$= 4.13\times10^{6}\ \mathrm{NC^{-1}}\text{along BP.}$$

Field at P due to charge −10 μC

$$= \frac{10^{-5}\ \mathrm{C}}{4\pi(8.854\times10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}})} \times \frac{1}{[15+0.25]^2\times10^{-4}\ \mathrm{m^2}}$$

$$= 3.86\times10^{6}\ \mathrm{NC^{-1}}\text{ along PA.}$$

The resultant electric field at P due to the two charges at A and B is

$$= 2.7\times10^{5}\ \mathrm{NC^{-1}}\text{ along BP.}$$

In this example, the ratio OP/OB is quite large (=60). Thus we can expect to get approximately the same result as above by directly using the formula for electric field at a far-away point on the axis of a dipole. For a dipole consisting of charges $\pm\,q, 2a$ distance apart, the electric field at a distance r from the centre on the axis of the dipole has a magnitude

$$E = \frac{2p}{4\pi\epsilon_0 r^3}\ \text{for } r/a \gg 1,$$

where $p = 2aq$ is the magnitude of the dipole moment.

The direction of electric field on the dipole axis is always along the direction of the dipole moment vector (i.e. from $-q$ to q). Here

$$p = 10^{-5}\ \text{C} \times 5 \times 10^{-3}\ \text{m} = 5 \times 10^{-8}\ \text{Cm}$$

Therefore

$$\begin{aligned} E &= \frac{2 \times 5 \times 10^{-8}\ \text{Cm}}{4\pi(8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2})} \times \frac{1}{(15)^3 \times 10^{-6}\text{m}^3} \\ &= 2.6 \times 10^5\ \text{NC}^{-1} \end{aligned}$$

along the dipole moment direction AB which is close to the result obtained earlier.

(b) Field at Q due to charge $+10\,\mu$C at B

$$\begin{aligned} &= \frac{10^{-5}\ \text{C}}{4\pi(8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2})} \times \frac{1}{[15^2 + (0.25)^2] \times 10^{-4}\ \text{m}^2} \\ &= 3.99 \times 10^6\ \text{NC}^{-1} \text{ along BQ.} \end{aligned}$$

Field at Q due to charge $-10\,\mu$C at A

$$\begin{aligned} &= \frac{10^{-5}\ \text{C}}{4\pi(8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2})} \times \frac{1}{[15^2 + (0.25)^2] \times 10^{-4}\ \text{m}^2} \\ &= 3.99 \times 10^6\ \text{NC}^{-1} \text{ along QA.} \end{aligned}$$

Clearly, the components of these two forces with equal magnitudes cancel along the direction OQ but add up along the direction parallel to BA. Therefore, the resultant electric field at Q due to the two charges at A and B is

$$\begin{aligned} &= 2 \times \text{cosine of angle QBO} \times 3.99 \times 10^6\ \text{NC}^{-1} \text{ along BA} \\ &= 2 \times \frac{0.25}{\sqrt{15^2 + (0.25)^2}} \times 3.99 \times 10^6\ \text{NC}^{-1} \text{ along BA} \\ &= 1.33 \times 10^5\ \text{NC}^{-1} \text{along BA.} \end{aligned}$$

As in (a), we can expect to get approximately the same result by directly using the formula for dipole field at a point on the normal to the axis of the dipole:

$$\begin{aligned} E &= \frac{p}{4\pi\epsilon_0 r^3} \text{ for } r/a \gg 1 \\ &= \frac{5 \times 10^{-8}\ \text{Cm}}{4\pi(8.854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\text{m}^{-2})} \times \frac{1}{15^3 \times 10^{-6}\ \text{m}^3} \\ &= 1.33 \times 10^5\ \text{NC}^{-1}. \end{aligned}$$

The direction of electric field in this case is opposite to the direction of the dipole moment vector. Again the result agrees with that obtained before.

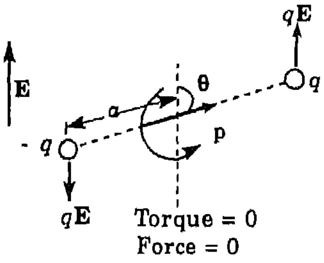


Figure 1.22: Dipole in a uniform field

The dipole in an electric field: Now let us consider the second aspect mentioned at the start of this Section. If a dipole is placed in an electric field (due to some other sources), what forces and torques act on it?

For comparison, recall that on a point charge the force is its charge times the electric field. This is the content of Eq. (1.16). What we now want is the generalization of this if we have a dipole instead of a point charge.

Let us assume we have a uniform electric field **E** (constant direction and magnitude) in some region of space, and a dipole is placed there. Suppose the dipole moment vector makes an angle θ with the field **E**, as shown in Fig.1.22. The positive charge feels a force $q\mathbf{E}$, while the negative charge feels a force $-q\mathbf{E}$. So *the net force on a dipole in a uniform field is zero:*

Uniform electric field:

$$\text{Force on dipole} = q\mathbf{E} - q\mathbf{E} = 0. \qquad (1.32)$$

Does this mean that the dipole does not 'feel' anything? Not at all! Even though the two forces on the ends of the dipole cancel as free vectors, *they act at different points*. This means that they give rise to a *torque* on the dipole. We can see from Fig. 1.22 that the turning effect of this torque is to reduce the angle θ towards zero, and make the dipole moment vector become parallel to the field.

We can compute the magnitude of the torque with respect to the centre of the dipole It is the sum of the two forces times their lever arms

$$|\tau| = 2q\,|\mathbf{E}|\,a\sin\theta = |\mathbf{p}|\,|\mathbf{E}|\sin\theta. \qquad (1.33)$$

Keeping track of the right hand corkscrew rule to fix the direction of τ (in Fig.1.22 it comes out of the page), the full expression is

$$\tau = \mathbf{p} \times \mathbf{E}. \qquad (1.34)$$

We can summarise and say in a uniform electric field, a dipole feels only a torque but no net force.

Since the effect of the field is to make the dipole align itself parallel to the field, when the orientation is some non zero angle θ there must be potential energy stored in the dipole. This is because if you want to rotate the dipole from the preferred orientation $\theta = 0$ to some nonzero θ, you have to oppose the torque due to the field and do some work. We will examine the form of this potential energy in Section 2.5 in Chapter 2.

★ If a dipole is placed in a *nonuniform field*, it experiences a net force in addition to a torque. We consider only the simplest situations, to bring out the basic ideas most easily.

Suppose that in some region of space, to a good accuracy, the electric field has a constant direction, say the x direction Also suppose its magnitude increases steadily as we move in that direction. We consider only two cases· either the dipole moment $\mathbf{p}$ is parallel to $\mathbf{E}$, or points in the opposite direction.

The former is shown in Fig. 1 23a You can see that the force on the positive charge (acting towards the right) is greater in magnitude than the force on the negative charge (acting towards the left) So the net force on the dipole is to the right, that is, towards the region of stronger field This means the dipole is accelerated in that direction.

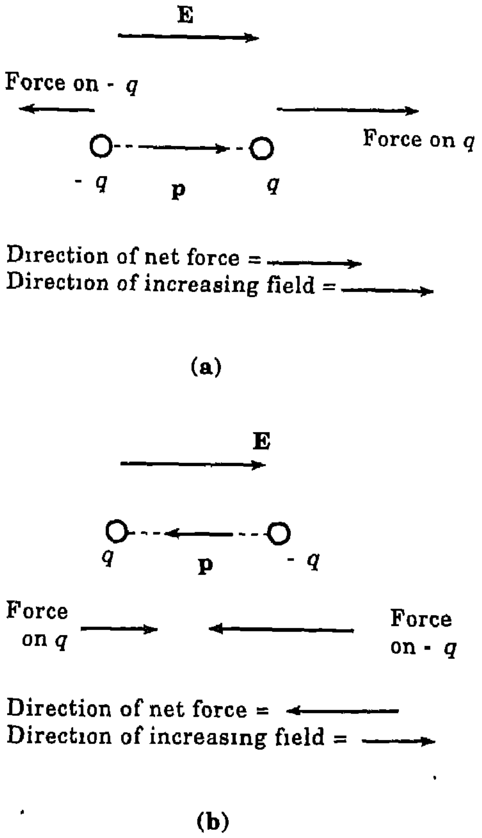


Figure 1.23: Dipole in a nonuniform field

On the other hand, if $\mathbf{p}$ is opposite to the field direction, we have the situation of Fig 1 23b. Now the force on the positive charge is less in magnitude than that on the negative charge The net result is that there is a force towards the left on the dipole, that is, towards the region of smaller field This is opposite to the previous case!

Thus in a nonuniform field we see that a dipole experiences a nonzero force It depends on the orientation of the dipole moment $\mathbf{p}$ with respect to the direction in which the field increases Apart from this force, there is a torque as well on the dipole. ★

Summary

1 Electric and magnetic forces determine the properties of atoms, molecules and bulk matter.

2 From simple experiments on frictional electricity, one can infer that there are two types of charge in nature; and that like charges repel and unlike charges attract. By convention, the charge on a glass rod rubbed with silk is positive; that on a rubber rod rubbed with wool is then negative.

3. The leaf electroscope is a simple apparatus to detect charge on a body It consists of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergence is an indicator of the amount of charge

4. Electric charge has three basic properties. Quantization, Additivity and Conservation

   Quantization of electric charge means that total charge (Q) of a body is always an *integral* multiple of a basic quantum of charge (e) i.e. $Q = ne$, where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ Proton and electron have charges $+e$, $-e$ respectively. For macroscopic charges for which n is a very large number, quantization of charge can be ignored.

   Additivity of electric charge means the total charge of a system is the algebraic sum (i e. the sum taking into account proper signs) of all individual charges in the system.

   Conservation of electric charge means that the total charge of an isolated system remains unchanged with time. This means that when bodies are charged through friction, there is a transfer of electric charge from one body to another, but no creation or destruction of charge.

5. Conductors allow large scale movement of electric charge through them, insulators do not. In metals, the mobile charges are electrons; in electrolytes both positive and negative ions are mobile

6. *Coulomb's Law* : The mutual electrostatic force between two point charges q_1 and q_2 is proportional to the product q_1q_2 and inversely proportional to the square of the distance r separating them. Mathematically,

$$\mathbf{F}_{21} \equiv \text{force on } q_2 \text{ due to } q_1 = \frac{k(q_1q_2)}{r^2}\hat{\mathbf{r}}_{21}$$

   where $\hat{\mathbf{r}}_{21}$ is a unit vector in the direction from q_1 to q_2 and $k = 1/4\pi\epsilon_0$ is the constant of proportionality. In the same way

$$\mathbf{F}_{12} \equiv \text{force on } q_1 \text{ due to } q_2 = \frac{k(q_1q_2)}{r^2}\hat{\mathbf{r}}_{12}$$

   where $\hat{\mathbf{r}}_{12}$ is a unit vector in the direction from q_2 to q_1. $\mathbf{F}_{21} = -\mathbf{F}_{12}$ i.e. Coulomb's law agrees with Newton's Third law of motion.

   In SI units, the unit of charge is *coulomb*. The experimentally determined value of k in coulomb's law is,

$$k = 1/4\pi\epsilon_0 = 8.98755 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

7 The ratio of electric force to gravitational force between proton and electron is

$$\frac{ke^2}{Gm_em_p} \simeq 2.4 \times 10^{39}$$

8 **Superposition Principle**

This principle embodies the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge For an assembly of charges q_1, q_2, q_3 .. the force on any charge, say q_1, is the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on For each pair, the force is given by the Coulomb's law for two charges stated earlier.

9 The electric field **E** at a point due to a charge configuration is the force on a small positive test charge q_0 placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude $|q|/4\pi\epsilon_0 r^2$, it is radially outwards from q, if q is positive, and radially inwards if q is negative Like Coulomb's force, electric field also satisfies superposition principle

10 A line of force is a line drawn in such a way that the tangent at each point on the line gives the direction of electric field at that point The relative closeness of lines of force indicates the strength of electric field at various places, they crowd near each other in regions of strong electric field and are far apart where the electric field is weak. In regions of constant electric field, the lines of force are parallel straight lines

11. Some of the important properties of lines of force are (a) Lines of force are continuous curves without any breaks (b) Two lines of force cannot cross at a common point (c) Lines of force for *electrostatic* fields start at positive charges and end at negative charges - they cannot form closed loops.

12 An electric dipole is a pair of equal and opposite charges q and $-q$ separated by some distance $2a$ Its dipole moment vector **p** has magnitude $2qa$ and is in the direction of the dipole axis from $-q$ to q

13. Field of an electric dipole in its equatorial plane (i e the plane perpendicular to its axis and passing through its.centre), at a distance r from the centre.

$$= \frac{-\mathbf{p}}{4\pi\epsilon_0} \frac{1}{(a^2+r^2)^{3/2}} \simeq \frac{-\mathbf{p}}{4\pi\epsilon_0 r^3} \text{ for } r \gg a$$

Dipole electric field on the axis at a distance r from the centre,

$$\simeq \frac{2\mathbf{p}}{4\pi\epsilon_0 r^3} \text{ for } r \gg a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charge.

14 In a *uniform* electric field **E**, a dipole experiences a torque τ given by,

$$\tau = \mathbf{p} \times \mathbf{E}$$

but experiences no net force.

Exercises

1.1 State Coulomb's law of force between two charges at rest. What is the force of repulsion between two charges of 1C each, kept 1m apart in vacuum?

1.2 What is the force between two small charged spheres having charges of 2×10^{-7}C and 3×10^{-7}C placed 30 cm apart in air?

1.3 The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge - 0.8 μC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

1.4 Check that the ratio $ke^2/G\ m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

1.5 (a) Explain the meaning of the statement "electric charge of a body is quantized".

(b) Why can one ignore quantization of electric charge when dealing with macroscopic i.e. large scale charges?

1.6 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

1.7 State the superposition principle for electrostatic force on a charge due to a number of charges

1.8 Four point charges $q_A = 2\mu$C, $q_B = -5\mu$C, $q_C = 2\mu$C, and $q_D = -5\mu$C are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1μC placed at the centre of the square?

1.9 Define electric field **E** at a point in space due to a distribution of charges. A point charge q is placed at the origin. How does the electric field due to the charge vary with distance r from the origin?

1.10 (a) An electrostatic line of force is a continuous curve. That is, a line of force cannot have sudden breaks. Why not?

(b) Explain why two lines of force never cross each other at any point.

1.11 Two point charges $q_A = 3\mu$C and q_B = -3μC are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-9}C is placed at this point, what is the force experienced by the test charge?

1.12 An electric dipole is placed in a uniform external electric field **E**. Show that the torque on the dipole is given by

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

where **p** is the dipole moment of the dipole. What is the net force experienced by the dipole?

1.13 A system has two charges $q_A = 2.5 \times 10^{-7}$C and $q_B = -2.5 \times 10^{-7}$C located at points A:(0,0,-15cm) and

B (0,0,+15cm) respectively What is the total charge and electric dipole moment of the system?

1.14 An electric dipole with dipole moment 4×10^{-9}Cm is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 NC^{-1}$. Calculate the magnitude of the torque acting on the dipole.

1.15 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7}C (a) Estimate the number of electrons transferred (from which to which?)

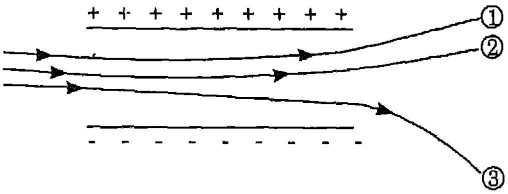
(b) Is there a transfer of mass from wool to polythene?

1.16 (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50cm What is the mutual force of electrostatic repulsion if the charge on each is $6\ 5 \times 10^{-7}$C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

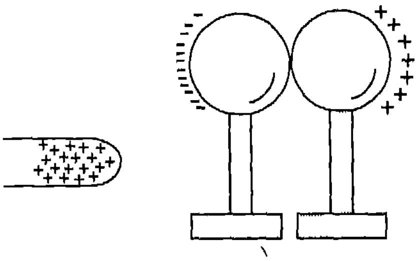
1.17 Suppose the spheres A and B in 1 16 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

1.18 Figure below shows tracks of three charged particles in a uniform electrostatic field Give the signs of the three charges Which particle has the highest charge to mass ratio?



Additional Exercises

1.19 A glass rod rubbed with silk is brought close to two uncharged metallic spheres in contact with each other, inducing charges on them as shown in the figure Describe what



happens when

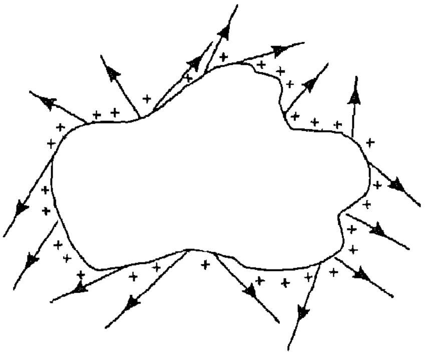
(i) the spheres are slightly separated, and

(ii) the glass rod is subsequently removed, and finally

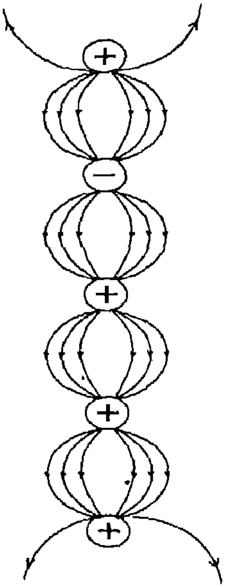
(iii) the spheres are separated far apart.

1.20 An oil drop of 12 excess electrons is held stationary under a constant electric field of $2\ 55 \times 10^4$ NC^{-1} in Millikan's oil drop experiment. (This experiment is described in Chapter 12)

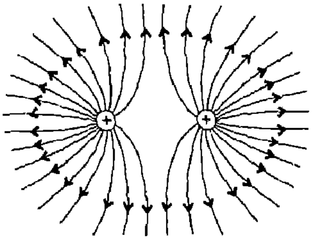
The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81 ms^{-2}$, $e = 1.60 \times 10^{-19}C$).



(a)



(b)



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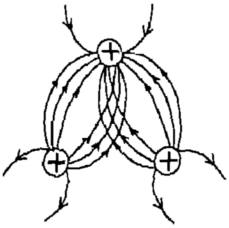

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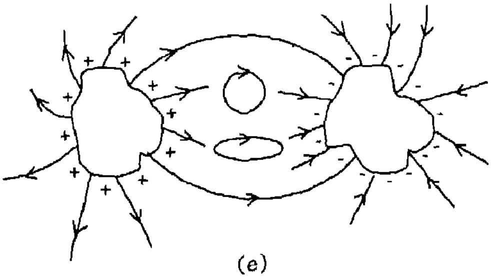

(e)

1.21 Which of the above cannot possibly represent electrostatic field lines?

1.22 In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of $10^5 NC^{-1}$ per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7}Cm in the negative z-direction?

Henry Cavendish (1731-1810)

Outstanding physicist and chemist, he became wealthy by inheritance at age 40, but remained a recluse all his life His results on electricity were unpublished and remained unknown till Maxwell edited and published them around 1875 A part from the law of electrostatic attraction and repulsion, he invented the torsion balance to measure the Newtonian gravitational constant by a laboratory experiment, and thus weighed the earth! He also did experiments on the composition of air, and properties of hydrogen The Cavendish laboratory of Cambridge University was set up in his honour in 1871

Charles Augustin de Coulomb (1736-1806)

Spent nine years as a military engineer in the West Indies. After the French Revolution, he retired to a small estate to do his scientific research His experiments on the electrostatic force, to find the quantitative form of the law of repulsion earlier stated by Priestley, were done in 1786-89 This led to Coulomb's Law He also found the inverse square law of attraction and repulsion among unlike and like magnetic poles; and worked on friction, windmills, elasticity of metal and silk fibres among other things.

Karl Friedrich Gauss (1777-1855)

He was a child prodigy and was gifted in mathematics, physics, engineering, astronomy and even land surveying The properties of numbers fascinated him, and in his work he anticipated major mathematical development of later times Along with Wilhelm Welser, he built the first electric telegraph in 1833 His mathematical theory of curved surface laid the foundation for the later work of Riemann.

CHAPTER 2

Electrostatics

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2.1 Introduction

In the previous Chapter we studied some of the basic properties of electric forces and charges. We also developed the idea of the electric field. We assumed throughout that the charges producing the field were stationary. Such electric fields belong to the subject of electrostatics. We continue our study of electrostatics in this Chapter.

First we take up the concept of electric flux and Gauss's Theorem. This theorem describes a basic property of electric fields. As we will see, we may think of it as a counterpart to Coulomb's Law In certain situations Gauss's Theorem gives us a much easier way to calculate electric fields than directly using Coulomb's Law.

After this we go on to develop the concept of the electric potential. In Chapter 1 we learnt how to pass from electric forces among charges to the idea of the electric field. Now we go one step further from the field to its potential We can think of it as a convenient and mathematically simple way to describe the field. Whereas the electric field is a vector, the potential is a scalar; so it is easier to deal with. We will see how to find the potential due to given charges. The potential is also closely related to the concept of potential energy of systems of charges, and to the work done on charges in various situations.

All this is quite similar to the passage from gravitational forces among masses to the gravitational potential produced by given masses. The similarity is enhanced by the fact that both Newton's Law of Gravitation and Coulomb's Law are inverse square laws.

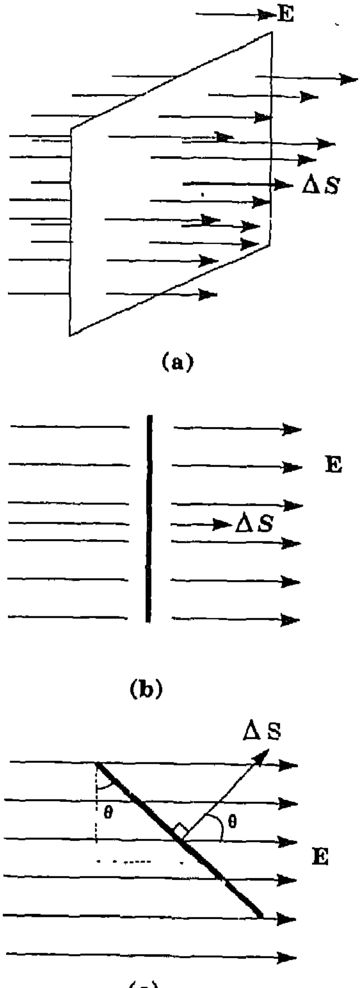
Gauss's Theorem on the one hand, and the concept of the potential on the other, together teach us many interesting properties of charges and fields in and near conductors After a look at these aspects, we turn to the subject of capacitors and capacitance. Capacitors are very important practical devices for storing electrical charge as well as energy. They are used in many appliances such as radios, television sets etc. The effect of insulators or dielectrics in capacitors is very striking. Its understanding is based on the phenomenon of polarization, which was already briefly described in the previous Chapter.

At the end of the Chapter we take up two items of general interest. One is a study of the atmosphere as an electrical system. It is quite exciting to see how all that we learn about electricity can be applied to such an enormous practical system as the atmosphere, which covers the whole earth like a blanket. The other is a simple account of man-made machines for creating extremely strong electric fields These are used to accelerate charged particles to very high speeds, for various purposes.

2.2 Electric flux and Gauss's theorem

Qualitative definition of electric flux: In Section 1.7 we saw that an electric field can be pictured by drawing lines of force. The field is strong where these lines crowd together, weak where they are far apart. Using this picture we first develop a simple qualitative understanding of the idea of *electric flux*, and later improve upon it

Consider first the simple case of a uniform electric field, and imagine a (small) plane area ΔS, say in the shape of a square or a rectangle, exactly perpendicular to the field. This is shown in Figs 2.1a,b first in perspective and then in an edge-on or cross sectional view We then define the electric flux through ΔS to be the number of lines of force passing through ΔS, that is, cutting ΔS



(a)

(b)

(c)

Figure 2.1: Flux through a small area in a uniform field

Now suppose we tilt ΔS by an angle θ, to the position shown in Fig. 2.1c. Since the field is uniform, the lines of force too are uniformly spaced So we can see easily that the number of lines of force cutting ΔS in this tilted position is less than before, by the factor $\cos\theta$. Indeed, as we keep increasing the angle of tilt until $\theta = 90^\circ$, the flux reduces to zero: in this position, no lines of force cut the area element at all!

This definition of electric flux is unsatisfactory for a simple reason – it is given in terms of the number of lines of force cutting an area, but it is up to us to choose how many lines of force to draw. We may double, or halve, this number; then the flux too would be doubled or halved.

Quantitative definition of electric flux: We can nicely solve this problem, and at the same time take care of the $\cos\theta$ factor described above, by expressing the flux directly in terms of the electric field vector $\mathbf{E}$. For this we represent the area element ΔS too by a vector $\Delta\mathbf{S}$: it is drawn perpendicular to the area, and its magnitude $|\Delta\mathbf{S}|$ is just the area ΔS. This is shown in Fig. 2.1c. The angle of tilt θ is then just the angle between the vectors $\mathbf{E}$ and $\Delta\mathbf{S}$. We write $\Delta\Phi$ for the electric flux and define:

$$\begin{aligned}\Delta\Phi &= \left\{\begin{array}{l}\text{flux through small area } \Delta\mathbf{S}\\ \text{in a uniform field } \mathbf{E}\end{array}\right.\\ &= \mathbf{E}\cdot\Delta\mathbf{S} \qquad (2.1)\end{aligned}$$

Using the definition of the scalar product of two vectors, we can write this improved expression for flux in two ways: either in terms of the magnitude of the electric field and the angle θ, or in terms of the component of $\mathbf{E}$ along $\Delta\mathbf{S}$, that is, perpendicular to the area element. This component is written as $E_\perp$ and is given by

$$E_\perp = |\mathbf{E}|\cos\theta. \qquad (2.2)$$

So we have two convenient expressions for the flux $\Delta\Phi$:

$$\begin{aligned}\Delta\Phi &= |\mathbf{E}|\Delta S\cos\theta\\ &= E_\perp\Delta S \qquad (2.3)\end{aligned}$$

The expected $\cos\theta$ dependence, the proportionality to the area $\Delta\mathbf{S}$, and to the number of lines of force representing $\mathbf{E}$, are all clearly seen. The situation is illustrated in Fig. 2.2.

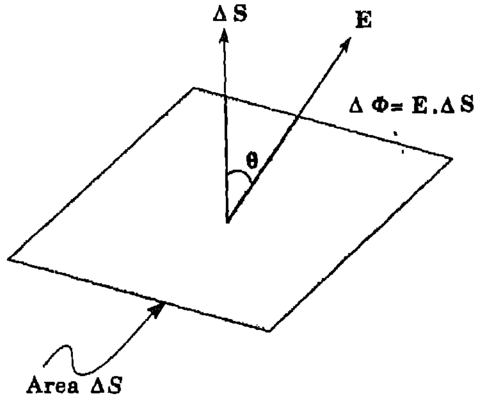


Figure 2.2: Precise definition of flux through an area element

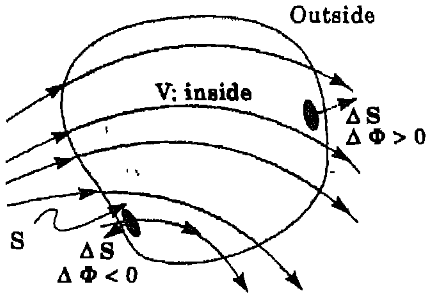


Figure 2.3: Flux through a closed surface in a general field

So far we assumed the field was uniform, and the area ΔS was small. We can now generalize in both respects. We can consider an electric field which may vary from point to point in space. We also imagine a general finite surface S drawn in the region of the field. This surface may be open or closed, but for our purpose we will assume it is closed. This means that there is an enclosed volume V (or the space *inside* S, and the space *outside* S) separated by S, as in Fig. 2.3. We want to know: how shall we define the eletric flux Φ through S in this situation?

We proceed in this way. We divide S into a large number of small area elements $\Delta\mathbf{S}$, such that over each one the field is practically uniform. The vector $\Delta\mathbf{S}$ at each element is chosen to point along the *outward* normal to S there: from the inside towards the outside. Then the total flux Φ is just the algebraic sum of small contributions $\Delta\Phi$ of the form (2.1), one from each area element:

$$\begin{aligned}\Phi &= \text{flux through general closed surface S}\\ &\quad\ \text{in general electric field}\\ &\simeq \sum_{\substack{\text{all area}\\ \text{elements}}} \mathbf{E}\cdot\Delta\mathbf{S}\\ &= \sum_{\substack{\text{all area}\\ \text{elements}}} (|\mathbf{E}|\ \Delta S\ \cos\theta \text{ or } E_{\perp}\ \Delta S)\end{aligned} \tag{2.4}$$

In the second line of Eq. (2.4), we used an approximate equality sign because we supposed that over each element $\Delta\mathbf{S}$ the field $\mathbf{E}$ is approximately uniform.

Let us now make some general remarks and then give some examples:

(a) The SI unit for electric flux is

$$\begin{aligned}[\Phi] &= \text{newton} \times (\text{meter})^2/\text{coulomb}\\ &= \text{joule} \times \text{meter}/\text{coulomb}\end{aligned} \tag{2.5}$$

(b) We chose the direction of each small vector $\Delta\mathbf{S}$ to be along the outward normal to S at that area element. This means that wherever the field $\mathbf{E}$ happens to point from the inside of S to the outside, the angle θ is less than 90° and the contribution to the flux is positive. On the other hand from those places where $\mathbf{E}$ points from the outside to the inside we get negative contributions to Φ, since θ is more than 90°. In the lines of force picture this means that lines of force *entering* a closed surface S are counted as contributing *negatively* to the flux Φ; while those *leaving* S contribute *positively*. (This choice of flux entering as negative and flux leaving as positive is a matter of convention, because we chose to draw $\Delta\mathbf{S}$ along the outward normal. But the fact that they are of opposite signs is not a matter of convention!)

(c) If all over S the angle between $\mathbf{E}$ and $\Delta\mathbf{S}$ is less than 90°, then the total flux is positive. If it is throughout more than 90°, the flux is negative. In general we can expect different area elements to make contributions of varying sign to Φ, so the total flux could be positive or negative, or sometimes zero.

★ We said that we used an approximate equality sign rather than an exact equality in Eq. (2.4) because we supposed that the field $\mathbf{E}$ is approximately constant over the area element $\Delta\mathbf{S}$. As we make these elements smaller and smaller in size, while increasing their total number, we get progressively more accurate values for the total flux Φ through the surface S. Ultimately, as each $\Delta\mathbf{S}$ approaches zero, the sum in Eq. (2.4) becomes a *surface integral* over S. So the precise definition of the total flux is written in this way

$$\begin{aligned}\Phi &= \text{total electric flux through a } S\\ &= \int_{\text{over } S} \mathbf{E}.\mathrm{d}\mathbf{S} = \int_{\text{over } S} E_{\perp}\mathrm{d}S\end{aligned} \tag{2.6}$$

★

Examples of calculating fluxes: Let us look at a couple of examples which are simple enough so that we can calculate fluxes without using integrals. Simplifications occur if the total surface S can be divided into a few parts, over each of which $E_{\perp}$ has a constant value. Then each such part contributes its $E_{\perp}$ times its area, and we simply add up such contributions.

As a first example take a cylinder of length L and uniform circular cross-sectional area A, in a uniform field. Let the axis of the cylinder be parallel to the field, as shown in Fig. 2.4. We want to calculate the total flux through the entire surface of the cylinder. By drawing the outward normals we see that $\theta = 0°$ on the flat right end face, $\theta = 180°$ on the flat left end face, and $\theta = 90°$ all over the curved surface. So the corresponding values of $E_{\perp}$, the component of $\mathbf{E}$ along the outward normal, are $E, -E$ and zero. The total flux gets equal and opposite contributions from the two end faces, and a vanishing contribution from the curved part of the cylinder. Adding these we find that the total flux vanishes

$$\Phi_c = AE - AE + 0 = 0. \tag{2.7}$$

The above example dealt with a uniform field. Now let us next look at the field of a point charge q, given in Eq. (1.19). Assume

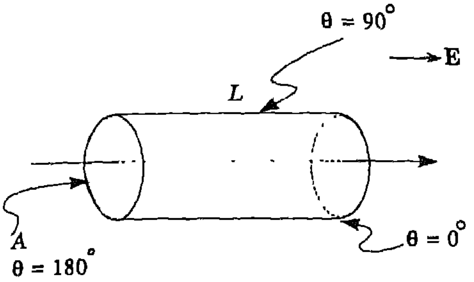


Figure 2.4: Example of flux calculation in a uniform field

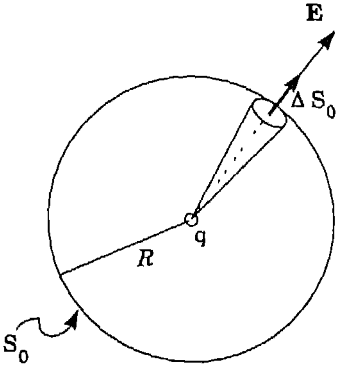


Figure 2.5: Flux through a sphere around a point charge

q is positive Let the surface S be a sphere S_0 of radius R centered on the charge. This is shown in Fig. 2.5. At each point of the sphere the electric vector points radially outward, so it is parallel to the outward normal there. This means $\theta = 0^\circ$, and the normal component $E_\perp$ is constant and equal to the field strength all over the sphere The total flux is then very easy to calculate:

$$\begin{aligned}
\Phi &= \left\{ \begin{array}{l} \text{total flux through sphere of} \\ \text{radius } R \text{ due to point charge } q \\ \text{at the centre} \end{array} \right. \\
&= \sum_{\substack{\text{all area}\\ \text{elements}}} E_\perp \Delta S \\
&= E_\perp \times \text{total area of sphere} \\
&= \frac{q}{4\pi\epsilon_0}\,\frac{1}{R^2}\,4\pi R^2 \\
&= \frac{q}{\epsilon_0} \qquad (2.8)
\end{aligned}$$

We have the remarkable result that the total flux is independent of the radius R of the sphere this is because the Coulomb's Law is an inverse *square* law

In case the charge q is negative, at all points of the sphere the field points radially inwards This means $\theta = 180^\circ$ throughout, and the flux Φ is negative However, the formula (2.8) is unchanged, as q is now negative

Gauss's theorem: We have given first an intuitive, and then a more precise, definition of electric flux We have calculated it in a few simple cases. Now let us state Gauss's Theorem.

Let S be any closed surface, and V be the enclosed volume. As usual we think of V as the inside of S, and the rest of space as the outside. Then we have

Gauss's Theorem:

Total outward electric flux Φ through closed surface S

$$= \frac{1}{\epsilon_0} \times \left\{ \begin{array}{l} \textbf{total electric charge} \\ \textbf{contained in volume} \\ V\ \textbf{enclosed by}\ S \end{array} \right\}. \qquad (2.9)$$

For the very simple case of a single point charge q at the centre of a sphere S of any radius, we have seen in Eq (2.8) that the theorem holds We will forego giving a proof of this theorem in this book. Later we will use it to calculate the electric fields in simple situations. At this point it is more useful to clearly appreciate its meaning, and look at ways to use it For these purposes, we now present several comments·

(a) According to Eq. (2.4), the calculation of the total flux Φ through a surface S involves breaking S into many small elements $\Delta\ S$, calculating the flux $\mathbf{E} \cdot \Delta \mathbf{S}$ through each, and then adding them all up Therefore at each point of S we must know the electric field $\mathbf{E}$. By the superposition principle, this $\mathbf{E}$ gets contributions from *all* source charges, both those which may be in the volume V inside S, and those which may be outside But the theorem says that as far as the *total* flux through S is concerned, only the total charge *inside* S matters! This means that charges *outside* S *do* contribute to the field $\mathbf{E}$ at each point of S, and so also to the flux $\mathbf{E}\cdot\Delta\mathbf{S}$ through each small portion of S, *but not to the total flux* Φ *at all!*

(b) All kinds of charges inside S must be taken into account on the right hand side of Gauss's Theorem, Eq. (2.9), whether they are point charges or continuous distributions.

(c) Since the total flux Φ through S involves only the total charge inside S, the locations of the charges inside do not matter. So, if we move these charges to new positions inside S, the individual flux contributions $\mathbf{E}\cdot\Delta\mathbf{S}$ may change, *but the total flux* Φ *will not change*.

(d) As we know from Coulomb's Law, the electric field of a point charge gets stronger and stronger as we move closer and closer to the charge In fact it becomes infinitely strong right at the charge. For this reason it is always necessary to ensure that the surface S does not pass through point charges. It may however cut through continuous distributions of charge, and Gauss's Theorem will be valid.

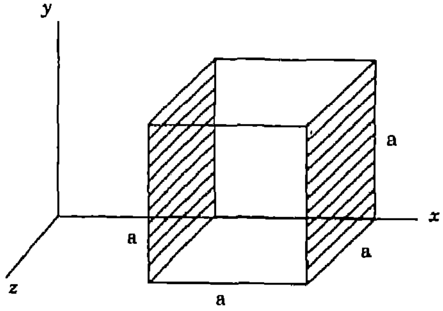
(e) In any given situation, we may choose any closed surface S we please, depending on convenience, and apply Gauss's Theorem to it. (The only restriction to remember is (*d*) above). Any such surface is called a *Gaussian surface*

At this point let us go back to the lines of force picture of electric flux and ask this question: is there any useful property of lines of force that we can learn from Gauss's Theorem? There is – it is the property of continuity of lines of force. We describe it briefly.

Suppose the total charge inside a Gaussian surface S is zero. Then the total flux Φ through S is also zero Now in the lines of force picture we saw that the lines *entering* S make a *negative* contribution to Φ, while those *leaving* S make a *positive* contribution. So if the total enclosed charge is zero, we can say that there are as many lines of force entering S as there are leaving S.

Now this is true for *any* Gaussian surface S enclosing zero total charge. The only way there can be a net outflow (or inflow) of lines of force is if the total charge inside S is positive (or negative). This leads to the following intuitive and useful property: lines of force always *begin* at *positive* charges and *end* at *negative* charges; *otherwise they are always continuous*.

Example 2.1: The electric field components in the following Figure are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 800\mathrm{N/Cm}^2$. Calculate (a) the flux Φ_E through the cube and (b) the charge within the cube Assume that $a = 0.1\mathrm{m}$



Answer: (a) Since the electric field has only an x component, for faces perpendicular to x direction, the angle between $\mathbf{E}$ and $\Delta\mathbf{S}$ is $\pm\pi/2$. Therefore, the flux $\Phi = \mathbf{E}\cdot\Delta\mathbf{S}$ is separately zero for each face of the cube except the two shaded ones. Now the magnitude of the electric field at the left face is

$$E_L = \alpha x^{1/2} = \alpha a^{1/2}$$
$$(x = a \text{ at the left face}).$$

The magnitude of electric field at the right face is

$$E_R = \alpha x^{1/2} = \alpha(2a)^{1/2},$$
$$(x = 2a \text{ at the right face}).$$

The corresponding fluxes are

$$\Phi_L = \mathbf{E}_L\cdot\Delta\mathbf{S} = E_L\Delta S\cos\theta$$

$$
\begin{aligned}
&= -E_L \Delta S \text{, since } \theta = 180^\circ \\
&= -E_L a^2 \\
\Phi_R &= \mathbf{E}_R \cdot \Delta \mathbf{S} = E_R \Delta S \cos\theta \\
&= -E_R \Delta S \text{, } \quad \text{since } \theta = 0^\circ \\
&= E_R a^2
\end{aligned}
$$

Net flux through the cube

$$
\begin{aligned}
&= \Phi_L + \Phi_R = E_R a^2 - E_L a^2 \\
&= a^2(E_R - E_L) \\
&= \alpha a^2[(2a)^{1/2} - a^{1/2}] \\
&= \alpha a^{5/2}[\sqrt{2} - 1] \\
&= 800(0.1)^{5/2}(\sqrt{2} - 1) \\
&= 1.05\,\mathrm{Nm^2/C}.
\end{aligned}
$$

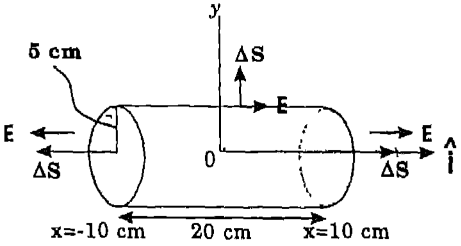
(b) We can use Gauss's theorem to find the total charge q inside the cube. We have $\Phi = q/\epsilon_0$ or $q = \Phi\epsilon_0$ Therefore

$$
\begin{aligned}
q &= 1.05 \times \frac{1}{4\pi \times 9 \times 10^9}\,\mathrm{C} \\
&= 9.27 \times 10^{-12}\,\mathrm{C}.
\end{aligned}
$$

Example 2.2 : An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x It is given that $\mathbf{E} = 200\hat{\mathbf{i}}$ N/C for $x > 0$ and $\mathbf{E} = -200\hat{\mathbf{i}}$ N/C for $x < 0$. A right circular cylinder of length 20cm and radius 5cm has its centre at the origin and its axis along the x axis so that one face is at $x = +10$cm and the other is at $x = -10$cm. (a) What is the net outward flux through each flat face? (b) What is the flux through the side of the cylinder? (c) What is the net outward flux through the cylinder? (d) What is the net charge inside the cylinder?

Answer : (a) We can see from the figure, that on the left face E and ΔS are parallel. Therefore, the outward flux is

$$
\begin{aligned}
\Phi_L &= \mathbf{E} \cdot \Delta\mathbf{S} = -200\hat{\mathbf{i}} \cdot \Delta\mathbf{S} \\
&= -200\,\Delta S \text{, since } \hat{\mathbf{i}} \cdot \Delta\mathbf{S} = \Delta S \\
&= -200 \times \pi(0.05)^2 = -1.57\,\mathrm{Nm^2/C}.
\end{aligned}
$$



On the right face, E and ΔS are parallel and therefore

$$
\begin{aligned}
\Phi_R &= \mathbf{E} \cdot \Delta\mathbf{S} \\
&= +1.57\,\mathrm{Nm^2/C}
\end{aligned}
$$

(b) For any point on the side of the cylinder $\mathbf{E}$ is perpendicular to $\Delta\mathbf{S}$ and hence $\mathbf{E} \cdot \Delta\mathbf{S} = 0$. Therefore, the flux out of the side of the cylinder is zero.

(c) Net outward flux through the cylinder

$$
= 1.57 - (-1.57) + 0 = 3.14\,\mathrm{Nm^2/C}.
$$

(d) The net charge within the cylinder can be found by using Gauss's theorem which says that

$$
\begin{aligned}
\Phi &= \frac{q}{\epsilon_0} \text{ or } q = \epsilon_0\Phi \\
&= 3.14 \times \frac{1}{4\pi \times 9 \times 10^9}\,\mathrm{C} \\
&= 2.78 \times 10^{-11}\,\mathrm{C}
\end{aligned}
$$

2.3 Applications of Gauss's theorem

For certain simple kinds of sources, the electric field can be found very easily using Gauss's Theorem. In these cases, the direct use of Coulomb's Law to find the field is more complicated. In this Section we look at some examples to see how the method works.

What are the conditions on the sources that allow us to use Gauss's Theorem in this way? The requirement is that the source charges should be distributed, or arranged in space, in a highly symmetrical way. Of course, Gauss's Theorem is true for *any* source distribution, and for *all choices* of Gaussian surface. But if there are enough simplifying features in the situation, we may be able to easily determine the *direction* of the field at each point in space. Then by an appropriate choice of Gaussian surface S we may be able to use Gauss's Theorem to find the magnitude as well We need to be able to choose S so that it is made up of a few simple portions, over each of which the perpendicular field component $E_\perp$ is known in advance to be constant This is similar to the discussion in the previous section for the calculation of electric fluxes in simple situations.

From a physical point of view we may say that Coulomb's Law and Gauss's Theorem complement one another. This can be understood as follows. If the locations and amounts of all source charges are known, then by using Coulomb's Law and the superposition principle we can (at least in principle) find the electric field at every point in space. On the other hand, suppose we happen to know the field all over space. Then Gauss's Theorem helps us to find where the source charges are located, and what their amounts are! This is because, by choosing various Gaussian surfaces S and calculating the corresponding total fluxes Φ, we would know how much charge is contained inside each S But the choice of S is completely in our hands. We can change its shape and size and location as we wish, and so finally we can "zero in" on all the source charges, whether they are point like or continuous distributions

Expressed concisely we can say: Coulomb's Law takes us from sources to their field; Gauss's Theorem takes us from the field to its sources

Here is another point worth mentioning. Though we have not done so in this book, it is possible to prove Gauss's Theorem starting from Coulomb's Law and the superposition principle. Conversely, Coulomb's Law can be derived from Gauss's Theorem and a special property of electrostatic fields which we describe later in Section 2 4. Thus we can regard either Coulomb's Law or Gauss's Theorem as basic, and then derive the other as a consequence.

In the previous Chapter (see Section 1.6) we briefly described the electric fields produced by some simple sources. uniform thin straight line of charge; uniform plane sheet of charge, uniform thin spherical shell, and solid sphere, of charge Now we shall see how Gauss's Theorem leads to those results.

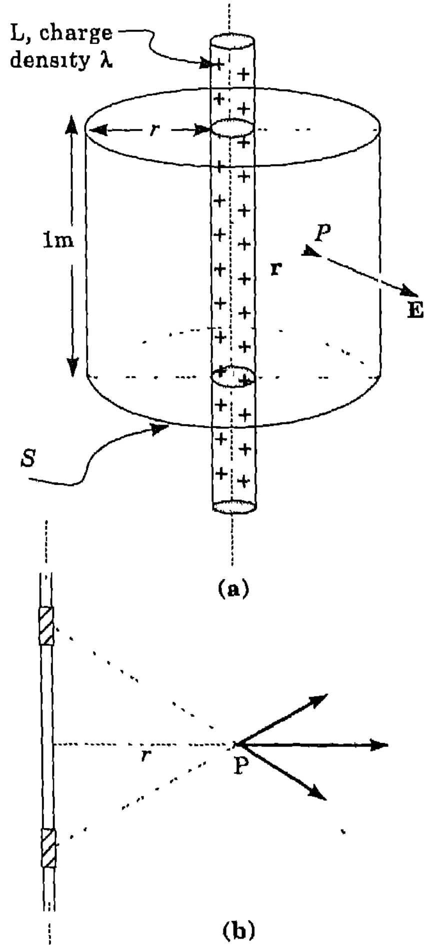


Figure 2.6: Field of a straight line of charge using Gauss's Theorem: (a) Line charge and Gaussian surface (b) Construction for showing that the electric field due to a line charge is in the radial direction

Uniform straight line of charge. This is our first example. Consider a thin infinitely long straight line L of charge, as pictured in Fig 1.12a and drawn again in Fig. 2.6a. Let the uniform charge density be λ Cm^{-1} The problem is clearly two-dimensional· conditions in all planes perpendicular to L are the same (no matter where these parallel planes cut the line of charge) Thus no physical quantity depends on the position along the length of the infinite line of charge. At any point P a perpendicular distance r away from L, we can see that the electric field **E** lies in the plane containing P, and that it points radially outwards or inwards according as the charge density is positive or negative. These properties of **E** at P are evident from Coulomb's Law. Imagine dividing the line L into many small equal segments. Then each pair of segments at equal distances above and below the plane of P will produce fields at P which add up to a resultant with the above properties, as indicated in Fig. 2.6b.

Clearly the magnitude of **E** at P, $E(r)$, can depend only on r. To find it we choose a Gaussian surface S as shown in Fig. 2.6: the surface of a circular cylinder of radius r with L as axis, and of unit length. On the flat top and bottom faces, over each small area element, **E** and $\Delta\mathbf{S}$ are perpendicular to one another so there is no electric flux through those faces. On the sides they are parallel if λ is positive, antiparallel if λ is negative. Therefore, the total outward flux Φ through S is

$$\Phi = \pm 2\pi r E(r) \text{ according as } \lambda \gtrless 0, \quad (2.10)$$

since $E(r)$ is the same at all points on the cylindrical surface, whose area is $2\pi r$ for unit height of the cylinder. As the length of the cylinder is one meter the total charge contained inside S is λ. Gauss's Theorem then gives

$$\begin{aligned} \pm 2\pi r E(r) &= \frac{\lambda}{\epsilon_0}\text{according as } \lambda \gtrless 0, \\ \text{i.e., } E(r) &= \frac{|\lambda|}{2\pi\epsilon_0}\frac{1}{r} \qquad (2\ 11) \end{aligned}$$

Thus we have found the magnitude of **E** at P (For positive λ, this result was mentioned in Eq. (1.21).) If **r** is the two-dimensional outward radial vector at P, perpendicular to L, as in Fig 2 6, we have the complete vector result

$$\mathbf{E} \text{ at P} \equiv \mathbf{E}(\mathbf{r}) = \frac{\lambda}{2\pi\epsilon_0}\frac{\mathbf{r}}{r^2}. \qquad (2.12)$$

The sign of λ fixes the direction of **E** properly.

We could have obtained this result using Coulomb's Law by adding the electric field at P due to the many small bits of charge making up the line charge L Here we have found it much more simply using Gauss's Theorem

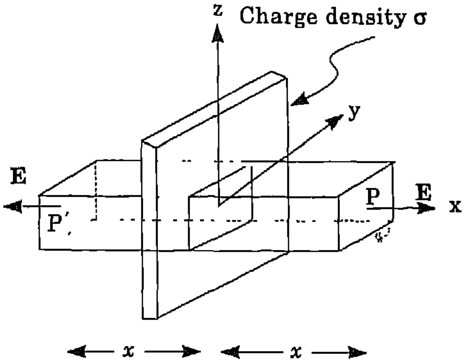


Figure 2.7: Field of a uniform plane charge sheet Only a finite segment of the charge sheet is shown

Uniform plane sheet of charge· Now we consider our second example, the field of an infinite thin plane sheet of charge Let the uniform charge density be σ Cm^{-2}. Now the problem is essentially one-dimensional since physically any point on a plane parallel to the sheet of charge is indistinguishable from any other point on it. Thus the only relevant dimension or coordinate is the distance of this plane from the sheet

As depicted in Fig. 2.7, let x be the distance from the sheet to a point P where we wish to find the field. We measure x from left to right, so it is positive (negative) for points to the right (left) of the plane (The other two coordinates y and z lie in the plane sheet of the charge.)

From the many simplifying features in this situation, we can see the following: the field at any point P has only an x–component, i.e., it is perpendicular to the plane sheet; its magnitude can depend only on the co-ordinate x; on both sides of the sheet the field points away from the sheet if $\sigma > 0$, towards the sheet if $\sigma < 0$ From all this we can conclude that the fields at points P and P′ equal distances away from the plane, on the two sides, must be equal in magnitude and opposite in direction.

As a Gaussian surface S we take a rectangular parallelipiped of unit cross–sectional area, with axis perpendicular to the sheet of charge, and extending a distance $|x|$ on either side of the sheet Now only the two end faces (shown shaded) contribute to the total outward flux through S, while the sides contribute nothing The field $E(x)$ is parallel to the outward normals of the two unit area end faces, and is constant in magnitude over them,, so the total flux is

$$\begin{aligned}\Phi &= \pm(2E(x)) \times (1)\\ &\quad \text{according as } \sigma \gtrless 0,\\ &\quad \text{since } |\mathbf{E} \text{ at P}| = |\mathbf{E} \text{ at P}'|\\ &= E(x) \qquad (2\,13)\end{aligned}$$

The total charge contained inside S is exactly σ, so Gauss's Theorem gives

$$\begin{aligned}\pm 2E(x) &= \frac{\sigma}{\epsilon_0}\\ &\quad \text{according as } \sigma \gtrless 0,\\ \text{i.e.}, E(x) &= \frac{|\sigma|}{2\epsilon_0}. \qquad (2\,14)\end{aligned}$$

Let $\hat{\mathbf{x}}$ be a unit vector perpendicular to the sheet pointing in the direction of increasing x, i.e., from left to right. Then we can write the field in vector form as:

$$\begin{aligned}\mathbf{E} &= \frac{\sigma}{2\epsilon_0}\hat{\mathbf{x}}, \quad x > 0;\\ \mathbf{E} &= -\frac{\sigma}{2\epsilon_0}\hat{\mathbf{x}}, \quad x < 0. \qquad (2.15)\end{aligned}$$

We can check that the direction of the field is correctly given by taking into account the sign of the charge density σ (For positive σ, these results agree with Eq. (1.22)).

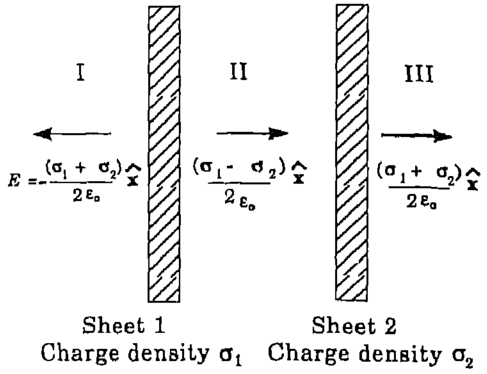

I
II
III
$E = -\frac{(\sigma_1 + \sigma_2)}{2\varepsilon_0}\hat{x}$
$\frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}\hat{x}$
$\frac{(\sigma_1 + \sigma_2)}{2\varepsilon_0}\hat{x}$
Sheet 1
Charge density σ_1
Sheet 2
Charge density σ_2

Figure 2.8: Field due to two infinite plane parallel sheets of charge ($\sigma_1 > \sigma_2 > 0$ case) Only a section of a finite part of the two sheets is shown

As another example similar to the above, suppose you have two thin plane parallel sheets of charge. Let them carry uniform charge densities σ_1 and σ_2 C m^{-2}. Suppose sheet 1 is to the left of sheet 2, as in Fig 2.8; what will the field be? We can find it using Eq (2 15) and the superposition principle, that is, adding the fields separately produced by each sheet

There are clearly three regions, and in each the field has a constant magnitude and direction. region I to the left of sheet 1; region II between the two sheets; and region III to the right of sheet 2. As in the previous Fig. 2.7, here again we measure the coordinate x from left to right Then using Eq. (2.15) for field due to each sheet, we get the total field in each region.

$$\begin{aligned}\text{Region I}\cdot \mathbf{E} &= -\frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)\hat{\mathbf{x}};\\ \text{Region II}: \mathbf{E} &= \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)\hat{\mathbf{x}};\\ \text{Region III}: \mathbf{E} &= \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)\,\hat{\mathbf{x}}. \qquad (2.16)\end{aligned}$$

Each charge density σ_1 and σ_2 could independently be positive or negative. For definiteness in Fig 2.8 we have supposed that $\sigma_1 > \sigma_2 > 0$.

An interesting special case arises if $\sigma_1 = \sigma$,

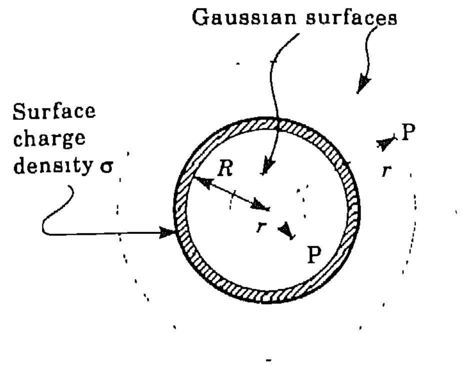


Figure 2.9: Field of a thin uniform spherical shell of charge

$\sigma_2 = -\sigma$, say. Then we have two infinite plane parallel sheets with equal and opposite uniform charge densities. In this case we find that the field is non-zero only between the two sheets, and there it is constant, and points from the positive to the negative sheet. The strength of this constant field, $|\sigma|/\epsilon_0$, is independent of the distance between the sheets. This is then a practical way of producing such uniform fields, at least over a small region of space. We must just make sure that in practice the sheets are much larger than the linear dimensions of this region.

Uniform spherical shell and solid sphere of charge: As another set of applications of Gauss's Theorem, we look at two cases having spherical symmetry. Consider first a thin spherical shell of radius R with a uniform surface charge density $\sigma \,\mathrm{C\,m^{-2}}$. The situation is depicted in Fig. 2.9. We can appreciate that at any point P the field is purely radial. If the charge density σ is positive, at least outside the shell we expect the field to point radially outwards, while if σ is negative, it would point inwards. In either case, the magnitude of the field can only depend on the distance r of the point P from the centre of the shell, so we write $E(r)$ for it. All this results from the spherical symmetry: sitting at the centre of the shell, we see that all directions are alike.

We have to consider two regions: points P inside the shell, for which $r < R$, and points outside, where $r > R$. In either case we choose the Gaussian surface S to be a sphere of radius r, concentric with the shell. All over the sphere, the field and the vector $\Delta\mathbf{S}$ representing each small portion of S are either parallel or antiparallel. So the total outward flux through S is

$$\Phi = \pm 4\pi r^2 E(r) \quad \text{according as } \sigma \gtrless 0,$$

$$E(r) = |\mathbf{E}| \left\{ \begin{array}{l} \text{at any point P distance } r \\ \text{from the centre of shell} \end{array} \right. \tag{2.17}$$

(Actually, the signs given here for Φ are proper only if $r > R$, but we will see in a moment that there is no problem if $r < R$.) Next, the total charge contained inside S is clearly zero if $r < R$, while it is $4\pi R^2 \sigma$ if $r > R$. Therefore Gauss's Theorem gives us $E(r)$ in each region as:

Interior points, $r < R$: $E(r) = 0$,

Exterior points, $r > R$:

$$\pm 4\pi r^2 E(r) = \frac{4\pi R^2 \sigma}{\epsilon_0}$$

according as $\sigma \gtrless 0$,

$$\text{i.e., } E(r) = \frac{|\sigma|}{\epsilon_0}\frac{R^2}{r^2} \tag{2.18}$$

We see that *inside* the shell the field vanishes! For points outside, it is better to express the field in terms of the total charge on the shell, $q = 4\pi R^2 \sigma$, rather than the density σ. We then see that in complete vector form the electric field is given by

$$r < R \;;\; \mathbf{E} = 0,$$

$$r > R \;;\; \mathbf{E} \text{ at P} = \mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}}{r^3}. \tag{2.19}$$

The inward or outward radial direction of $\mathbf{E}$ is correctly given by the sign of the charge q. We find that at points *outside* the shell of charge, the field is as though the entire charge q had been concentrated at the origin! (This was mentioned in connection with Eq. (1.22).)

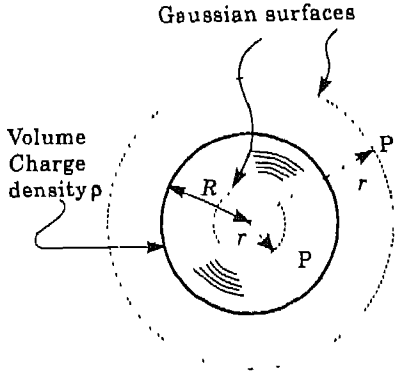


Figure 2.10: Field due to solid uniformly charged sphere

This result that the field inside a uniformly charged spherical shell vanishes identically is a particular and simple example of what is called "electrostatic shielding". It is a consequence of the precise inverse *square* law nature of electrostatic forces. We shall study shielding in more detail in Section 2 6.

Next, we look at the case of a solid sphere of radius R. Let a total charge q be uniformly distributed throughout its volume. This is shown in Fig. 2.10. The volume density of charge is $\rho = q/\frac{4}{3}\pi R^3$ Cm^{-3}. As in the spherical shell case, we expect the field to be purely radial, and its magnitude to depend on the radial distance r alone. The total flux through any spherical Gaussian surface S of radius r is given again by Eq. (2.17) (with σ replaced by ρ) The difference is this: both for $r < R$ and for $r > R$, there is a non zero amount of charge enclosed by S . So now Gauss's Theorem tells us, since the charge enclosed is the charge density times the volume of the sphere of radius r:

$$
\begin{aligned}
r < R : \pm\ 4\pi r^2 E(r) &= \frac{1}{\epsilon_0}\frac{4}{3}\pi r^3 \rho \\
&\quad \text{according as } \rho \gtrless 0, \\
\text{i.e., } E(r) &= \frac{|\rho|}{3\epsilon_0} r, \\
r \geq R .\ \pm\ 4\pi r^2 E(r) &= \frac{1}{\epsilon_0}\frac{4}{3}\pi R^3 \rho \\
&\quad \text{according as } \rho \gtrless 0, \\
\text{i.e., } E(r) &= \frac{|\rho|}{3\epsilon_0}\frac{R^3}{r^2} \\
&= \frac{|q|}{4\pi\epsilon_0}\frac{1}{r^2} \qquad (2.20)
\end{aligned}
$$

For points outside the sphere, we have given the magnitude of the field both in terms of density ρ and total charge q These results can next be converted into complete vector expressions:

$$
\begin{aligned}
r < R .\ \ \mathbf{E} \text{ at P} &= \mathbf{E}(\mathbf{r}) \\
&= \frac{\rho}{3\epsilon_0}\mathbf{r}; \\
r \geq R \ \ \mathbf{E} \text{ at P} &= \mathbf{E}(\mathbf{r}) = \frac{\rho}{3\epsilon_0} R^3 \frac{\mathbf{r}}{r^3} \\
&= \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}}{r^3} \qquad (2.21)
\end{aligned}
$$

These expressions match at the surface.

If we compare the spherical shell and the solid sphere, Eqs (2.19 and 2 21), we see that in both cases the field at points outside is the same – it is as if the entire charge were concentrated at the centre. But for interior points the two cases differ. For the shell, the field vanishes For the solid sphere, the field decreases gradually from its value at the surface to zero at the centre.

★ It is quite generally true that if we have a spherically symmetric charge distribution more general than a shell or a solid uniform sphere, the field at outside points is as if the entire charge were concentrated at the centre. What we are allowing here is a dependence of the volume charge density on the radial distance r. It took Newton almost twenty years to prove the analogous result for gravitation This caused a corresponding delay in the publication of his great book, the *Principia*!

In the example of a line charge treated earlier, we assumed we had a *thin* line of charge. This is of course in comparison to the distances at which we calculate the field. Now a simple extension of the previous treatment helps us to tackle the case of an infinitely long straight uniformly charged cylinder of *finite* thickness, of radius R. Let the

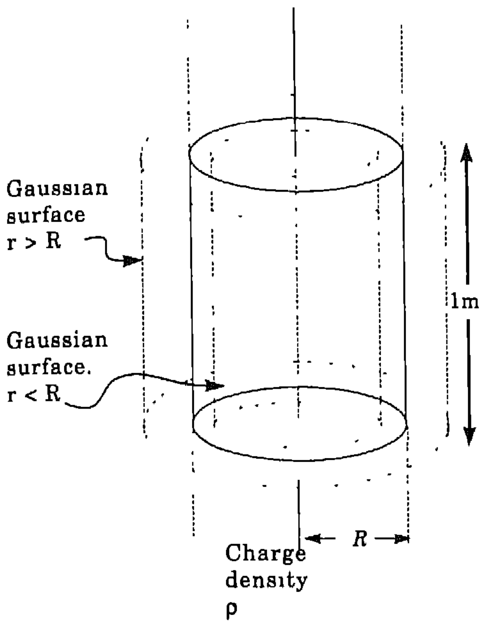


Figure 2.11: Field due to infinite cylinder of charge choice of Gaussian surfaces

volume density of charge now be ρ C m^{-3} The problem is again essentially two–dimensional, and the field again purely radial

We would like to find $\mathbf{E}(\mathbf{r})$ both inside and outside the cylinder, i.e , for $r < R$ as well as for $r \geq R$ As shown in Fig 2 11, in either case we use a Gaussian surface S of the same cylinder kind as before of radius r and of unit length. In each situation, the total outward flux through S and the charge enclosed by S, are easily calculated Using Gauss's Theorem we then find

$$r \leq R \qquad \Phi = \pm 2\pi r E(r) = \frac{1}{\epsilon_0}\pi r^2 \rho ,$$

$$\text{i e , } E(r) = \frac{|\rho|}{2\epsilon_0} r, \qquad (2\,22\text{a})$$

$$r \geq R \qquad \Phi = \pm 2\pi r E(r) = \frac{1}{\epsilon_0}\pi R^2 \rho ,$$

$$\text{i e , } E(r) = \frac{|\rho|}{2\epsilon_0}\frac{R^2}{r} . \qquad (2\,22\text{b})$$

Here we use the upper sign for positive ρ and the lower sign for negative ρ In vector form the complete electric field is given by

$$r \leq R \qquad \mathbf{E}(\mathbf{r}) = \frac{\rho}{2\epsilon_0}\mathbf{r}, \qquad (2.23\text{a})$$

$$r \geq R \qquad \mathbf{E}(\mathbf{r}) = \frac{\rho}{2\epsilon_0}\frac{R^2}{r^2}\mathbf{r} \qquad (2\,23\text{b})$$

In contrast to Eqs (2 11, 12) for the case of a thin line of charge, here the field is finite at all points, even as r approaches zero. This is because we have smeared out the charge into a volume distribution with finite density. For $r \geq R$, outside the cylinder, we may identify $\pi R^2 \rho$ with the previous line charge density λ Then the fields in Eqs (2 12, 23b) agree This is physically reasonable ★

Let us conclude this Section with a remark. If we have a single point charge, then as a particular case of the results for a spherical shell or a solid sphere, we see that spherical symmetry plus Gauss's Theorem immediately gives us the Coloumb Law But already for two point charges there is not enough symmetry to allow efficient use of Gauss's Theorem! We have to fall back on Coulomb's Law and the superposition principle to get the complete electric field

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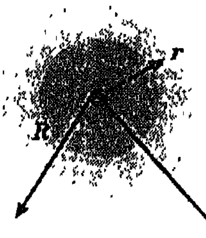
Example 2.3 : An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge upto a radius R The atom as a whole is neutral For this model, what is the electric field at a distance r from the nucleus?



Answer : The charge distribution for this model ~~of the atom~~ is as shown in the figure

of Education

The total negative charge in the uniform spherical charge distribution of radius R must be $-Ze$, since the atom (nucleus of charge Ze + negative charge) is neutral This immediately gives us the negative charge density ρ, since we must have

$$Ze + \frac{4\pi R^3}{3}\rho = 0$$

or

$$\rho = -\frac{3Ze}{4\pi R^3} \qquad (1)$$

To find the electric field $\mathbf{E}(\mathbf{r})$ at a point P which is a distance r away from the nucleus, we use Gauss's theorem Because of the spherical symmetry of the charge distribution, the magnitude of the electric field $\mathbf{E}(\mathbf{r})$ depends only on the radial distance, no matter what the direction of $\mathbf{r}$. Its direction is along (or opposite to) the radius vector $\mathbf{r}$ from the origin to the point P The obvious Gaussian surface is a spherical surface centred at the nucleus We consider two situations, namely, $r < R$ and $r > R$

(a) $r < R$. The electric flux Φ enclosed by the spherical surface is

$$\Phi = E(r)\ \ 4\pi r^2 \qquad (2)$$

where $E(r)$ is the magnitude of the electric field at r. This is because the field at any point on the spherical Gaussian surface has the same direction as the normal to the surface there, and has the same magnitude at all points on the surface.

The charge Q enclosed by the Gaussian surface is the positive nuclear charge and the negative charge within the sphere of radius r, i e ,

$$Q = Ze + \frac{4\pi r^3}{3}\rho$$

Substituting from Eq. (1) above, we have the total charge to be

$$Q = Ze - Ze\frac{r^3}{R^3} \qquad (3)$$

Thus, from Gauss theorem, Eq. (2.9) and Eqs. (2) and (3), we find

$$E(r) = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right), \qquad r < R$$

The electric field is directed radially outward

(b) $r > R$. For $r > R$, the total charge enclosed by the Gaussian spherical surface is zero since the atom is neutral. Thus from Gauss's theorem,

$$E(r)4\pi r^2 = 0$$

or

$$E(r) = 0; \quad r > R. \qquad (4)$$

2.4 The electrostatic potential

We know that the electric field is a vector. At each point in space it has a magnitude and a direction The use of lines of force helps us to visualize its nature and variations in space in a qualitative way.

Now we develop the idea of the *electrostatic potential* This is a simpler concept than the electric field It is a scalar. So at each point in space the potential is just a single number. We will, however, see that in the static situation the potential contains just as much information as the field. The reason for this is a special property of electrostatic fields to be described later. This property is as fundamental as that expressed by Gauss's Theorem

We will first learn how to define and find the potential if the field is known Then at the next stage we will see how the field can be determined if the potential is known.

Potential as work done on test charge: Let us suppose an electrostatic field, produced by some sources, is given. We want to define two things: the potential difference between any two points in space; and the potential at each point in space

Let A and B be any two points in space Imagine carrying a unit test charge from point B to point A along some continuous path. At each point along the path an electric force acts on the charge. It is the product of the field there and the amount of charge. We have to *just overcome* this force, by applying an opposite force ourselves, and then make the

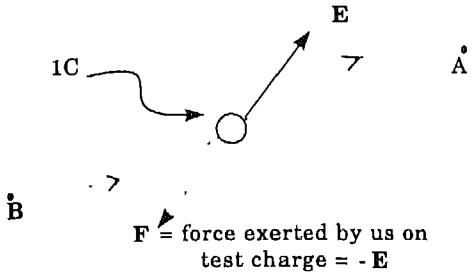


Figure 2.12: Transporting a unit charge from point B to point A in a given field

charge travel along the path from B to A. In this process we have to do a certain amount of work. The potential difference between points A and B is defined in terms of this work. The situation is illustrated in Fig. 2.12.

We use the Greek letter φ, (and sometimes later the letter V) to denote the potential. As the test charge is exactly one coulomb, we say Electrostatic potential difference between point A and point B

$$\begin{aligned} &\equiv \varphi(\mathrm{A}) - \varphi(\mathrm{B}) \\ &= \text{work done in carrying} \\ &\quad \text{unit test charge from B to A.} \qquad (2.24) \end{aligned}$$

In practice we know that one coulomb of charge is a really enormous amount. So we re-express our definition of potential difference in another way. We take a test charge of amount q_0, which can be as small as we wish. In taking it from B to A let the amount of work done by us be W_{BA}. It is clear that at each point along the path the force we have to overcome is proportional to q_0. Therefore the work done, W_{BA}, is also proportional to q_0. Then the potential difference between points A and B can be also expressed this way:

$$\begin{aligned} W_{\mathrm{BA}} &= \text{work done in carrying} \\ &\quad \text{test charge } q_0 \\ &\quad \text{from B to A} \Longrightarrow \\ \varphi(\mathrm{A}) - \varphi(\mathrm{B}) &= W_{\mathrm{BA}}/q_0. \qquad (2.25) \end{aligned}$$

It is important to stress that in the above process of transporting a charge and seeing how much work we do, we must keep it in equilibrium all along. We must *just* counter balance the electric force acting on it at each point, to the extent of gently nudging it along the path we want it to follow. There should be no wasted effort! The work done by us may be positive or negative–physically, in the former case we really exert ourselves, in the latter, we could extract work from the charge to our advantage. Later examples will illustrate both situations.

The definitions we have given in Eqs (2.24, 25) give the difference in potentials at any two points. To set up the idea of the potential at a point, we need a convention. We must agree to say that the potential is zero at some conveniently chosen point. Then the potential will have a definite value at every other point. Usually we agree that the potential vanishes at points in space infinitely far away from all sources. (There are some cases where this may not be possible, namely when the sources themselves extend to infinity. But these can be handled separately.) Let us suppose we are able to do so. Then by imagining the point B in Eqs (2.24, 25) to be gradually taken away to infinity, we get the potential $\varphi(\mathrm{A})$ at a general point A in space

$$\begin{aligned} &\text{Electrostatic potential at a point A} \equiv \varphi(\mathrm{A}) \\ &\quad = \text{work done per unit test charge} \\ &\quad\quad \text{in transporting it from infinitely} \\ &\quad\quad \text{far away to the point A} \qquad (2.26) \end{aligned}$$

This too could in general be either positive or negative.

We can easily understand why the potential is a scalar quantity: it is because work is such a quantity. Thus for a given arrangement of sources which produces a certain field, the potential at each point is just a number, unlike the electric field. Expressed concisely: the electric field at a point in space tells us what force a test charge would feel if it were placed there, the potential tells us how much work we have to do to bring it there from very far away.

SI unit for the potential–the volt· The dimensions of the potential φ are determined by the definitions (2.25, 26): since we have to calculate work per unit charge, we get joules per coulomb for φ Equally well, since one joule is the product of one newton by one metre, we get the dimensions (newtons per coulomb) × metre for φ, which is the product of electric field and distance or length. This SI unit for potential is called the *volt*, after the Italian scientist Alessandro Volta·

$$1 \text{ volt} = 1 \text{ Nm C}^{-1} = 1 \text{ JC}^{-1} \qquad (2.27)$$

So, we measure electric potential and differences in potential in volts The symbol for volt is V.

Suppose we have a six-volt battery This means we would have to do six joules of work to carry one coulomb of charge from the negative to the positive terminal of the battery.

Path independence of work and potential· Now we come to an important point which may have puzzled you. We have defined potential difference (and potential) in terms of work done on a test charge while moving it along a path from one point to another. How can we be sure that this work does not depend on the particular path we choose, but depends only on the end points of the path? Indeed in Eqs (2 24, 25, 26) we have anticipated that this work is the difference of something defined at the point A and something at the point B, quite independent of the intermediate points along which we carry the test charge!

Now this is the very special property of electrostatic fields which we mentioned at the beginning of this Section. It can be shown that in such a field the work done in taking a test charge from a point B to another point A *does not depend at all on the path we follow*. It depends only on the end points A and B, and we *can* express it as the difference of two terms as written in Eqs (2 24, 25) For the simple case of the field produced by a single point charge, we will outline a proof of this later on. It is because of this property that we are able to define the electrostatic potential, and differences of potential, the way we have done above.

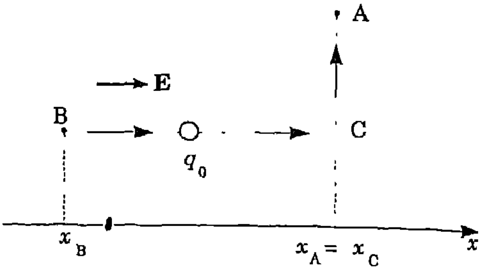


Figure 2.13: Calculating potential for uniform field

Examples of calculating the potential. We shall consider now two examples: a uniform electric field, and the radial field of a point charge. Later on we shall look at some other important examples.

Uniform field: Let a uniform field of strength E point in, say, the x-direction So in three-dimensional vector form the components of the field are

$$\mathbf{E} = (E, 0, 0) \qquad (2.28)$$

Take any two points A and B in space, as shown in Fig. 2.13. Let their x-coordinates i e x_A and x_B respectively. (We will see that the values of the other coordinates do not matter.) Imagine carrying a test charge q_0 from B to A along the path shown in the figure. first go to point C in a straight line parallel to **E**, then on to A along another straight line perpendicular to **E** (Thus C has the same x-coordinate as A, namely x_A) We choose this path from B to A since we are free to choose any one; and this one is the simplest for calculating the work done.

Along BC we have to apply a constant force $-q_0$ **E**, and this is opposite to the actual displacement from B to C Along CA the force we apply is perpendicular to the displacement. So we find:

Work done from B to C =

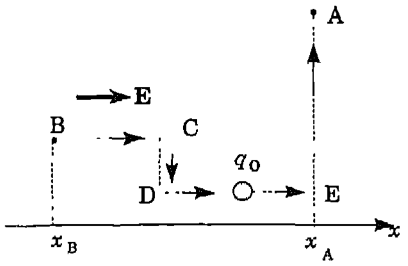


Figure 2.14: Alternative path in uniform field

$$-q_0 E(x_A - x_B),$$

Work done from C to A = 0;

$$W_{BA} = \left\{ \begin{array}{l} \text{work done from B to A along} \\ \textit{any} \text{ path} \end{array} \right.$$

$$= -q_0 E(x_A - x_B). \qquad (2.29)$$

We compare this with Eqs (2.24, 25), divide by q_0, and conclude that we may take the potential at the general point A to be

$$\varphi(A) = -E x_A \qquad (2.30)$$

This is a very simple result: the potential *increases* steadily as we move *against* the field, it *decreases* if we move *along* the field Notice that we have in effect agreed to set $\varphi = 0$ all over the plane $x = 0$ which is perpendicular to $\mathbf{E}$ It is clear that in this case we *cannot* make the convention that the potential vanishes at points infinitely far away!

Another sample path from point B to point A is the path BCDEA shown in Fig. 2.14. You can see that no work is done on the stretches CD and EA, while the total work done on BC and DE is the same amount, W_{BA}, found in Eq. (2.29). This should reinforce the statement that the work done in an electrostatic field does not depend on the path at all.

Point charge field: Let a point A be located at a distance r_0 from a point charge q. We want to find the potential at A For this we must calculate how much work we have to do, W_A say, to bring in a test charge q_0 from infinitely far away to the point A. As depicted in Fig 2.15, we bring q_0 in from infinity to A along the straight radial line connecting q and A, since this is geometrically the simplest path from infinity to A So we travel radially inwards along the dotted line

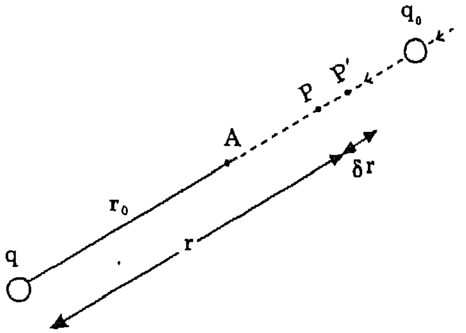


Figure 2.15: Calculating the potential for the field of a point charge

We break up the (semi infinite) straight line from infinity to A into small stretches, and find out how much work is done over each stretch Let P and P′ be two nearby points on the way, at distances r and $r + \delta r$ respectively from q. The electric field $\mathbf{E}$ at the point P, the force $\mathbf{F}$ applied by us when the test charge is at this segment, and the contribution to W_A from this portion, are all familiar by now

$$\begin{aligned} \mathbf{E} \text{ at P} &= \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}}{r^3}; \\ \mathbf{F} \text{ over segment P}' \text{ P} &= -q_0\mathbf{E} \\ &= -\frac{q_0 q}{4\pi\epsilon_0}\frac{\mathbf{r}}{r^3}; \end{aligned}$$

$$\begin{aligned} &\delta W_A \text{ (P}' \text{ to P)} \\ &= \text{work done over the segment P}' \text{ to P} \\ &= \mathbf{F} \cdot \mathbf{P'P} \\ &= \left(-\frac{q_0 q}{4\pi\epsilon_0}\right)\left(\frac{\mathbf{r}}{r^3}\right)\cdot\left(-\frac{\mathbf{r}}{r}\delta r\right) ; \\ &= \frac{q_0 q}{4\pi\epsilon_0}\frac{\delta r}{r^2} \qquad (2.31) \end{aligned}$$

Here we have (naturally!) denoted the vector from q to P by $\mathbf{r}$ We can reassure ourselves that the sign here is correct: suppose q and q_0 are both positive (or both negative). Then they repel one another, and we surely have to exert ourselves and do a positive amount of work, to bring them nearer each other!

Now this small contribution δW_{A} (P$'$ to P) to W_{A} should be the difference of two terms, one dependent on the point P and the other on P$'$. Guided by this, and also remembering that δr is a very small distance compared to r, we try to express $\delta r/r^2$, which occurs in Eq. (2.31), as approximately a difference. This is interesting and easy. We write the numerator as a difference, $\delta r = r + \delta r - r$. Then as δr is very small compared to r, we can replace r^2 in the denominator approximately by the product $r(r+\delta r)$. This then gives the result

$$\begin{aligned}\frac{\delta r}{r^2} &\simeq \frac{r+\delta r - r}{r(r+\delta r)}\\ &= \frac{1}{r} - \frac{1}{r+\delta r} \qquad (2.32)\end{aligned}$$

Using this in Eq. (2.31) we see that the small amount of work done in bringing this test charge q_0 from P$'$ to P is the difference of two terms.

$$\begin{aligned}\delta \mathrm{W}_A(\mathrm{P}' \text{ to } \mathrm{P}) &= \frac{q_0 q}{4\pi\epsilon_0}\frac{\delta r}{r^2}\\ &\simeq \frac{q_0 q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r+\delta r}\right) \quad (2.33)\end{aligned}$$

Apart from the factor $q_0 q/4\pi\epsilon_0$, this is the reciprocal of the distance from q to P minus the reciprocal of the distance from q to P$'$. To get the total work W_{A} done in travelling all the way from infinity to the point A, we have to add up infinitely many such small amounts, one for each little segment. And each such piece is a difference expression. When we do this we see that "neighbouring terms" coming from "neighbouring segments" neatly cancel, two at a time. That was the whole point of expressing δW_{A} (P$'$ to P) as a difference! We choose a series of closely separated points P$_1$, P$_2$, P$_3$, .. from A radially outwards all the way to infinity, at distances $r_1, r_2, r_3,$ .. from q. Then repeatedly using Eq. (2.33) we end up with

$$\begin{aligned}W_{\mathrm{A}} &= \left\{\begin{array}{l}\text{work done in bringing a}\\ \text{test charge } q_0 \text{ from infinity}\\ \text{to A along any path}\end{array}\right.\\
&= \sum_{\substack{\text{all segments along}\\ \text{inward radial path}}} \delta W_{\mathrm{A}}(\mathrm{P}'\text{to P})\\
&= \delta W_{\mathrm{A}}(\mathrm{P}_1\text{to A}) + \delta W_{\mathrm{A}}(\mathrm{P}_2\text{to P}_1) +\\ &\quad \delta W_{\mathrm{A}}(\mathrm{P}_3\text{to P}_2) + \ldots\\
&= \frac{q_0 q}{4\pi\epsilon_0}\left(\frac{1}{r_0} - \frac{1}{r_1} + \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2}\right.\\ &\quad \left. - \frac{1}{r_3} + \ldots\right)\\
&= \frac{q_0 q}{4\pi\epsilon_0}\frac{1}{r_0} \qquad (2.34)\end{aligned}$$

And dividing this by q_0 as well as replacing r_0 by r now for simplicity, we get the potential φ at A:

$$\begin{aligned}\varphi(\mathrm{A}) &= \left\{\begin{array}{l}\text{electrostatic potential at a}\\ \text{point A distance } r\\ \text{from point charge } q\end{array}\right.\\ &= \frac{q}{4\pi\epsilon_0}\frac{1}{r} \qquad (2.35)\end{aligned}$$

As anticipated, this potential vanishes if the point A is infinitely far away.

Comparing the potential and the field of a point charge, some interesting features can be seen. We know that the field decreases as the inverse square of the distance as we move away from the source, in contrast the potential decreases more slowly, only as the inverse of the distance. Conversely, as we come close to the source, both field and potential become very large in magnitude. However, the field increases more rapidly than the potential.

We can see from Eq. (2.35) that for a positive source, the potential is positive everywhere. For a negative source it is negative everywhere. These make good physical sense. For, if q is greater than zero and we bring in a unit positive test charge from infinity to a distance r from q, we have to overcome the force of repulsion between the two charges and definitely do some work. So in this case φ is positive. On the other hand, if q were negative, the test charge would be continually attracted to the source, so we can benefit from and "get work out of it" while bringing it in from infinity. In other words in this case the work done by us is negative!

Proof of path independence - case of field of point charge - general field. We now sketch the proof of the path independence of work done in moving a test charge in the field of a point charge, by looking at some sample paths. As shown in Figs 2.16, we can clearly assume that the source charge q, and the points B and A between which we carry the test charge, all lie in the plane of the paper.

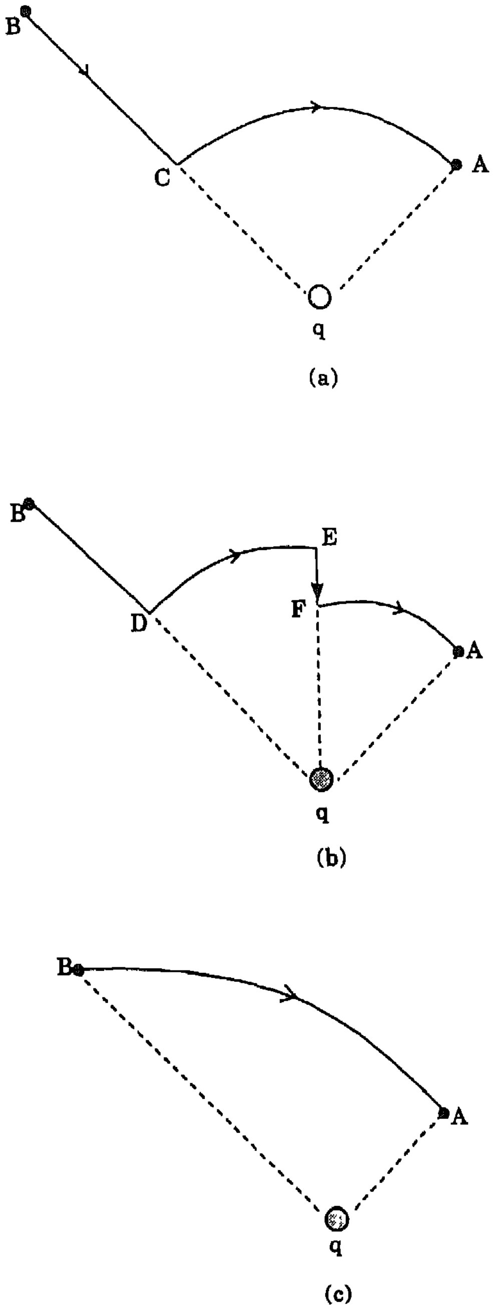
Fig. 2.16a shows a very simple path: a radial segment BC, then a circular arc CA. Work is done only on the former, not on the latter segment since there the force we apply is perpendicular to the displacement. Another possible path is BDEFA in Fig. 2.16b: here we have two radial portions BD and EF, two circular arcs DE and FA. Again on the latter no work is done. On the other hand, appealing to our results while deriving Eqs (2.33, 34, 35) we see that the work done along the radial segment BC equals the work done along BD plus that along EF. So we have

$$\begin{aligned}&\text{Work done along CA or DE or FA} = 0;\\ &\text{Work done along BC} =\\ &\quad\begin{cases}\text{work done along BD}+\\ \text{work done along EF,}\end{cases}\\ &\text{work done along BCA} =\\ &\quad\text{work done along BDEFA.} \qquad (2.36)\end{aligned}$$

We can keep increasing the number of alternately radial and circular segments, and the above kind of argument applies again and again. Finally, any smooth path from B to A as shown in Fig. 2.16c can be approximated by a series of short radial and circular pieces, and we will find that the work done along such a path is just the same as along BCA!

For simplicity we assumed in Figs 2.16 that the paths from B to A all lay in the plane determined by q, A and B. But with a little thought you can convince yourself that the argument will work for *any* path from B to A, even one that leaps out of the page at you, or ducks under it away from you!

This proof of path independence of work done can be extended to the field produced by



(a)

(b)

(c)

Figure 2.16: Path-independence of work done - field of point charge

any distribution of source charges, by making use of the superposition principle Without giving a detailed proof, we indicate the main statements to give you an idea of the argument'

$$W_{\mathrm{BA}} = \left\{ \begin{array}{l} \text{work done in going from B to} \\ \text{A along a given path, in field} \\ \text{of any source distribution} \end{array} \right.$$

$$= \sum_{\substack{\text{all point} \\ \text{source} \\ \text{charges}}} \left\{ \begin{array}{l} \text{work done in going} \\ \text{from B to A along} \\ \text{given path, in field} \\ \text{of each single point} \\ \text{source charge} \end{array} \right\} \tag{2.37}$$

It is in this statement that we need the superposition principle But we have just seen that each contribution on the right hand side here is independent of the path So this property holds also for the total work W_{BA}! We express it explicitly as follows

$$W_{\mathrm{BA}} = \left\{ \begin{array}{l} \text{work done in going from B to} \\ \text{A along a given path, in field} \\ \text{of any source distribution} \end{array} \right.$$

$$= \left\{ \begin{array}{l} \text{work done in going from B to} \\ \text{A along } \textit{any} \text{ path, in field} \\ \text{of that source distribution} \end{array} \right. \tag{2.38}$$

Potential of an electric dipole: Next to a point electric charge, the electric dipole is a physically very important system It is therefore useful to find the potential produced by a dipole.

As a first step let us find the potential due to two point charges Let the charges q_1 and q_2 be located at points P_1 and P_2 in space, as shown in Fig 2.17. We want to find the potential at a general point A. Let the distances of A from P_1 and P_2 be r_1 and r_2 respectively. Then the potential at A is the sum of the potentials there due to q_1 alone and due to q_2 alone.

$$\varphi(\mathrm{A}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) . \tag{2 39}$$

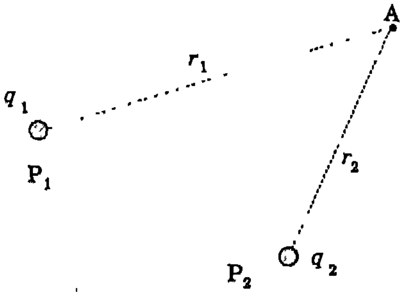


Figure 2.17: Potential due to two point charges

Here we used the result (2 35) for the potential due to each charge; and the fact that we have to simply add these individual contributions is due to the superposition principle as expressed in Eq. (2.37).

Now let us turn to the case of a dipole As we learnt in Chapter 1, a dipole is described by a *vector* **p** representing its *moment* Unlike a point charge, it has both a built-in magnitude and a direction. And as we also saw, the field at a general point in space with position vector **r**, due to a dipole **p** situated at the origin, has a dependence not only on the magnitude r of **r**, but also on the angle between **r** and **p**

It is naturally slightly easier to find the dipole potential, as compared to its field, since the former is a scalar We have to deal with a special case of the result of Eq. (2.39), giving the potential due to two general point charges. For finding the field at points in the equatorial plane, we depicted the dipole in Fig. 1.20 as pointing "vertically upwards" Let us draw a similar figure for finding the potential at a general point

As drawn in Fig. 2.18, we have charges q and $-q$ separated by a distance $2a$. The dipole moment **p** has magnitude $2aq$. We want to calculate the potential φ at a general point A The vector from the dipole centre to A is **r**. it has magnitude r and makes an angle θ with the dipole axis

To use Eq. (2 39), we must take $q_1 = q$ and $q_2 = -q$. We need expressions for the distances marked r_1 and r_2 in Fig. 2 18 These will depend on r, a and θ We may of course

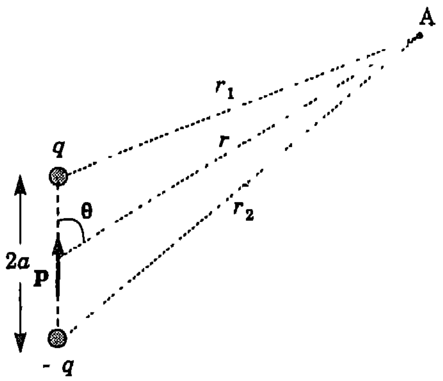


Figure 2.18: Potential due to a dipole

treat a as very small compared to r. From the two triangles shown in Fig. 2.18, we have expressions for ${r^2}_1$ and ${r_2}^2$

$$\begin{aligned} {r_1}^2 &= r^2 + a^2 - 2ar\cos\theta, \\ {r_2}^2 &= r^2 + a^2 + 2ar\cos\theta. \end{aligned} \qquad (2.40)$$

We neglect a^2 in these expressions and retain only terms upto linear in a. Then, after taking square roots, we have

$$\begin{aligned} r_1 &\simeq (r^2 - 2ar\cos\theta)^{1/2} \\ &= r(1 - \frac{2a}{r}\cos\theta)^{1/2}, \\ r_2 &\simeq (r^2 + 2ar\cos\theta)^{1/2} \\ &= r(1 + \frac{2a}{r}\cos\theta)^{1/2} \end{aligned} \qquad (2.41)$$

For their reciprocals we then get the approximate expressions

$$\begin{aligned} \frac{1}{r_1} &\simeq \frac{1}{r}(1 - \frac{2a}{r}\cos\theta)^{-1/2} \\ &\simeq \frac{1}{r}(1 + \frac{a}{r}\cos\theta), \\ \frac{1}{r_2} &\simeq \frac{1}{r}(1 + \frac{2a}{r}\cos\theta)^{-1/2} \\ &\simeq \frac{1}{r}(1 - \frac{a}{r}\cos\theta) \end{aligned} \qquad (2.42)$$

With the help of all these, we can use Eq. (2.39) to say that the dipole potential at the general point A is given by

$$\begin{aligned} \varphi(\mathrm{A}) &= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\ &\simeq \frac{q}{4\pi\epsilon_0}\,\frac{1}{r}\,\frac{2a\cos\theta}{r} \\ &= \frac{|\mathbf{p}|}{4\pi\epsilon_0}\,\frac{\cos\theta}{r^2} \end{aligned} \qquad (2.43)$$

This becomes more compact if we use the vector **r** and its scalar product with **p**. Since

$$\mathbf{p}\cdot\mathbf{r} = |\mathbf{p}|\, r\,\cos\theta \qquad (2.44)$$

we have the result.

$$\begin{aligned} \varphi(\mathrm{A}) &= \left\{ \begin{array}{l} \text{electrostatic potential at} \\ \text{point A with position vector} \\ \mathbf{r} \text{ due to dipole of moment} \\ \mathbf{p} \text{ at the origin.} \end{array} \right. \\ &= \frac{1}{4\pi\epsilon_0}\,\frac{\mathbf{p}\cdot\mathbf{r}}{r^3} \end{aligned} \qquad (2.45)$$

It is natural to compare this with the potential (2.35) due to a point charge. We can see that (a) in a fixed direction, i.e., fixed θ, the potential varies as $1/r^2$ here rather than as $1/r$; (b) even for a fixed distance r, there is now a dependence on direction, i.e., on θ. These are the ways in which the fact that we are dealing with a dipole, with an inbuilt sense of direction, shows up in the electrostatic potential.

★ We can easily extend the result (2.39) to find the potential due to any number, N say, of point charges $q_1, q_2, \ldots, q_N$ located at points $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_N$ in space. If a general point A is at distances $r_1, r_2, \ldots, r_N$ from $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_N$ respectively, the potential there is

$$\varphi(\mathrm{A}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots + \frac{q_N}{r_N}\right) \qquad (2.46)$$

At the next step we can ask for the potential due to any continuous source distribution. The answer is simple in principle: divide the source into many small portions, treat each one as a point charge, then use the result (2.46)! In the limit as the portions become indefinitely small, we have an integral rather than a sum. In this way we could find the potential due to a uniform line charge, uniform plane sheet etc., but we will omit the details.

One case worth explicit mention, however, is that of a spherically symmetric distribution of charge. We have seen that at points *outside* the

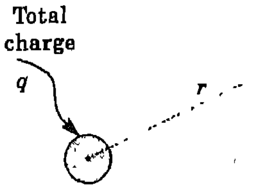


Figure 2.19: Potential due to spherically symmetric source at outside points

source the field is the same as if all the charge, q say, were concentrated at the geometrical centre of the distribution. So, while calculating the work done on a test charge in bringing it in from infinity to some point A *outside* the source, we would never notice the difference between the actual source and a point charge q sitting at the centre! Then, if as in Fig 2.19 we denote by r the distance from the centre of the source to the point A, we have

$$\varphi(\text{A}) = \begin{cases} \text{Potential at a point A \textit{outside}} \\ \text{spherically symmetric source} \\ \text{of total charge } q \end{cases}$$

$$= \frac{q}{4\pi\epsilon_0}\,\frac{1}{r} \qquad (2.47)$$

★

Equipotential surfaces – from the potential to the field. As we have said often, the potential φ being a scalar is simpler than the field **E**. Upto now we learnt how to find φ from **E**: just see how much work has to be done to carry a unit test charge along a path in the field. If conversely **E** could be found from φ, that would greatly simplify the calculation of the field due to a given source: first find the potential φ, then use it to obtain the field **E**.

Towards this, we develop the idea of *equipotential surfaces*, or *equipotentials*. In a given field produced by some source distribution, an equipotential surface is a (two-dimensional) surface all over which the electrostatic potential φ has a constant value. We illustrate this with two simple examples.

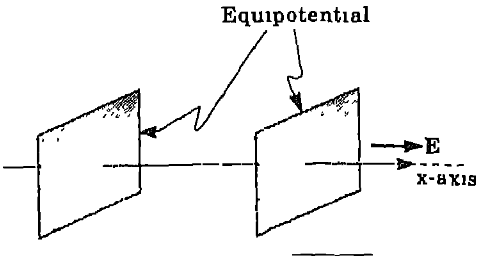


Figure 2.20: Equipotentials in a uniform field

Take the case of a uniform field of strength **E** in the x-direction. We found in Eq (2.30) that the potential φ at any point A depends only on the x-co-ordinate of A. So all over a plane with *constant* x-coordinate but *varying* y and z coordinates, the potential has a *constant* value. For a uniform field, then, the equipotential surfaces are *planes perpendicular to the field direction*. These are indicated in Fig 2.20.

Take next the case of the field of a point charge. We found in Eq (2.35) that the potential at a point A distance r from the charge is proportional to the reciprocal of r. This immediately tells us: the equipotential surfaces here are surfaces of spheres (of varying radii) centred on the charge. Now we know that the field is purely radial. So we see, as in the previous example, that again the field at any point is perpendicular to the equipotential passing through that point. In Fig 2.21 we show a few equipotentials for a point charge, chosen so that the potential changes by equal amounts as we go from one surface to the next one.

For more complicated sources — two or more point charges, dipole, etc. — the equipotential surfaces are more complicated. But some important general physical properties can be easily understood.

Firstly, at any point A in a general field, the electric vector **E** must be perpendicular to the equipotential surface passing through A. This is because if we move a test charge at A to any nearby point A′ *on the equipotential*, we do no work as the potential has not changed. But then this means that **E** has no

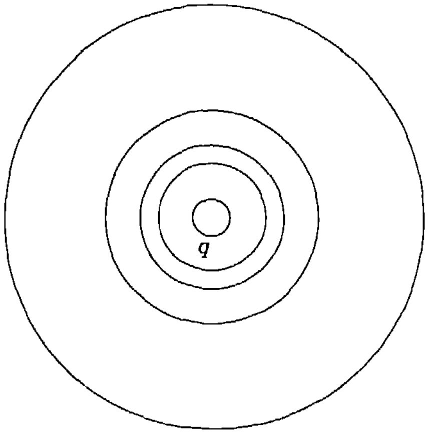


Figure 2.21: Equipotentials around point charge

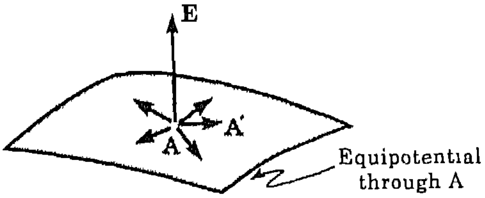


Figure 2.22: Relating equipotential to direction of E

component along any tangential direction at A lying within the equipotential surface So E, as shown in Fig. 2 22, is perpendicular to the equipotential. We have seen this explicitly in the simple cases of a uniform field, and the field of a point charge

Secondly, as illustrated in Fig. 2.23, let us imagine moving a unit test charge from A to a nearby point B (lying on a neighbouring equipotential surface) in the direction of the field E at A. We can equate the work done to the change in the potential·

$$\begin{aligned} \text{Potential at A} &= \varphi, \\ \text{Potential at B} &= \varphi + \delta\varphi, \\ \text{Work done} &= -\mathbf{E}\cdot\delta\mathbf{l} \\ &= -|\mathbf{E}|\,\delta l; \\ -|\mathbf{E}|\delta l &= \delta\varphi. \end{aligned} \tag{2.48}$$

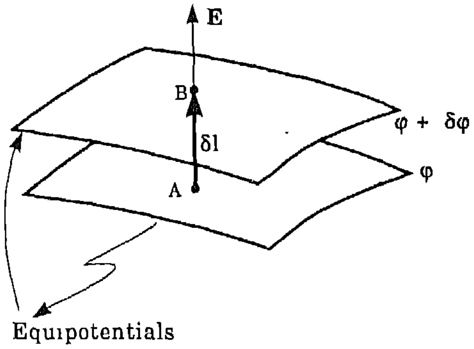


Figure 2.23: From the potential to the field

Here δl is the distance between points A and B. The negative sign is understandable as we saw in the case of a uniform field, Eq. (2 30), the potential *decreases* as we move in the field direction And for a small displacement we can suppose the field to be approximately uniform We can now interpret Eq (2 48) by saying that the magnitude of the field is given by the change in potential per unit distance perpendicular to the equipotential surface.

To sum up· the field at any point is perpendicular to the equipotential surface there, and points in the direction of decreasing potential Its magnitude is the magnitude of the rate of change of potential as we move normal to the equipotential These general rules to obtain the field **E** from knowledge of the potential φ are seen to work in the two simple cases of the uniform field, and the field of a point charge, which we studied in detail earlier.

2.5 Relation between potential and potential energy

In our study of mechanics, and motion under gravity, we learnt about kinetic energy and potential energy These are two forms of energy possessed by a body moving under the influence of given forces While the kinetic energy is more easily "visible" in terms of the actual movement of the body, the potential energy is a kind of "stored" energy — the body has more or less of it depending on its location.

The close connection between the electrostatic potential, which we have studied in the preceding Section, and potential energy due to electrical forces, is now easy to trace It follows from the way we defined the potential as work done in moving a unit test charge from place to place. We introduced the symbol φ for the potential. When needed, and to clearly distinguish one from the other, we shall write U for electrostatic potential energy

Potential energy for a system of point charges Let us consider a simple system of just two point charges, q_1 and q_2. Suppose they are initially far away (infinitely separated) from one another. We want to know how much work has to be done to start from this situation and gradually bring q_1 to a location P_1 (position vector $\mathbf{r}_1$) and q_2 to a location P_2 (position vector $\mathbf{r}_2$) We can ask this in another way: keeping q_1 fixed at P_1, how much work do we have to do to bring in q_2 from infinity to P_2? This is easy to answer.

The charge q_1 at P_1 leads to an electrostatic potential φ all around it. At the point P_2 this potential has the value (see Eq. (2 35))

$$\varphi(P_2) = \begin{cases} \text{value at } P_2 \text{ of potential} \\ \text{due to } q_1 \text{ at } P_1 \end{cases}$$

$$= \frac{q_1}{4\pi\epsilon_0} \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}. \qquad (2\,49)$$

The work we have to do on q_2 in the process described above is then the product $q_2\,\varphi(P_2)$. In slightly different words we can say·

Work done in assembling two point charges q_1 and q_2 from infinite separation to the positions $\mathbf{r}_1$ and $\mathbf{r}_2$

$$= U_{12}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|}. \qquad (2.50)$$

We must interpret this expression as the *potential energy stored in the system of charges* We can reassure ourselves of this by considering, for example, the case $q_1 q_2 > 0$ when the charges *repel* one another. If we have brought them near one another from infinitely far away, we have certainly done some work in the process We could recover this work or energy by allowing the charges to move apart on account of their repulsion, then their stored potential energy gets converted into kinetic energy. Of course, if $q_1 q_2 < 0$, we could have extracted work from the charges by allowing them to come close together from infinite separation

To appreciate again that one coulomb of charge is really immense, we can now see that the work we must do to bring two such charges together from infinity to a separation of one metre is 9×10^9J, quite a lot by human standards. If we are willing and able to do this much work, it will not be lost but will be stored as potential energy!

Where is the potential energy located? Is it at the location $\mathbf{r}_1$ of q_1, or $\mathbf{r}_2$ of q_2, or may be somewhere in between? Actually the symmetry of the energy Eq. (2.50) in $\mathbf{r}_1$ and $\mathbf{r}_2$ indicates the answer: we *cannot locate* this potential energy at any precise place, it *belongs to the system as a whole* For instance, if $q_1 q_2 < 0$, this potential energy is negative we have to supply energy from the outside (i.e., do work) to pull the two charges apart.

★ It is easy to generalize the result, Eq. (2 50), and get an expression for the potential energy of a system of point charges $q_1, q_2, q_3, \ldots$ at positions $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots$ It is just the total work we have to do to bring them to these positions from infinitely great mutual separations We do this in a step by step manner To begin, consider just three charges. At first bring q_1 to $\mathbf{r}_1$ and q_2 to $\mathbf{r}_2$. this requires the amount of work U_{12} given in Eq (2 50). Then bring in q_3 from infinity to $\mathbf{r}_3$: we have to work against the forces that q_1 and q_2 exert on it. This work is clearly the product of q_3 and the total potential at $\mathbf{r}_3$ due to q_1 and q_2 combined. For the latter we have an expression in Eq (2 39). We only need to replace r_1 and r_2 there by the distances $|\mathbf{r}_3 - \mathbf{r}_1|$ and $|\mathbf{r}_3 - \mathbf{r}_2|$ respectively So the work done in this second stage is

$$q_3 \frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{|\mathbf{r}_3 - \mathbf{r}_1|} + \frac{q_2}{|\mathbf{r}_3 - \mathbf{r}_2|}\right\} \qquad (2\,51)$$

Adding this to the previous expression (2.50) we see that the total work done in assembling the system of three charges, i.e., its potential energy, is

Potential energy of system

of three point charges =

$$\frac{1}{4\pi\epsilon_0}\left\{\frac{q_1q_2}{|\mathbf{r}_1-\mathbf{r}_2|}+\frac{q_1q_3}{|\mathbf{r}_1-\mathbf{r}_3|}\right. ,$$
$$\left.+\frac{q_2q_3}{|\mathbf{r}_2-\mathbf{r}_3|}\right\}. \quad (2.52)$$

From this expression the pattern becomes clear: we have *one* contribution for *each distinct pair* of point charges; each such contribution has the general form we see in Eq. (2.50). So for a system of four point charges, there are six contributions to the potential energy from the six distinct pairs 12, 13, 14, 23, 24, 34:

Potential energy of system

of four point charges =

$$\frac{1}{4\pi\epsilon_0}\left\{\frac{q_1q_2}{|\mathbf{r}_1-\mathbf{r}_2|}+\frac{q_1q_3}{|\mathbf{r}_1-\mathbf{r}_3|}+\frac{q_1q_4}{|\mathbf{r}_1-\mathbf{r}_4|}\right.$$
$$\left.+\frac{q_2q_3}{|\mathbf{r}_2-\mathbf{r}_3|}+\frac{q_2q_4}{|\mathbf{r}_2-\mathbf{r}_4|}+\frac{q_3q_4}{|\mathbf{r}_3-\mathbf{r}_4|}\right\} \quad (2.53)$$

And so on for more charges. ★

Potential energy of a charge in a given field: Next we consider a sightly different situation. Let some system of sources create a certain field with a potential φ. (We suppose that φ vanishes at infinity). If we now bring in a charge q from infinity to a point A with position vector $\mathbf{r}$ (here we do not view q any more as a test charge!), the work we do is stored as potential energy of this charge:

Potential energy of charge

q at position $\mathbf{r}$ in a given field

$$= q\,\varphi(\mathbf{r}) \quad (2.54)$$

This is on account of the interactions between the charge q and the sources producing the potential φ.

We can appreciate this result with the help of some remarks. Suppose the charge q is positive, and we move it from a place of lower to a place of higher potential. We definitely exert ourselves and do a certain amount of work in the process. But this effort is never wasted, it is stored as potential energy of the charge q. This is similar to increasing the gravitational potential energy of a massive body by raising it through some height above the earth's surface. For $q < 0$, of course, the potential energy increases if it moves from higher to lower potential — this has no analogue in the case of gravity.

The result (2.50) for the potential energy of a pair of point charges is a particular case of the present discussion. We can view it as the potential energy of q_2 because it is in the field created by q_1, or the other way around. But as we said earlier, it is more useful to regard this potential energy as being "jointly owned" by the pair of charges!

A charge of one coulomb, if it "moves up" by one volt, increases its potential energy by one joule. In atomic physics and chemistry, a more convenient unit of energy is the *electron volt*, written as eV. It is the potential energy gained or lost by an electron in moving through a potential difference of one volt. Since the charge on an electron is -1.6×10^{-19} C (see Eq. (1.8)), we have

$$1\ \text{eV} = 1.6\times10^{-19}\ \text{J}. \quad (2.55)$$

Related units are the milli (10^{-3}), kilo (10^3), mega (10^6) and giga (10^9) electron volts, written as meV, keV, MeV, GeV, respectively.

We can easily extend the case of one charge in a given field to two charges in the field. What then is their total potential energy? We see that there are three distinct contributions: (i) the potential energy of q_1 due to its interaction with the sources; (ii) that of q_2 due to its interaction with the sources; (iii) the potential energy due to the interaction between q_1 and q_2. If q_1 and q_2 are at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, as shown in Fig. 2.24, and the potential due to the sources is φ, we get:

Potential energy of two point

charges in a given field =

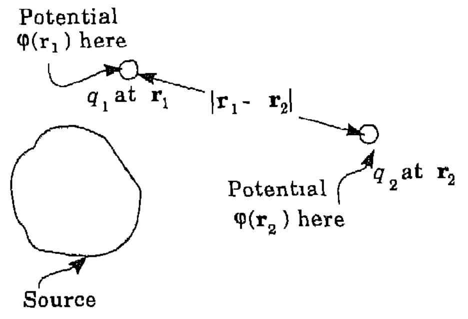


Figure 2.24: Potential energy of two charges in a given field

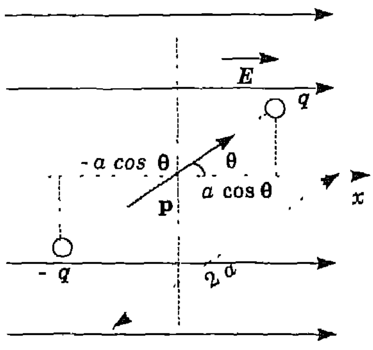


Figure 2.25: Potential energy of a dipole in a uniform field

$$q_1\ \varphi(\mathbf{r}_1) + q_2\ \varphi(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}. \tag{2.56}$$

As a useful special case of this result we can find the potential energy of a dipole in a given field! For simplicity let us suppose that the field is uniform, and points in the x-direction. Then its potential φ is given by Eq. (2.30) The situation is pictured in Fig. 2.25. The dipole, consisting of charges $\pm q$ a distance $2a$ apart, is in general aligned at some angle θ with the field. So the x-coordinates of the two ends $\pm q$ are $\pm a\cos\theta$

In the complete expression (2.56) we omit the last term since it has a fixed value for a given dipole, which is not affected by the external electric field (This is the energy liberated in the process of forming the dipole) So the result we get is

Potential energy of a dipole in a uniform field

$$\begin{aligned}
&= q\varphi(\text{at } x = a\cos\theta) - \\
&\quad q\varphi(\text{at } x = -a\cos\theta) \\
&= q(-Ea\cos\theta) - q(+Ea\cos\theta), \\
&\qquad \text{using Eq } (2.30) \\
&= -2aq\,E\ \cos\theta \\
&= -|\mathbf{p}|\,|\mathbf{E}|\ \cos\theta \\
&= -\mathbf{p}\cdot\mathbf{E}
\end{aligned} \tag{2.57}$$

We have brought in the magnitude $2aq$ of the moment of the dipole, and finally expressed the potential energy directly in terms of the two vectors **p** and **E**.

We can nicely connect this result with what we learnt in Section 1.8 of the preceding chapter There we saw that in a uniform electric field a dipole experiences a torque, given by Eqs (1 33, 34). The torque tends to align the dipole along the field, or in other words to bring the angle θ towards zero. The torque thus acts in such a way as to reduce the potential energy of the dipole to its minimum value, which occurs at $\theta = 0$.

Contrasting the effects of an electric field on point charges and on dipoles, we can concisely express what we learnt in the preceding Chapter and in the present one: the field vector **E** determines *forces* on point charges, *torques* on dipoles, and *potential energies* of dipoles For *forces on dipoles*, we need a nonuniform electric field, or the rate of change of **E** with position!

2.6 Electrostatics of Conductors

In Section 1.4 of Chapter 1 we briefly studied atomic structure, and the different electrical properties of bulk materials. In particular, we saw that materials can be broadly and usefully classified into conductors and insulators (also called dielectrics) The former have freely moveable electrons inside them, the latter do not. Even in a conductor, while electrons do move freely *inside* the material, they normally cannot escape from the conductor by passing through its surface, unless there is contact with something external

We have also studied properties of electrostatic fields produced by given (static) sources We assumed that the source charges are placed at definite locations, and are then "held there" by some means. To some extent these are ideal situations Nevertheless, through them, we have learnt the fundamental properties of electrostatic fields *which are valid under all circumstances*. These include Gauss's Theorem, the path independence of work done, the concept of the potential, and equipotential surfaces.

Now we discuss briefly the behaviour of conducting bodies when placed in an electrostatic field. We would like to know: what are the general features of this behaviour; and what are the conditions that are always obeyed in such situations? What can we say about the field, the charge distribution, and the potential, in and near a conducting body in static conditions?

Behaviour of field, charges, potential: Now the word *static* means that *nothing* changes with time. In the present context it means that there is *no movement of charges*, no current, in the interior or on the surface of each conductor. But we know that because there are free charges in a conductor, if there were any nonzero field in the interior, there would surely be a current and things would not be static! Similarly at the surface too, or just outside, the electric field can have no tangential component. If it did, there would have been a surface current (Both in the interior and at the surface, presence of a field leads to a current. Conversely, existence of a current requires presence of a field – otherwise there is no cause for the free charge to move!) The knowledge then that in static conditions there is no current allows us to say· the field must vanish in the interior of every conductor. And just outside the surface it can only have a normal component.

These facts lead via Gauss's Theorem to an important conclusion: there can be no charge in the interior of a conductor. For, if we imagine *any* closed surface S *inside* a conductor, the total outward flux through S is zero, because the field vanishes all over S Therefore the total charge enclosed within S is zero. But we are free to choose S as we please, provided it is totally inside the conductor. Then the statement made above immediately follows.

The vanishing of the field inside a conductor also tells us something about the potential throughout its volume. For, if we imagine trying to move a test charge from one place to another within the conductor, we do not have to do any work at all: there is no electric force to overcome. Therefore all points within (and on the surface of) a conductor are at a common constant potential. (Of course different conducting bodies could be at different constant potentials.) So each conducting body is a (three-dimensional) equipotential *volume*, rather than just a (two-dimensional) equipotential *surface*!

Where then can charges reside on a conducting body? Since the interior is ruled out, the only place is the surface! Suppose a conducting body (which may be overall neutral, or which may carry some net nonzero charge which we have given to it) is brought into an already existing field. At first, all the conditions we have deduced above may not be obeyed. In that case, the freely moveable charges within the conductor (and any excess charges we may have added) will quickly move and redistribute themselves. They will create additional fields such that after a very short time the conditions described above are all obeyed, and everything is static once more.

We have learnt several things about fields, charges and potentials in relation to conductors in an electrostatic field We list them now, and illustrate them pictorially in Fig 2.26.

(i) In the interior of each conductor, the electric field and the volume charge density both vanish.

(ii) Therefore charges in a conductor can

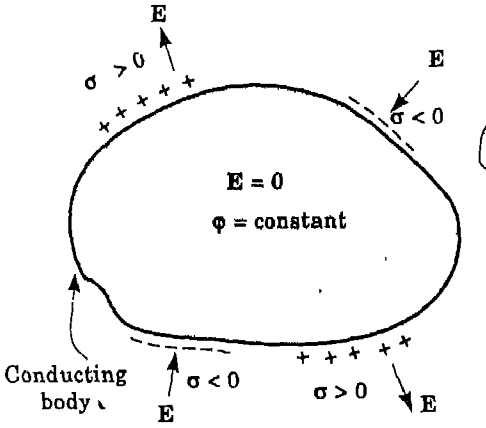


Figure 2.26: Characteristics of a conducting body in electrostatics

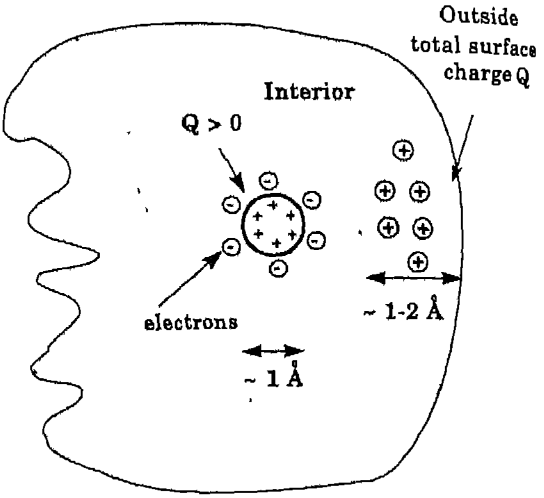


Figure 2.27: Screening and attaining static condition in a conductor.

only be at the surface.

(iii) Immediately outside the surface of each conductor, the electric field is perpendicular to the surface, pointing either outwards or inwards.

(iv) The entire body of each conductor, including its surface, is at a constant potential

We have described these general features in a macroscopic way, ignoring details on the atomic level. In fact, the surface region of a conductor is normally about two atoms thick, or some 2×10^{-10} m (i.e 2 Å) The surface charge density σ need not be constant. It will always adjust itself, being positive in some places and negative in others if necessary, so that the basic conditions we have found are fulfilled. If there is any excess charge on the conductor (either electrons added or taken away or positive ions injected), the total surface charge will be equal to it.

★ Here is a somewhat more detailed picture of what happens in a conducting body. Suppose for example by some means we inject some positive charge Q *into* a metal-for instance, by exposing it to a beam of charged particles produced by an accelerator But we said there cannot be any charge inside a metal! So what has happened? The free electrons roaming around inside the metal get attracted to Q, and move in and around it This takes place over a certain (short) period of time At some intermediate stage the total electronic charge surrounding Q may not yet balance it exactly, so there is still a net positive charge there Then more free electrons will be attracted and crowd in (to watch the fun) until there are enough to exactly neutralise Q This is called "screening" - the positive charge Q injected into the metal from the outside has got perfectly neutralised As indicated in Fig 2.27, in good metals this takes place over a length scale of about one Å(10^{-10}m)

Now the electrons screening Q have come from elsewhere in the metal. Those regions thus become positively charged In turn electrons from yet elsewhere move in there, and so on The process stops at the surface, which therefore finally has a deficit of electrons. Thus in the end a surface layer of positive charge, of total amount exactly Q, gets created The thickness of this layer (absence of electrons) is, as we have said in the main text, about one or two Å

This description explains in a physical manner why in a system with free electrons (a metal), under static conditions

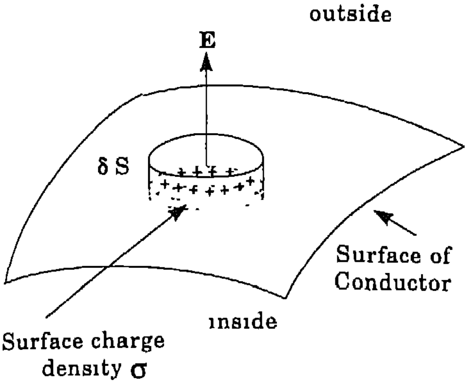


Figure 2.28: Applying Gauss's Theorem across a conducting body

(i) there is no net charge in the interior,

(ii) there is no electric field in the interior,

(iii) charges can reside only at the surface.

A simple use of Gauss's Theorem tells us the relation between the electric field **E** at any point just outside a conducting surface, and the surface charge density σ there. We show in Fig. 2.28 a portion of the surface, and a suitable Gaussian surface S a short cylinder (pill-box) of small cross-sectional area δS and negligible height, half inside and half outside the conductor. The total outward flux through S is $\pm|\mathbf{E}|\delta$S, according as **E** just outside the conductor points normally outwards or inwards. There is no contribution to the flux from the sides of S, or from the "face" inside the conductor. The total charge enclosed within S is $\sigma\delta$S. So Gauss's Theorem (2.9) gives

$$\pm|\mathbf{E}|\delta S = \frac{1}{\epsilon_0}\sigma\delta \mathrm{S}$$

according as **E** is outwards or inwards,

i.e.,

$$|\mathbf{E}| = \frac{|\sigma|}{\epsilon_0},$$

E points outwards or inwards according as $\sigma \gtrless 0$ (2.58)

This connection between the direction of **E** and the sign of σ has been used in Fig. 2.26. ★

Electrostatic Shielding We have contrasted the conditions existing just inside a conducting body from those just outside. Now suppose we have a cavity – a hollow volume or region of space – embedded *inside* a conductor. In a sense, this region too is *outside the conductor*! What more can be said about the field inside a cavity, and the charge distribution on the inner walls of the conductor bounding the cavity?

We know from Section 2.3 (see Eq. (2.19)) that in the interior of a uniform spherical shell of charge the electric field vanishes. (In particular if the shell had been a conductor with some net charge placed on it, this charge would have spread itself uniformly over the sphere, and the field inside would be zero.) But for this result to be true, the spherical symmetry was *essential*. So if the shell were even a tiny bit nonspherical, there is the possibility that the field inside may not exactly vanish.

Fortunately it is a remarkable fact that inside a cavity *of any shape* inside a *conductor*, if there is no charge there, the field is *strictly zero*. This property is called *electrostatic shielding*. As an immediate consequence, using Gauss's Theorem, we find that there are no surface charges at all along the inner walls of the conductor surrounding the cavity. This is shown in Fig. 2.29. We can also see that the same constant value that the potential has in the body of the conductor, say $\varphi = \varphi_0$, persists all over the cavity region too.

A practically important consequence of shielding is that in a thunderstorm accompanied by lightning (more on that in Section 2.9), it is safest to be inside a bus or car, rather than near a tree on open ground!

★ It is an interesting historical fact that before Cavendish and Coulomb had experimentally established the inverse square law, Benjamin Franklin had found that the field inside a conducting shell of charge was zero. He consulted J. Priestley about it. Priestley was aware of a similar result in gravitation inside a spherical shell

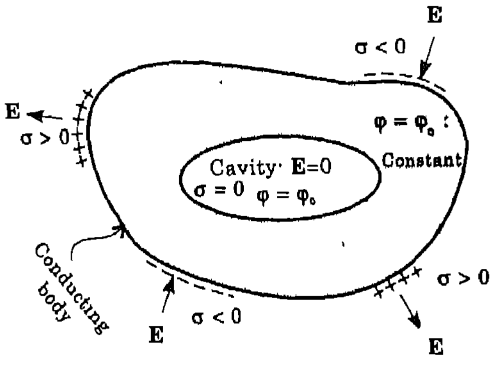


Figure 2.29: Illustrating electrostatic shielding

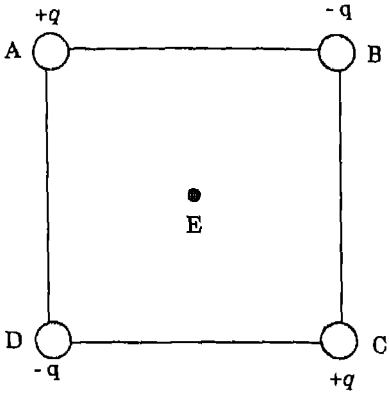
of mass the gravitational field vanishes. So he suggested that here too an inverse square law of force may be at work. Cavendish's experiments were done five years later (but results not disclosed for a long time!), and Coulomb's after another thirteen years!

★

Let us conclude this Section with a remark which we explore in more detail in Section 2.10. While it is true that charges on a conductor (in static conditions) can reside on its (outer) surface alone, they need not generally be spread uniformly all over the surface with a constant density. In fact what happens is that surface charges tend to accumulate near sharp points or corners on the surface, rather than over smooth portions. But as we said, we shall come back to this later.

Example 2.4: Four charges are arranged at the corners of a square ABCD of side as shown. (a) Find the work required to put together this arrangement. (b) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?

Answer: (a) Since the work done depends on the final, actual arrangement of the charges, and not on how they are put together, we calculate the work needed for one way of putting the charges at A, B, C and D.



Suppose first the charge $+q$ is brought to A, and then the charges $-q, +q$, and $-q$ are brought to B, C and D respectively. The total work needed can be calculated in steps.

(i) Work needed to bring charge $+q$ to A when no charge is present elsewhere. This is zero.

(ii) Work needed to bring charge $-q$ to B when $+q$ is at A. This is given by (charge at B) × (electrostatic potential at B due to charge $+q$ at A)

$$= -q \times \left(\frac{q}{4\pi\epsilon_0 d}\right) = -\frac{q^2}{4\pi\epsilon_0 d}, \text{ and}$$

(iii) Work needed to bring charge $+q$ to C when $+q$ is at A and $-q$ is at B. This is given by (charge at C) × (potential at C due to charges at A and B)

$$= +q\left(\frac{+q}{4\pi\epsilon_0 d\sqrt{2}} + \frac{-q}{4\pi\epsilon_0 d}\right)$$
$$= \frac{-q^2}{4\pi\epsilon_0 d}\left(+1 - \frac{1}{\sqrt{2}}\right).$$

(iv) Work needed to bring $-q$ to D when $+q$ is at A, $-q$ at B, and $+q$ at C. This is given by (charge at D) (potential at D due to charges at A, B and C)

$$= -q\left(\frac{+q}{4\pi\epsilon_0 d} + \frac{(-q)}{4\pi\epsilon_0 d\sqrt{2}} + \frac{q}{4\pi\epsilon_0 d}\right)$$
$$= -\frac{q^2}{4\pi\epsilon_0 d}\left(2 - \frac{1}{\sqrt{2}}\right)$$

Add the work done in processes (i), (ii), (iii) and (iv). The total work required is

$$= \frac{-q^2}{4\pi\epsilon_0 d}\left\{(0)+(1)+(+1-\frac{1}{\sqrt{2}}) + (2-\frac{1}{\sqrt{2}})\right\}$$

$$= -\frac{q^2}{4\pi\epsilon_0 d}(4-\sqrt{2}).$$

The work done depends only on the arrangements of the charges, and not on how they are assembled By definition, this is the total electrostatic energy of the charges.

(b) The extra work necessary to bring a charge q_0 to the point E when the four charges are at A, B, C and D is just (q_0) (electrostatic potential at E and D is just due to charges at A, B, C and D)

$$= q_0\left\{\frac{q}{4\pi\epsilon_0(d/\sqrt{2})} + \frac{(-q)}{4\pi\epsilon_0(d/\sqrt{2})} + \frac{(+q)}{4\pi\epsilon_0(d/\sqrt{2})} + \frac{(-q)}{4\pi\epsilon_0(d/\sqrt{2})}\right\} = 0$$

2.7 Capacitors and Capacitance

We have now gathered enough background knowledge of the basic principles of electrostatics to understand and appreciate the properties of a practically important electrical device — a *capacitor*. This device is used for many purposes — to store electric charge, to store electrical energy; to either absorb or release large amounts of charge without developing unduly high potential differences, and so on. It is used in these ways in, for instance, radio and television sets, electronic instruments, and computers.

It is a good idea to first grasp the concept of a capacitor in general, and then to learn about simple examples of it. In this Section, in accordance with usual custom, we shall use V rather than φ for the electrostatic potential

What is a Capacitor?: In most general terms, a capacitor is *any combination of two conducting bodies separated by an insulator*. They may be of any shapes and sizes, and should be in some definite position relative to one another To begin with, in this Section we assume they are situated in vacuum The effects, and advantages, of being immersed in or separated by some dielectric substance, rather than being in vacuum, will be taken up in the next Section

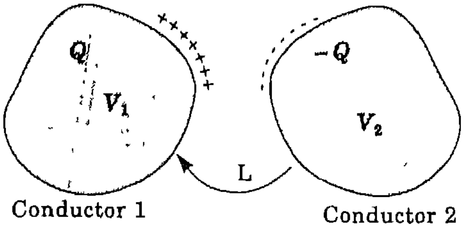


Figure 2.30: General concept of a capacitor

How exactly do we use, or exploit, such a system? Let us call the two conducting bodies as 1 and 2 – they are often referred to as the "plates" of the capacitor Suppose we put a total charge Q on conductor 1, and an opposite charge -Q on conductor 2. (This may be done by briefly connecting them to the two poles or terminals of a battery.) From the preceding Section we learn that these charges will spread themselves on the (outer) surfaces of the conductors, in such a way that the whole of each conductor will be at a certain constant potential. Suppose the potentials of the two conductors are V_1 and V_2 (relative to the potential being zero at infinity). A certain electrostatic field will also be built up in the space outside the two conductors. The potential difference $V = V_1 - V_2$ is just the work done in carrying 1 C of charge from any point on the surface of body 2 to any point on the surface of body 1, along any path L in space. All this is indicated in Fig 2.30.

Now the *capacitance* of the capacitor is defined as the ratio of charge put on each conductor to the potential difference developed between them:

$$C = Q/V \text{ coulomb/volt} \qquad (2.59)$$

In SI units this is called the *farad*, written F, in honour of Faraday:

$$1\ \mathrm{F} \equiv 1\ \text{farad} = 1\ \text{coulomb/volt}. \qquad (2.60)$$

We can easily see an important property of the capacitance of a capacitor. Suppose we double (or halve) the magnitude of the charges $\pm Q$ we put on the two bodies. Then from the superposition principle we can see that the potential at each point becomes double (or half) its previous value. So also for the potentials V_1 and V_2 at the two conductors, and their difference V. In fact, changing the amount of charge Q by *any* factor makes V_1, V_2, V change by the *same* factor. So the capacitance does not change at all! Thus for a given capacitor, we can find its capacitance by putting *any* charges $\pm Q$ on it, however small; finding the potential difference V that is produced, and taking the ratio. The capacitance of a capacitor is determined *purely geometrically* by the shapes, sizes, and relative positions of the two "plates", and by nothing else. (Of course if they are separated by a dielectric, there is an important effect; we come to that, as we said, in Section 2.8.)

The pictorial symbol for a capacitor with *fixed* capacitance is ⊣⊢. If the capacitance is *variable*, it is shown as ⊣↗⊢

Desirability of large capacitance - immensity of the farad: The capacitance of a capacitor tells us how much potential difference V will be created if we wish to store a definite amount of charge Q on it:

$$V = \frac{Q}{C} \qquad (2.61)$$

It is a matter of practical importance to avoid too high "voltages" V appearing when we store this charge. From this point of view we naturally like to have capacitors with large capacitance. it can then "hold" a large amount of charge Q without "running up" too high a potential difference V.

The reason for this concern is this: in the presence of very high potential differences, and accompanying high electric fields, the air surrounding the capacitor gets ionized. The outermost electrons in some of the atoms and molecules get torn away, may be by collisions with a stray electron or ion which has been accelerated to great speed by the intense field. This creates a pathway by which the charges we have put on the capacitor plates can leak away. Such "breakdown" of air by ionization (loss of its insulating power) occurs at fields of about $3 \times 10^6 V/m$. Remember that a charge of one C produces a field of $9 \times 10^9 V/m$ one meter away, so though these *are* large fields, they are not unimaginably large! The maximum electric field that a dielectric material can tolerate, beyond which it loses its insulating property and starts conducting electricity through partial ionization, is called its *dielectric strength*.

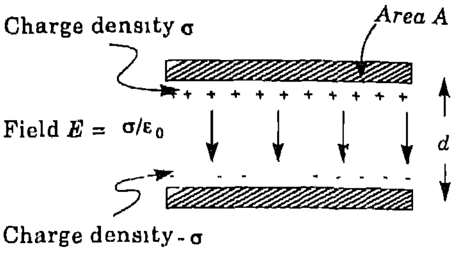


Figure 2.31: Parallel plate capacitor arrangement non-zero field only between plates

Still, while large capacitances are desirable, the SI unit of the farad is *immensely large*. A practical illustration of this will be seen very soon. And in an optional Section we describe why this is so. So for practical purposes one makes capacitors with capacitances in the micro farad ($\mu\mathrm{F} = 10^{-6}$ F) or pico farad ($p\mathrm{F} = 10^{-12}$ F) ranges.

The Parallel Plate Capacitor: Now we look at the simplest example of a capacitor, namely a *parallel plate* capacitor shown in Fig. 2.31. It consists of two (large) plane parallel conducting plates, each of area A say, separated by a (small) distance d. If the plates had been infinite in extent, and if we had put uniform charge densities $\pm\sigma$ on them, then we know from Section 2.3 (see Eq. (2.16) and the following discussion) that there would be a *uniform* field from the positive to the negative plate. Elsewhere the field would vanish. The strength of the field is given in Eq. (2.16):

$$E = \frac{\sigma}{\epsilon_0}. \tag{2.62}$$

For very large area A, we expect that except at the very edges the field remains uniform and perpendicular to the plates, with the above value. Edge effects would be small provided $A \gg d^2$. Then, since the field is uniform, the potential difference between the plates is

$$V = Ed = \frac{\sigma d}{\epsilon_0}. \tag{2.63}$$

The total charge on each plate is $\pm Q = \pm\sigma A$ Taking the ratio we get the result.

Capacitance of a parallel plate capacitor

$$\begin{aligned} C &= \frac{Q}{V} \\ &= A\sigma/\frac{\sigma d}{\epsilon_0} \\ &= \epsilon_0\frac{A}{d} \\ &\simeq 8.85\times 10^{-12}\times\frac{(\text{A in m}^2)}{(d\text{ in m})}\text{ F} \end{aligned} \tag{2.64}$$

You can see clearly that the capacitance is determined purely geometrically In fact, for any capacitor, the capacitance C has the form of ϵ_0 multiplied by some length.

Incidentally, by taking $A = 1$ m^2 and $d = 1$ m we see that we can express the constant ϵ_0 in this way. $\epsilon_0 \simeq 8.85\times10^{-12}$ farads per metre. This is merely a consequence of the way the farad is defined!

Now we can directly see how immense the farad is Suppose we want a parallel-plate capacitor with one farad capacitance Let the gap between the plates be $d = 1$ cm $= 10^{-2}$ m – a reasonable gap Let us estimate the area A of each plate. We find from Eq. (2.64)

$$\begin{aligned} A &= \frac{Cd}{\epsilon_0} \\ &= (1\times10^{-2}\times10^{12}/8.85)\text{m}^2 \\ &\simeq 10^9\text{ m}^2 \\ &\simeq (3\times10^4\text{ m})^2 \\ &= (30\text{ km})^2 \end{aligned} \tag{2.65}$$

So, if each plate was a square, each side must be 30 km long! *That* is how large one farad is! We could decrease the area A somewhat by reducing the gap d, but just cannot escape the enormity of the farad.

★ Why is it that the farad is so inconveniently large? You may be surprised to find that we can trace it all the way to this fact in *any* system of length and time units convenient for daily human needs, *the speed of light is enormously large* In SI units this speed is about 3×10^8m/s, in CGS units it is 3×10^{10} cm/s

We have to use a two step argument to see the connection First, we have appreciated the fact that the SI unit of charge, the coulomb, is *very large* by daily standards As a result, since the joule is a "reasonable" unit of energy, and the SI unit of potential difference, the volt, is one joule per coulomb, the volt is correspondingly *very small* So the farad, the ratio of the coulomb to the volt, is "doubly large" More precisely, its largeness goes as the square of the largeness of the coulomb

$$\text{One F} = \text{one C/V} = \text{one C}^2/\text{J}. \tag{2.66}$$

But then, why is the coulomb so large in the first place? This is where the speed of light comes in! The magnetic force between two charges in motion at moderate speeds is *much weaker* than the electric coulomb force, precisely by the product of the ratios of their speeds to the speed of light This is an experimental fact Now the coulomb has been defined so as to compensate, at least to some extent, for this smallness of magnetic effects It is chosen so that the force between two currents of one ampere (one coulomb per second) each, one meter apart, is of reasonable magnitude Thus this explains the largeness of the coulomb, the smallness of the volt, and the enormity of the farad! ★

The Spherical Capacitor: The next example we look at is the *spherical capacitor*. It consists of two concentric spherical conductors, as pictured in Fig 2.32 Denote the radii of the inner and outer spheres by r_a and r_b respec-

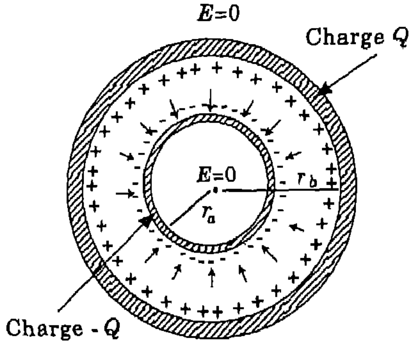


Figure 2.32: Spherical capacitor, non-zero field only between spheres

tively (We may suppose that the two spheres are held in place by insulating supports). In this case, unlike the previous parallel plate arrangement, there are no edge effects to worry about!

Imagine that we spread equal and opposite charges $\pm Q$ uniformly on the outer and inner spheres, respectively From the applications of Gauss's Theorem in Section 2.3, and especially Eq. (2.19), we know that *each* sphere creates *no* field *inside* itself; while at points *outside* it acts as a point charge placed at its centre. So we see that the magnitude of the field, which is in any case radial, is given in different regions thus:

$$\begin{aligned} r < r_a \cdot E(r) &= 0, \\ r_a < r < r_b \cdot E(r) &= +\frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}, \\ &\quad \text{radially inwards;} \\ r > r_b : E(r) &= 0. \end{aligned} \tag{2.67}$$

How about the potential? It is constant inside the inner sphere, $r < r_a$, and outside the outer one, $r > r_b$. In between, it has just the form (2.35) of a point charge as far as the contribution of the smaller sphere is concerned, plus a constant contribution from the large sphere:

$$r_a < r < r_b :$$

$$V(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r}\right). \tag{2.68}$$

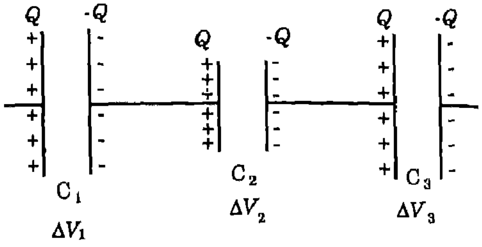


Figure 2.33: Series combination of capacitors - single lines are the connecting wires

So the potential difference between the two spheres is

$$\begin{aligned} V &= V(r_b) - V(r_a) \\ &= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \end{aligned} \tag{2.69}$$

This leads to the capacitance·
Capacitance of a spherical capacitor.

$$\begin{aligned} C &= \frac{Q}{V} \\ &= 4\pi\epsilon_0\frac{r_a r_b}{(r_b - r_a)} \end{aligned} \tag{2.70}$$

We see as expected that C is ϵ_0 times a geometrically determined length

Combinations of capacitors–Effective capacitance Upto now we understood a capacitor to be made up of *two conductors*. It is possible to combine several such capacitors to again form a capacitor, able to store charge and energy But the result is naturally not a system of two conductors alone. How do we find the resulting or *effective* capacitance C of such a combination? It depends on the way we combine the individual capacitors. We look at two simple possibilities.

Series Combination· Here the individual capacitors with capacitances C_1, C_2, C_3 are connected up as shown in Fig. 2.33 (for simplicity we consider just three capacitors). What is common to all the capacitors is the amount of charge, $\pm$ Q, on the plates of each one. The total potential difference or "drop" across the entire combination is the sum of the individual drops:

$$V = \Delta V_1 + \Delta V_2 + \Delta V_3 \qquad (2.71)$$

For the individual capacitors we have the relations

$$\Delta V_1 = \frac{Q}{C_1}, \Delta V_2 = \frac{Q}{C_2}, \Delta V_3 = \frac{Q}{C_3} \qquad (2.72)$$

So the effective capacitance C of the combination is

$$\begin{aligned} C &= Q/V \\ &= Q/(\Delta V_1 + \Delta V_2 + \Delta V_3) \\ &= 1/\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad (2.73) \end{aligned}$$

This is the rule for the series combination We can write it more conveniently as

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (2.74)$$

So the reciprocals of capacitances add in this combination, and C is less than the least of C_1, C_2, C_3. If we have four or more capacitors, we just add more terms in Eqs (2.73, 74)

Parallel Combination: Now the capacitors (again just three for simplicity) are put together as shown in Fig 2.34. There is a common potential drop V across each capacitor, but they carry different amounts of charge:

$$Q_1 = C_1 V, Q_2 = C_2 V, Q_3 = C_3 V \qquad (2.75)$$

The total charge stored is the sum

$$Q = Q_1 + Q_2 + Q_3, \qquad (2.76)$$

so the effective capacitance is

$$C = \frac{Q}{V} = C_1 + C_2 + C_3 \qquad (2.77)$$

This is just the sum of individual capacitances. For four or more capacitors in this combination, we add more terms in Eq (2.77).

Example 2.5: A network of four 10 μF capacitors is connected to a 500V supply as shown Determine the (a) equivalent capacitance of the network, and (b) charge on each capacitor (Note, the 'charge on a capacitor' is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential)

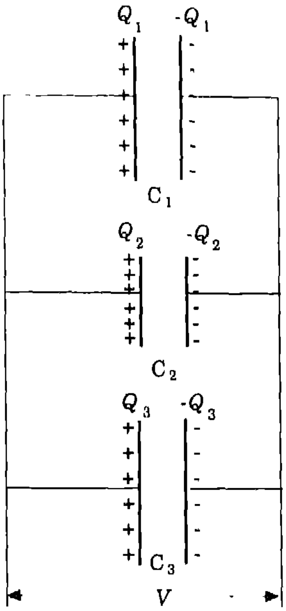


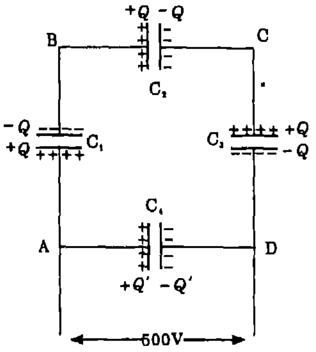
Figure 2.34: Parallel combination of capacitors

Answer: (a) In the given network, C_1, C_2 and C_3 are connected in series. The effective capacitance C' of these three capacitors is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

for $C_1 = C_2 = C_3 = 10\,\mu$F, $C' = 10/3\,\mu$F.

The network has C' and C_4 connected in parallel Thus the equivalent capacitance C of the network is



$$C = C' + C_4 = \left(\frac{10}{3} + 10\right)\mu\text{F} = 13.3\mu\text{F}$$

(b) Clearly, from the figure, the charge on each of the capacitors, C_1, C_2 and C_3 is the same, say Q. Let the charge on C_4 be Q'. Now, since the potential difference across AB is Q/C_1, across BC is Q/C_2, across CD is Q/C_3, we have

$$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500\text{V}.$$

Also $Q'/C_4 = 500\text{V}$.

This gives for the given value of the capacitances,

$$\begin{aligned} Q &= 500 \times \frac{10}{3}\mu\text{C} = 1.7 \times 10^{-3}\,\text{C and} \\ Q' &= 10 \times 500\mu\text{C} = 5.0 \times 10^{-3}\,\text{C} \end{aligned}$$

Single conductor as a capacitor: We have studied two situations – a single capacitor made up of two conducting bodies; and series or parallel combinations of capacitors where naturally several conductors or plates are involved. Sometimes it is useful to think of a *single conducting body* all by itself as a capacitor. In that case we imagine that the other plate is far away at infinity.

To find the capacitance of a single conductor, imagine placing a charge Q on it. Then suppose its potential is V, relative to zero potential at infinity. We call the ratio

$$C = \frac{Q}{V} \tag{2.78}$$

the *capacitance of the single conductor*.

As an example suppose the conductor is a (hollow or solid) sphere of radius R. When a charge Q is spread on it, its potential is $V = Q/4\pi\epsilon_0 R$. So the capacitance of the sphere is

$$C = \frac{Q}{V} = 4\pi\epsilon_0 R \tag{2.79}$$

The larger the radius, the greater the ability to hold a given amount of charge without running up too high a voltage!

Energy stored in a capacitor: We have said that a capacitor is a device for storing electrical energy, as much as for storing electric charge. Let us now study briefly how much energy is stored in a capacitor.

We take the simplest case of a single capacitor made up of two plates. Imagine "charging up" the capacitor in a series of small steps in each step we transfer a small amount of charge, $\delta Q'$ say, from the negative to the positive plate. Each such step involves our doing some work, which is then stored in the capacitor as potential energy. At the stage when the charges on the plates have reached the values $\pm Q'$ say, the potential difference between the plates would be $V' = Q'/C$. Now at the next step when a small additional of charge $\delta Q'$ is transferred from the negative to the positive plate, the work we have to do is

$$\begin{aligned} \delta W(Q' \text{ to} Q' + \delta Q') &= \\ &\left\{\begin{array}{l}\text{Work done by us in increasing} \\ \text{the charge from } Q' \text{ to } Q' + \delta Q'\end{array}\right. \\ &= \delta Q'\, V' \\ &= \frac{1}{C} Q'\, \delta Q'. \end{aligned} \tag{2.80}$$

This is because the charge $\delta Q'$ has to be carried from the plate at lower potential to the one at higher potential, up the potential difference V' which has already been built up.

To get the total energy stored in the capacitor at the end of many such small steps, we have to add up these contributions δW. It is easy to do this by realising that, since $\delta Q'$ is small, we can express $\delta W(Q' \text{ to} Q' + \delta Q')$ approximately as a difference.

$$\begin{aligned} \delta W(Q' \text{ to } Q' + \delta Q') &\simeq \\ \frac{1}{2C}\{(Q' + \delta Q')^2 - Q'^2\}. \end{aligned} \tag{2.81}$$

The term we have neglected is proportional to $(\delta Q')^2$. Now we can see that by the time the total charge has reached a value Q, the total work done is

$$W(Q) = \left\{\begin{array}{l}\text{work done in putting charges} \\ \pm Q \text{ on capacitor plates}\end{array}\right.$$

$$= \sum_{\substack{\text{many small}\\ \text{steps}}} \delta W(Q' \text{ to } Q' + \delta Q')$$

$$\begin{aligned}= \frac{1}{2C}\big[&\{(\delta Q')^2 - (0)^2\} \\ &+ \{(2\delta Q')^2 - (\delta Q')^2\} \\ &+ \{(3\delta Q')^2 - (2\delta Q')^2\} \\ &+ \dots + \{Q^2 - (Q - \delta Q')^2\}\big]\end{aligned}$$

$$= \frac{Q^2}{2C} \qquad (2.82)$$

This is then the energy stored in the capacitor. We can express it either in terms of charge Q or potential difference V.

$$\begin{aligned}\text{Energy stored in capacitor} &= \frac{Q^2}{2C} \\ &= \frac{1}{2}CV^2. \qquad (2.83)\end{aligned}$$

Thus for a given "voltage" V, the larger the capacitance the greater the energy stored.

This result can be also interpreted in a deeply physical way. For definiteness suppose we have a parallel plate capacitor, as in Fig. 2.31. The field strength between the plates and the total charge are given in terms of charge density σ and plate area A by Eq. (2.62)

$$E = \frac{\sigma}{\epsilon_0},$$

$$Q = A\sigma \qquad (2.84)$$

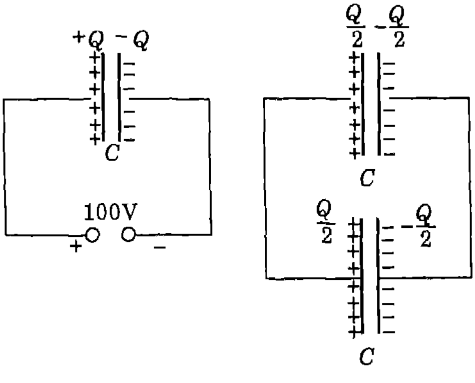
So the energy stored can be expressed in terms of the field strength.

Energy stored in parallel-plate capacitor

$$\begin{aligned}&= \frac{1}{2}\frac{Q^2}{C} \\ &= \frac{1}{2}(A\sigma)^2\frac{d}{\epsilon_0 A} \\ &= \frac{1}{2}(A\epsilon_0 E)^2\frac{d}{\epsilon_0 A} \\ &= \frac{1}{2}\epsilon_0 E^2 Ad \\ &\frac{\epsilon_0}{2}E^2 \times \begin{pmatrix}\text{volume between} \\ \text{plates}\end{pmatrix}\end{aligned} \qquad (2.85)$$

This leads to the interpretation: *where there is a non-zero electric field of strength E, there is an electrical energy density (= Electrical energy per unit volume) of amount* $(\epsilon_0/2)E^2$ J/m^3. While we have seen this result here in a special situation, it is a generally valid and very important statement.

Example 2.6: (a) A 900 pF capacitor is charged by 100 V battery. How much electrostatic energy is stored by the capacitor? The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system?



(a) (b)

Answer: The charge on the capacitor is

$$\begin{aligned}Q &= CV = 900 \times 10^{-12}\,\text{F} \times 100\,\text{V} \\ &= 9 \times 10^{-8}\,\text{C}.\end{aligned}$$

The energy stored by the capacitor is

$$\begin{aligned}&= \frac{1}{2}CV^2 = \frac{1}{2}QV \\ &= \frac{1}{2} \times 9 \times 10^{-8}\,\text{C} \times 100\,\text{V} \\ &= 4.5 \times 10^{-6}\,\text{J}.\end{aligned}$$

(b) In the steady situation, the two capacitors have their positive plates at the same potential, and their negative plates at the same potential. Let the common potential difference be V'. The charge on each capacitor is then $Q' = CV'$. By charge conservation, $Q' = Q/2$. The total energy of the system is

$$= 2 \times \frac{1}{2}Q'V' = \frac{1}{4}QV = 2.25 \times 10^{-6}\ \mathrm{J}$$

Thus in going from (a) to (b), though no charge in lost, the final energy is only half the initial energy. Where has the remainder of the energy gone?

There is a transient period before the system settles to the situation (b). During this period, a transient current flows from the first capacitor to the second Energy is lost during this time in the form of heat and electromagnetic radiation.

★ *Energy stored in combinations of capacitors*

If we put together many capacitors in the series or the parallel combination, we would expect that in both cases the total energy stored is additive This is indeed so. We can see this by using one or other of the two expressions (2.74, 77) for the effective capacitance in the two situations

For the series combination, we have $Q =$ constant. Using the result (2.74) for the effective capacitance, we have

Energy stored in series combination of capacitors

$$\begin{aligned} &= \frac{1}{2}Q^2\frac{1}{C} \\ &= \frac{1}{2}Q^2\left(\frac{1}{C_1}+\frac{1}{C_2}+\ \ldots\right) \\ &= \frac{1}{2}\frac{Q^2}{C_1}+\frac{1}{2}\frac{Q^2}{C_2}+\ldots \\ &= \text{Sum of energies stored in each capacitor} \end{aligned} \tag{2.86}$$

For the parallel combination, we have equal potential differences V, and the effective capacitance is given by Eq (2.77) So the energy is given by:

Energy stored in parallel combination of capacitors

$$\begin{aligned} &= \frac{1}{2}CV^2 \\ &= \frac{1}{2}(C_1+C_2\ldots)V^2 \\ &= \frac{1}{2}C_1V^2+\frac{1}{2}C_2V^2+\ldots \\ &= \text{Sum of energies stored in each capacitor} \end{aligned} \tag{2.87}$$

★

2.8 Effect of dielectrics in capacitors

Consider the parallel plate capacitor drawn in Fig 2.31. As we saw, if there is nothing between the plates – just vacuum – the capacitance is determined by the plate area A and separation d purely geometrically. Let us rewrite Eg (2.64) in this way:

Capacitance of parallel plate capacitor in vacuum:

$$C_0 = \epsilon_0\frac{A}{d} \tag{2.88}$$

Now suppose we fill the space between the plates with some dielectric substance – for example, glass or paraffin wax. We find that the capacitance becomes larger than the vacuum value C_0. The increase is by a (dimensionless) factor which depends on the dielectric. This factor is written as κ (Greek kappa) ; it is called the "dielectric constant" or "relative permittivity" of the dielectric. So we have:

Capacitance of parallel plate capacitor with dielectric.

$$\begin{aligned} C &= \kappa C_0 \\ &= \kappa\epsilon_0\frac{A}{d}, \\ \kappa &= \text{dielectric constant, or relative permittivity, of dielectric} \end{aligned} \tag{2.89}$$

Sometimes it is convenient to combine the factors κ and ϵ_0 into a single quantity ϵ, called "permittivity":

$$\epsilon = \kappa\epsilon_0 = \text{permittivity of dielectric.} \tag{2.90}$$

Now the name "permittivity of vacuum" for ϵ_0 (corresponding to $\kappa = 1$) is understandable. This was mentioned in the previous Chapter at Eq (1.6).

This increase in the capacitance of a capacitor when it is filled with a dielectric is true for any capacitor, not only in the parallel plate case. It has the following consequence. When given charges $\pm Q$ are put on the capacitor plates, the potential difference that develops between the plates will be less when there is a dielectric than when there is none. Thus the dielectric helps hold a larger amount of charge without running up too high a voltage. It also means that the electric field between the plates is less in the presence of a dielectric than in its absence, for the same plate charges.

The relative permittivities κ of some common dielectrics are.

Dielectric	Value of κ
Air	1.006
H_2 gas	1.00026
Glass	3 to 4
Mica	3 to 6
Paraffin wax	2 to 2.5
Transformer oil	2.24
Water	80

Since for air κ is quite close to unity, the capacitance of a capacitor measured under ordinary conditions in air is practically the same as the vacuum value. It is the latter that we were concerned with in the previous Section.

This effect of a dielectric on the capacitance of a capacitor is one of the many experimental discoveries made by Faraday. We sketch now a brief and qualitative explanation of it. We learnt in Chapter 1, in an optional part in Section 1.5, that in an electric field the atoms of a dielectric get polarized. Let us now look at this in slightly greater detail, to help understand Faraday's discovery.

There are in fact two types of dielectrics, i.e., insulating substances. nonpolar and polar. In the former type, in its normal state each atom (or molecule) of the dielectric has no dipole moment. That is to say, in each atom the "centre of charge" or average position of the electrons (the negative charges) is the same as that of the protons (the positive charges, in the nucleus). When such a substance is placed in an electric field (which is not too strong), within each atom the protons as a whole are pulled in the direction of the field, while the electrons are pulled the opposite way. This is what was shown in Fig. 1.5, and is shown again here in Fig. 2.35. Thus each atom gets distorted, two new centres of charge, separated by a certain distance proportional to the external field, get created. In effect, each atom becomes a tiny electric dipole, with a moment parallel to the external field, and proportional to it.

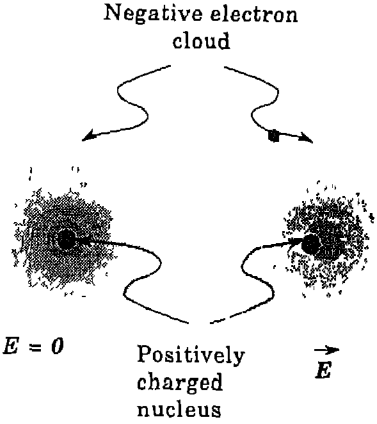


Figure 2.35: Polarization of an atom in an external electric field

On the other hand, in a polar dielectric the molecules of the substance have an unsymmetrical shape to begin with. So even in the normal condition, with no external field, each molecule has a nonzero spontaneous electric dipole moment. For example, a molecule of water, made up of two hydrogen atoms and one oxygen atom, has the unsymmetrical shape shown in Fig. 2.36. So it has a permanent dipole moment of its own. The effect of placing a polar substance in an electric field is then not so much the creation of any additional dipole moment, but of aligning the existing moments along the field because that is the state of lowest energy. We shall not deal with polar materials to any extent. We only mention in passing that crystals of such materials show small changes in dipole moment under pressure changes or mechanical stress, this leads to an electrical effect called piezoelectricity. It is the fact that water is a polar substance while, say, oil is not that explains their very different dielectric constants.

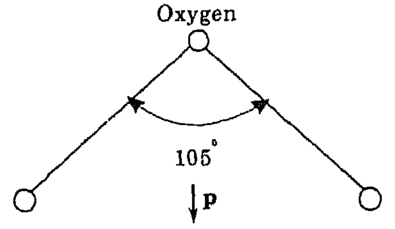


Figure 2.36: The water molecule – example of a polar substance

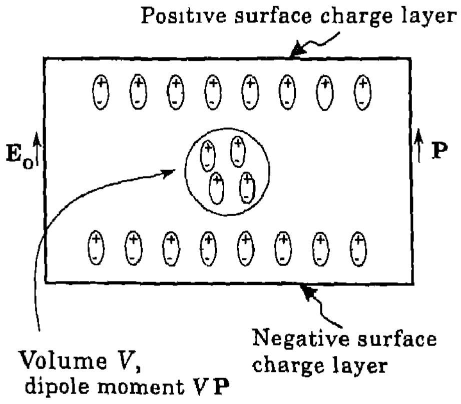


Figure 2.37: Uniformly polarized dielectric slab - equivalent surface charges

Now imagine for simplicity that a rectangular (non polar) dielectric slab is placed in a *uniform* electric field $\mathbf{E}_0$ parallel to two of its faces. We see that the atoms of the material get *uniformly polarized*. If we look at a small volume inside the slab, the dipole moments of all the atoms inside add up to an overall dipole moment proportional to the volume. All this is indicated in Fig. 2.37. This effect is proportional to the external field $\mathbf{E}_0$ and for constant $\mathbf{E}_0$, as we have assumed, it is also uniform all over the dielectric slab.

While each small volume in the slab thus develops a net dipole moment, we can see that as long as there are many atoms inside it, and it is in the interior, there is no net charge in the volume. The volume may be small in the macroscopic sense, but if it is large enough to contain many atoms this argument is valid. However at the "top" and "bottom" faces of the slab (the faces perpendicular to the field) the story is different! At the top face (head of $\mathbf{E}_0$) the positive ends of one layer of atomic dipoles stick out and remain uncancelled. And similarly at the bottom face (tail of $\mathbf{E}_0$) the negative ends of one layer of dipoles remain uncancelled. These have been depicted in Fig. 2.37. So, for macroscopic purposes a uniformly polarized rectangular dielectric slab behaves like (or is equivalent to) two uniform surface layers of charge over the two faces perpendicular to $\mathbf{E}_0$, with equal and opposite charge densities: positive at the head of $\mathbf{E}_0$, negative at the tail.

Now we can see why a dielectric increases the capacitance of a capacitor. Let the space between the plates be filled with a dielectric slab, as in Fig. 2.38. Initially the charges $\pm Q$ we put on the plates produce a uniform field between the plates. It points from the positive to the negative plate, and the strength E_0 is given by eq.(2.62) (with E rewritten as E_0):

$$E_0 = \frac{1}{\epsilon_0}\frac{Q}{A} \tag{2.91}$$

This field causes a uniform dipole moment distribution in the dielectric, in the same direction as the field between the plates. But we saw in the previous paragraph that the slab is then equivalent to a certain uniform negative surface charge density over the face adjoining the positive capacitor plate; and an equal and opposite positive surface charge density over the other face adjoining the negative capacitor plate. As far as the final electric field in the space between the plates is concerned, this means that the charges producing it have been reduced to values less than the charges $\pm Q$ we initially placed on the plates. So the field E in the region between the plates, i.e., throughout the volume of the dielectric, has a strength less than $\mathbf{E}_0$, the value in the absence of the dielectric. Correspondingly also

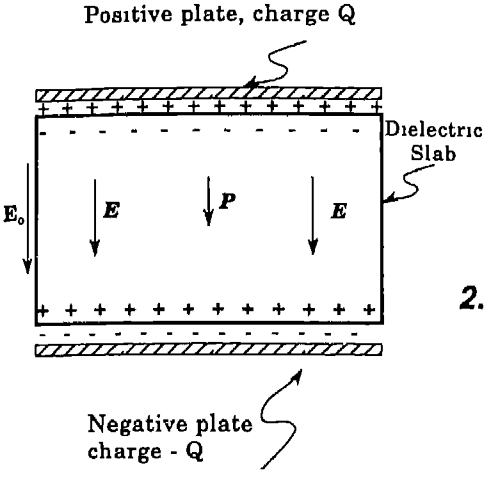


Figure 2.38: Effect of a dielectric in a capacitor

the potential difference between the plates is less than if the dielectric were absent It is this reduction that is expressed by the dielectric constant κ which appeared in Eq. (2.89)

$$\begin{aligned} \mathbf{E} &= \text{field strength between capacitor} \\ &\quad \text{plates in presence of dielectric} \\ &= \mathbf{E}_0/\kappa, \\ V &= \text{potential difference in} \\ &\quad \text{presence of dielectric} \\ &= (\text{potential difference in} \\ &\quad \text{absence of dielectric})/\kappa \end{aligned} \tag{2.92}$$

In this way, then, we see that the capacitance, being the ratio of Q to the potential difference, is increased by a factor κ when the dielectric is inserted.

The importance of this behaviour of dielectrics should be clear from this discussion. In particular it is worth emphasizing that while the dipole moment within the dielectric is caused by and is proportional and parallel to the initial electric field $\mathbf{E}_0$, its final effect is to *reduce* the strength of the field from $\mathbf{E}_0$ to $\mathbf{E}_0/\kappa$. this is because the surface charge layers to which it is equivalent create their own field in the opposite direction!

It is possible, and not too difficult, to connect the dielectric constant κ to the properties of individual atoms (or molecules) of the dielectric in an electric field, i e , to the magnitude of the dipole moment developed by each of them But we will not go into those details.

Coulomb's Law in a Dielectric Medium In older texts (especially in the days when the phenomenon of atomic polarizability was not very well understood and appreciated) the concept of the dielectric constant was introduced in this manner: in a dielectric medium, Coulomb's Law (1.26) is modified, the electric forces of attraction and repulsion among point charges q_1 and q_2, when surrounded by a dielectric medium, are reduced by the factor κ compared to the "vacuum value" given in Eq. (1.6).

Magnitude of Coulomb force between point charges q_1 and q_2 in dielectric medium, distance r apart,

$$\begin{aligned} &= \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}, \\ &= \frac{1}{\kappa} \times \text{vacuum value}, \\ \epsilon &= \kappa\epsilon_0 = \text{permittivity of dielectric}, \\ \kappa &= \text{dielectric constant of dielectric}. \end{aligned} \tag{2.93}$$

Thus the dielectric constant κ was introduced by a (macroscopic) modification of Coulomb's Law. But today it seems better and simpler to say that the Coulomb Law has not changed, and to develop a more physical picture of the effects of a dielectric by taking proper account of polarization at the atomic level.

Example 2.7: A slab of material of dielectric constant κ has the same area as the plates of a parallel-plate capacitor but has a thickness $3/4d$, where d is the separation of the plates How is the capacitance changed when the slab is inserted between the plates?

Answer: Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/\kappa$. The potential difference will then be

$$\begin{aligned} V &= E_0(\frac{1}{4}d) + \frac{E_0}{\kappa}\left(\frac{3}{4}d\right) \\ &= E_0 d\left(\frac{1}{4} + \frac{3}{4\kappa}\right) = V_0\frac{\kappa+3}{4\kappa} \end{aligned}$$

The potential difference decreases by the factor $(\kappa+3)/4\kappa$ while the free charge Q_0 on the plates remains unchanged. The capacitance thus increases:

$$C = \frac{Q_0}{V} = \frac{4\kappa}{\kappa+3}\,\frac{Q_0}{V_0} = \frac{4\kappa}{\kappa+3}\,C_0$$

★ *The electric displacement vector.* The effect of the polarization of the atoms in a dielectric when placed in an external field is described by the ***polarization density*** $\mathbf{P}$. This is the dipole moment per unit volume that gets built up due to the polarization. Apart from the polarization density $\mathbf{P}$, the effect of dielectrics in electrostatic problems is also described by another vector called the ***electric displacement vector***. It is denoted by the symbol $\mathbf{D}$. As we have seen, if a field $\mathbf{E}_0$ is produced by some source charges ***placed by us*** on some conducting or insulating bodies, then in any dielectric brought into this field the polarization density $\mathbf{P}$ gets built up. In the simplest substances, $\mathbf{P}$ is proportional to $\mathbf{E}_0$. The actual final electric field $\mathbf{E}$, taking into account both the sources set up by us and the equivalent surface charges describing the dielectric polarization $\mathbf{P}$, differs from $\mathbf{E}_0$. The displacement $\mathbf{D}$ is defined in terms of $\mathbf{E}$ and $\mathbf{P}$, and is the dielectric constant κ times $\mathbf{E}$.

$$\begin{aligned} \mathbf{D} &= \mathbf{E} + \frac{\mathbf{P}}{\epsilon_0} \\ &= \kappa\mathbf{E}. \end{aligned} \tag{2.94}$$

Comparing this with Eq. (2.92), we see that in the dielectric $\mathbf{D} = \mathbf{E}_0$. We also see that the proportionality of $\mathbf{P}$ to $\mathbf{E}_0$ is expressed by

$$\mathbf{P} = \epsilon_0\,\frac{\kappa-1}{\kappa}\,\mathbf{E}_0. \tag{2.95}$$

Usually, however, the polarization $\mathbf{P}$ is written as a constant of proportionality times the actual final electric field $\mathbf{E}$ (to which it has also contributed!); from Eq. (2.94),

$$\begin{aligned} \mathbf{P} &= \epsilon_0\,\chi\,\mathbf{E}, \\ \chi &= \kappa - 1 \end{aligned} \tag{2.96}$$

The factor χ that has been introduced here is called the ***electric susceptibility*** of the dielectric. like κ, it is also dimensionless.

The electric susceptibility χ is a natural measure of the polarizability of the medium, since the dipole moment developed in a small region of the dielectric depends on the ***total*** electric field $\mathbf{E}$ there, and is proportional to it, as well as to the volume of the region.

Such a description of the properties of a dielectric using constants κ and χ is quite good for many materials (and for not too large fields $\mathbf{E}$), they are then called ***linear dielectrics***.

In the example of the rectangular dielectric slab inserted into a parallel plate capacitor, we have noted from Eqs. (2.92, 94) that within the dielectric $\mathbf{D}$ has the value $\mathbf{E}_0$. Namely, this is the field produced by the charges $\pm Q$ we placed on the plates, unaffected by the polarization of the dielectric. If we write Eq. (2.94) as $\mathbf{E} = \mathbf{D} - \mathbf{P}/\epsilon_0$ we see vividly that the effect of the polarization $\mathbf{P}$ is to ***reduce*** the strength of the original field, due to the negative sign. This is generally true. The motivation for introducing the electric displacement vector is precisely to have something related directly to the charges we may place on conductors or insulators. Such charges are called "free" charges, unlike the "bound" charges within dielectrics. ★

2.9 The Atmosphere and its Electricity

In this Section we shall describe some very interesting features of the atmosphere covering the whole earth. Our main focus will be on the electrical properties and phenomena which go on all the time on a truly gigantic scale. We can also apply here some of the basic concepts we have been learning about, in a "real life" situation in nature.

Right away we must make some general points. Some amount of factual information and details will be presented here – they are

Table 2.1: Properties of the atmosphere

	Name of Layer	Extent in km above earth	Fall in density in the layer	Temperature variation in the layer
1	Troposphere	0 to 12	From 1 to 10^{-1}	Falls uniformly from 290K to 220K
2.	Stratosphere	12 to 50	From 10^{-1} to 10^{-3}	Rises uniformly from 220K to 280K
3	Mesosphere	50 to 80	From 10^{-3} to 10^{-5}	Falls uniformly from 280K to 180K
4.	Ionosphere	80 to 300	From 10^{-5} to 10^{-10}	Rises uniformly from 180K to 700K

quite fascinating, partly because much of the time most of us are unaware of them We must also always remember that *the atmosphere is a very variable system.* When we describe its properties it must be understood that we state its *average* behaviour – averages over large regions of space (sometimes the whole earth!), and over long periods of time (may be a day or more) When we give numerical magnitudes of various quantities we must be willing to give or take a factor of two or three in either direction All this must be kept in mind throughout this Section.

Major regions of the atmosphere· First let us look at some gross physical features of the atmosphere, and the names given to its different parts or layers. This is best done in the form of a table. We present the following information· the name and extent (height above ground level) of each major layer, the amount by which the density decreases in each layer as we ascend (density at any height being expressed as a fraction of ground level density); the nature and range of variation of temperature in each layer.

The earth's radius is about 6 4 × 10^6m We can say that the atmosphere extends to about 3 × 10^5m above the earth, i e., about one–twentieth the radius of the earth. To some extent this is a matter of definition, in any case, by this height the density is about 10^{-10} times its value at the sea level The parts and physical properties of the atmosphere are then as given in Table 2 1

The height ranges for the four layers are approximate and variable; in any case there are no sharp discontinuities between them We will not go here into the reasons why the temperature alternately decreases and then increases with height For orientation let us remember that Mt. Everest is 8 85 km high – well within the troposphere, and pretty cold.

Conductivity of the atmosphere – a giant spherical capacitor. For understanding the *electrical* properties of the atmosphere, we can limit ourselves to the height of 50 km – the troposphere and stratosphere The reason is this· the earth is a good conductor (so are our bodies) At low altitudes the atmosphere is a rather poor conductor. Whatever conductivity there is arises becauses of the presence of ions (molecules of gas with either an extra electron or an electron less), small "nuclei" of dirt, water vapour carrying static charge, etc

The ions are much more mobile than the dirt. Where do the ions come from? They are (mainly) created by a steady stream of cosmic rays constantly bombarding the earth from outer space These are very energetic charged particles which collide violently with the molecules of the atmosphere, especially at high altitudes, and ionize them The conductivity of the atmosphere is *very variable*. It changes from day to day at a given place, it depends *very much* on how much dust there is, how much humidity, etc. (So we repeat again that we speak only of average properties!) As we go up in the atmosphere, the conductivity gradually increases. There are more ions caused by the cosmic rays, and since the density is less their mean free path is more.

At about 50 km, the top of the stratosphere, *the atmosphere is a pretty good conductor!* The electrical phenomena we shall be describing take place between these two conducting spherical surfaces, the earth and the top of the stratosphere. (In between, as we said, the conductivity of the atmosphere increases gradually as we ascend.) Each of these surfaces is then an equipotential, so we really have a grand example of a spherical capacitor (except for the conductivity of the air). We can easily estimate its capacitance. We must use Eq. (2.70) with appropriate values for r_a and r_b, the inner (earth) and the outer (stratosphere) radii:

$$\begin{aligned} r_a &= 6.4 \times 10^6 \text{ m}, \\ r_b &= (6\,4 \times 10^6 + 5 \times 10^4) \text{ m}; \\ C &= 4\pi\epsilon_0 r_a\, r_b/(r_b - r_a) = 0\,1\text{F} \qquad (2.97) \end{aligned}$$

You see here again how immense one farad is!

Electrical properties of and phenomena in the atmosphere. Let us now briefly collect the main average electrical properties of the atmosphere, and the phenomena taking place in it. The following are experimentally observed facts:

(a) At ground level, there is a downward vertical electric field of about 100 V/m all over the earth. This field gets weak as we go to higher altitudes By 10 km it is quite small, at 50 km it is negligible

(b) There is a potential drop of about 4×10^5V from the conducting layer 50 km high to the earth below. Most of the potential drop occurs at low altitudes where the field is sizeable. This means the field strength is *not* the same at all heights; it decreases quite rapidly as we ascend.

(c) Since there is a downward electric field at ground level, and since the earth is a good conductor and an equipotential, there is a negative surface charge density all over the earth. We can estimate its magnitude by using Eq (2 58) for the field near a charged conducting surface·

$$\sigma = -\epsilon_0 E \simeq -10^{-9} \text{ C/m}^2 \qquad (2\,98)$$

Thus the total charge on the entire surface of the earth is

$$\begin{aligned} -Q &= 4\pi {r_a}^2 \sigma \\ &\simeq -0.5 \times 10^6 \text{ C}. \qquad (2\,99) \end{aligned}$$

As an order of magnitude, we can say this is about one million coulombs, negative of course.

(d) These figures are *very roughly* consistent with the picture of the earth and the top of the stratosphere forming a spherical capacitor. The latter surface must carry approximately $+10^6$ C of charge From the value (2.97) for the capacitance, we expect the potential difference and the electric field to be approximately

$$\begin{aligned} \Delta\varphi &= Q/C \simeq 10^7\ V, \\ E' &\simeq \Delta\varphi/(r_b - r_a) \\ &\simeq 200\text{V/m}. \qquad (2\,100) \end{aligned}$$

In actual fact $\Delta\varphi$ is smaller by a factor of about 25, and E is smaller at ground level by a factor of two These could be caused by the fact that the intervening air becomes a progressively better conductor as we go up.

(e) The constantly prevailing downward electric field causes a fairly stable current to flow into the earth. This includes both positive ions coming down and negative ones ascending. It amounts to about 3×10^{-12}C of charge being pumped into each square metre of the earth's surface each second One can express it in another way: each second, about +1800C of charge is being pumped into the earth as a whole This is roughly like having about 2×10^7 protons enter each square meter of the earth's surface each second.

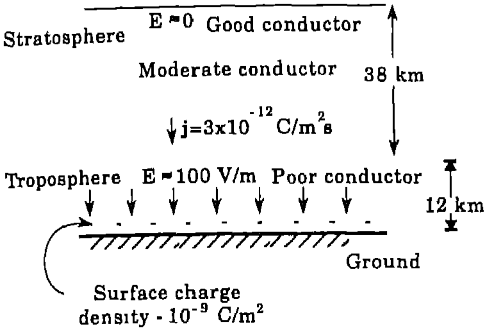


Figure 2.39: Average steady electrical properties of the atmosphere

All this information summarised above is depicted in Fig 2.39. It gives you a good picture of what is going on *all the time* But then, obviously, this cannot be the whole story! If the total charge on the earth is about -10^6C, and if each second +1800C is being deposited, then in about fifteen minutes the whole earth would become neutral. But that is not happening. The facts listed above do describe the *steady condition* of the earth and its atmosphere Something else must be happening all the time, to *maintain* the potential difference of 4×10^5V between stratosphere and earth, and the negative charge of -10^6C on the earth.

Thunder and Lightning: The "something else that must be happening" are thunderstorms and lightning flashes occurring world wide, and which we see in such spectacular fashion during the monsoon. The detailed mechanism of how this happens is quite complex, and is not yet very well understood. We only present a few important points here. We do this again as a series of statements based on observations:

(a) On the average there are about 40,000 thunderstorms per day worldwide. So we estimate that every two seconds a storm begins somewhere Typically each storm lasts about one hour

(b) Within each thundercloud (or thunderstorm cell as it is called), there is a separation of charged ions: positive charges are carried upwards to a height of about 6 km (above ground level), while negative charges collect at about 2 to 3 km above ground, at the bottom of the cloud The process is complex and not yet well understood.

(c) The total amount of this negative charge is quite large, may be −20 C to −30 C. This creates a potential difference of as much as 2×10^7V to 10^8V between ground and the bottom of the cloud, the former being at the *higher* potential. Thus, since the separation between the two is about 3 km, electric fields in the *upward* direction anywhere from 10^4 V/m to 3×10^4 V/m are created.

(d) Towards the end of a storm, when these conditions are built up, the high fields cause an "electrical breakdown" of the air It gets ionized and becomes able to conduct large amounts of negative charge in bursts along narrow pathways from cloud to earth

(e) Thus in the concluding stages of a storm, there are on the average about 200 bolts or flashes of lightning, each lasting about 2×10^{-3} s Combining with what we said in (a) above, this means that worldwide there are about 100 bolts of lightning each second It is estimated that 90% of them do carry negative charge to the earth.

(f) The peak electric current in each bolt of lightning is about -10^4 C/s in the downward direction Since each flash lasts about 2×10^{-3}s, we estimate that each bolt deposits −20 C of charge on the earth. After each bolt, the thunder cloud gets charged again, and is then ready for the next bolt. So with about 90 bolts per second doing this worldwide, we find that thunderstorms pump

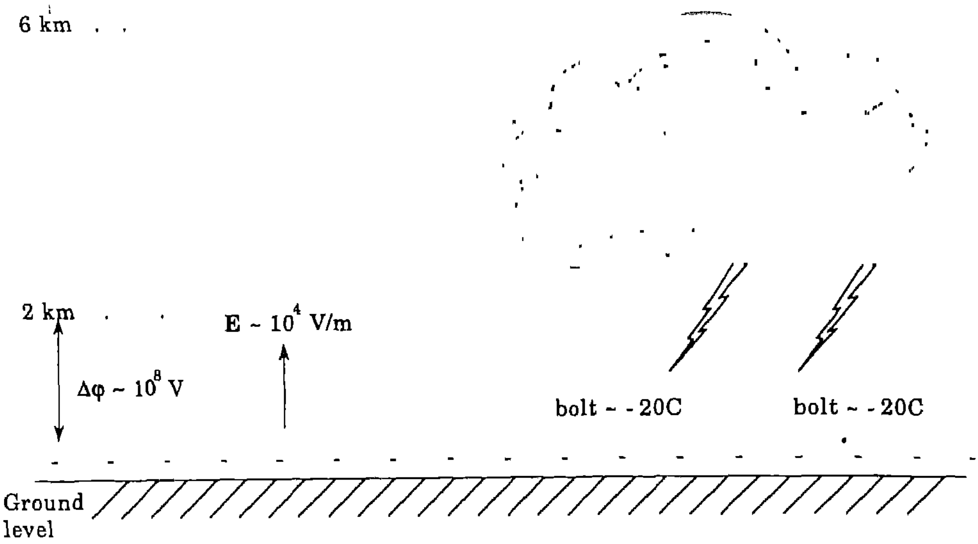


Figure 2.40: How lightning bolts recharge the earth negatively

−1800 C of charge into the earth each second. This is what is needed to counterbalance the steady inflow of +1800 C per second in the thunderstorm-free regions of the earth.

These salient features of thunder and lightning are shown in Fig. 2.40.

This is how this giant electrical machine works. One last intriguing fact: on the average, over each 24 hour period, the *maximum* of the steady downward positive current occurs *all over the earth* at 7 00 pm Greenwich Mean Time, which means half an hour past midnight Indian Standard Time. And the *minimum* occurs at 4.00 am Greenwich Mean Time, or 9.30 am Indian Standard Time. These extreme values differ from the average by about ±15%.

2.10 A man – made high voltage generator

We have seen how nature operates a mighty electrical machine which can create large charge concentrations, enormous potential differences and fields To be fair, let us describe briefly how humans too have built machines to produce high voltages.

Let us first explain the principle involved. Suppose we have a large spherical conducting shell of radius R, on which we place a charge Q. This charge spreads itself uniformly all over the sphere. As we have seen in Section 2.3 and also in Section 2.7 (see Eq. (2 19)), the field outside the sphere is just that of a point charge Q at the centre, while the field inside the sphere vanishes. So the potential outside is that of a point charge, and inside it is constant, namely the value at the radius R. We thus have.

Potential inside conducting spherical shell of radius R carrying charge Q

$$= \text{constant}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{R}. \tag{2.101}$$

Essentially this result was used in Eqs (2.68, 69) while discussing the spherical capacitor.

Now, as shown in Fig 2 41, let us suppose that in some way we introduce a small sphere of radius r, carrying some charge q, into the large one, and place it at the centre. The po-

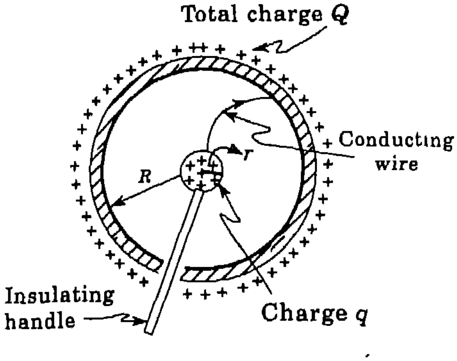


Figure 2.41: Illustrating the principle of the electrostatic generator

tential due to this new charge clearly has the following values at the radii indicated:
Potential due to small sphere of radius r carrying charge q

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r} \text{at surface of small sphere}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R} \text{at large shell of radius } R. \qquad (2.102)$$

Taking both charges q and Q into account we have for the total potential φ and the potential difference the values

$$\begin{aligned}\varphi(R) &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R}+\frac{q}{R}\right), \\ \varphi(r) &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right), \\ \varphi(r)-\varphi(R) &= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right) \qquad (2.103)\end{aligned}$$

Assume now that q is positive. We see that, independent of the amount of charge Q that may have accumulated on the larger sphere and even if it is positive, the inner sphere is always at a higher potential: the difference $\varphi(r) - \varphi(R)$ is positive. The potential due to Q is constant upto radius R and so cancels out in the difference!

This means that if we now connect the smaller and larger spheres by a wire, the charge q on the former will immediately flow onto the latter, even though the charge Q may be quite large. The natural tendency is for positive charges to move from higher to lower potential. Thus, provided we are somehow able to introduce the small charged sphere into the larger one, we can in this way keep piling up larger and larger amounts of charge on the latter. The potential (2.101) at the outer sphere would also keep rising, at least until we reach the breakdown field of air.

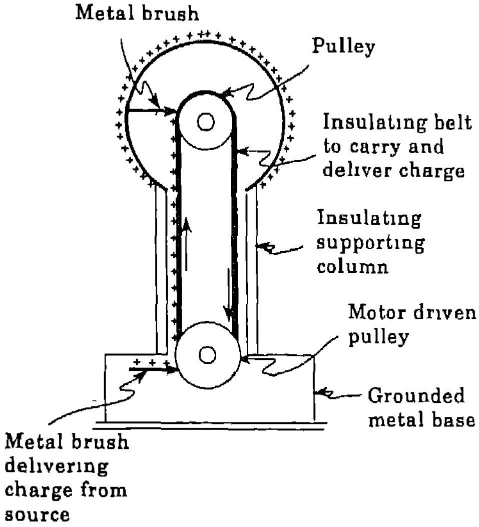

Metal brush
Pulley
Insulating belt to carry and deliver charge
Insulating supporting column
Motor driven pulley
Grounded metal base
Metal brush delivering charge from source

Figure 2.42: Principle of construction of van de Graaff generator

This is the principle of the van de Graaff generator. It is a machine capable of building up potential difference of a few million volts, and fields close to the breakdown field of air which is about 3×10^6 V/m. A schematic diagram of the van de Graaff generator is given in Fig. 2.42. A large spherical conducting shell (of few metres radius) is supported at a height several meters above the ground on an insulating column. A long narrow endless belt of insulating material, like rubber or silk, is wound around two pulleys – one at ground level, one at the centre of the shell. This belt is kept continuously moving by a motor driving the lower pulley. It continuously carries positive charge, sprayed on to it by a brush at ground level, to the top. There it transfers its positive charge to another conducting brush connected to the large shell. Thus positive charge is transferred to the shell, where it spreads out uniformly on the outer surface In this way voltage differences of as much as 6 or 8 million volts (with respect to ground) can be built up This is then used to accelerate charged particles (electrons, ions) to high speeds and energies for various experiments.

★ *Sharp points on conductors–the lightning arrester* We know that the field just outside a conducting surface is proportional to the surface charge density there – the connection is given in Eq (2 58). We also have seen that the potential to which a spherical conducting shell of radius R is raised by putting a charge on it is inversely proportional to R - see Eq (2 101). This means that a given amount of charge Q spread uniformly on a shell results in a *higher* voltage the *smaller* the radius From all this we can appreciate the following qualitative consequences mentioned at the end of Section 2 6 On the surface of any charged conductor, places which are smooth tend to have lower electric fields near them, and correspondingly lower surface charge densities. But sharp points and corners accumulate very high charge densities and have very high fields nearby

In any instrument intended to carry a large amount of charge, and not run the risk of breakdown of the air by ionization and leaking away of the charge, it is thus best to avoid sharp points This is why the conducting shell in the van de Graaff generator is made perfectly spherical

But the opposite is the case in the working of lightning rods! These are long metal rods with pointed tips, which are mounted on tall buildings to protect them from bolts of lightning In the event of a thunderstorm, which we have described in the previous Section, the air near the tip of the lightning rod gets ionized early because fields there become large. This then initiates and provides a pathway in the air along which the charge built up in a cloud can quickly travel and pass to the earth Thus other objects and places are saved ★

Summary

1. *Gauss's theorem:* The flux of electric field through any *closed* surface S is $1/\epsilon_0$ times the total charge *inside* S. The theorem is especially useful in determining electric field $\mathbf{E}$, when the source distribution has simple symmetry

   Thin infinitely long straight wire of uniform linear charge density λ.

   $$\mathbf{E}(\mathbf{r}) = \frac{\lambda}{2\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{r}$$

   where r is the perpendicular distance of the point from the wire and $\hat{\mathbf{r}}$ is a unit vector in the radial direction.

   Infinite thin plane sheet of uniform surface charge density σ

   $$\mathbf{E}(\mathbf{r}) = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}}$$

   where $\hat{\mathbf{n}}$ is a unit vector normal to the plane, outward on either side.

   Thin spherical shell of uniform surface charge density σ *and radius* R

   $$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{r^2}, \quad r > R$$

   $$= 0, r < R$$

   where r is the distance of the point from the centre of the shell q is the total charge of the shell; $q = 4\pi R^2\sigma$

   Solid Sphere of uniform volume charge density ρ *and radius* R

   $$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\ \frac{\mathbf{r}}{R^3}, \quad r \leq R$$

   $$= \frac{q}{4\pi\epsilon_0}\ \frac{\mathbf{r}}{r^3}, \quad r \geq R$$

   Here

   $$q = \frac{4\pi}{3}R^3\rho$$

   For both the shell and sphere cases, $\mathbf{E}$ *outside* is as though the total charge is concentrated at the centre.

2 Work done (by an external agency) in moving a charge δq from a point A to a point B in an *electrostatic* field is independent of the path taken from A to B. It is equal to

   $$\delta q[\varphi(\mathbf{r}_B) - \varphi(\mathbf{r}_A)]$$

   where $[\varphi(\mathbf{r}_B) - \varphi(\mathbf{r}_A)]$ is the difference in potential between B and A. Over any closed loop, work done is zero.

3. Potential at a point is the work done per unit charge (by an external agency) in bringing a charge from infinity to that point. Potential at a point is arbitrary to within an additive constant, since it is the potential *difference* between two points which is physically significant If potential at infinity is chosen to be zero, potential due to a point charge q at the origin is given by

$$\varphi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

and that due to two charges q_1, q_2 at $\mathbf{r}_1$ and $\mathbf{r}_2$ is given, using the superposition principle, to be

$$\varphi(\mathbf{r}) = \frac{q_1}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}-\mathbf{r}_1|} + \frac{q_2}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}-\mathbf{r}_1|}$$

4 The electrostatic potential at a point A with position vector $\vec{r}$, due to a dipole of moment $\vec{p}$ at the origin, is

$$\Phi(A) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\ \mathbf{r}}{r^3}$$

5 An equipotential surface is a surface over which potential has a constant value For a point charge, surfaces of spheres about the location of the charge are equipotentials. The electric field $\mathbf{E}$ at a point is perpendicular to the equipotential surface through the point. $\mathbf{E}$ is in the direction of steepest decrease of potential

6 Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations Potential energy of two charges q_1, q_2 at $\mathbf{r}_1$, $\mathbf{r}_2$ is equal to

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\mathbf{r}_1-\mathbf{r}_2|}$$

The potential energy of a charge q in an external potential $\varphi(\mathbf{r})$ is

$$q\,\varphi(\mathbf{r})$$

The potential energy of a dipole $\mathbf{p}$ in a uniform electric field $\mathbf{E}$ is

$$-\mathbf{p}\cdot\mathbf{E}.$$

7 In the static case, the electric field $\mathbf{E}$ is zero in the interior of a conductor, just outside its surface, $\mathbf{E}$ is normal to the surface Charges in a conductor can reside only at its surface. Potential is constant within and on the surface of a conductor In a cavity within a conductor, the electric field is zero

8 A capacitor is a system of two conductors Its capacitance C is defined by $C = Q/V$, where Q and $-Q$ are the charges on the two conductors and V is the potential difference between them C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. The unit of capacitance is the farad: $1\text{F} = 1\text{CV}^{-1}$

For a parallel plate capacitor,

$$C = \epsilon_0\, A/d$$

where A is the area of each plate and d is the separation between the plates. For a spherical capacitor,

$$C = \frac{4\pi\epsilon_0 r_a r_b}{(r_b - r_a)}$$

where r_a and r_b are the radii of the inner and outer spheres respectively

9 For capacitors in the *series* combination, the total capacitance C is given by,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} +$$

In the parallel combination, the formula for C is

$$C = C_1 + C_2 + C_3 + \ .$$

where C_1, C_2, C_3 are individual capacitances

10 The energy E stored in a capacitor of capacitance C and charge Q with voltage V is

$$E = Q^2/2C$$
$$= \frac{1}{2}CV^2$$

The electrical energy density (energy per unit volume) in a region with electric field $\mathbf{E}$ is

$$\frac{1}{2}\epsilon_0 \mathbf{E}^2$$

11. If the medium between the plates of a capacitor is filled with an insulating material, the electric field due to the charge on the plates polarizes the medium Polarization gives rise to a field in the opposite direction The net electric field and hence the potential difference between the plates is thus reduced Consequently, the capacitance C increases from its value C_0 when there is no medium (vacuum).

$$C = \kappa C_0$$

where κ is the dielectric constant of the medium

12 The earth has an electric field of about 100 $\mathrm{Vm^{-1}}$ at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} $\mathrm{Cm^{-2}}$ Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is a good conductor) about + 1800 C is pumped every second into the earth as a whole The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth

13 A Van de Graaff generator consists of a large spherical conducting shell (a few m in diameter) By means of a moving belt and suitable brushes, charge is continuously transferred to the shell, and potential differences of the order of several million volts are built up, which can be used for accelerating charged particles.

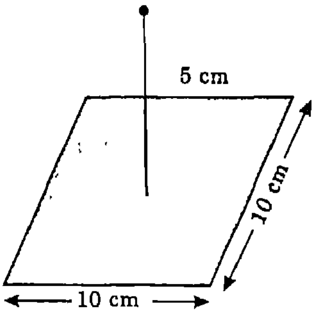
Exercises

2.1 Consider a uniform electric field $\mathbf{E} = 3 \times 10^3\,\hat{\mathbf{i}}\,\mathrm{N/C}$. (a) What is the flux of this field through a square of 10cm on a side whose plane is parallel to the yz plane? (b) what is the flux through the same square if the normal to its plane makes a 60° angle with the x axis?

2.2 What is the net flux of the uniform electric field of Exercise 2.1 through a cube of side 20cm oriented so that its faces are parallel to the coordinate planes?

2.3 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3\,\mathrm{Nm^2/C}$. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?

2.4 A point charge $+10\,\mu\mathrm{C}$ is a distance 5cm directly above the centre of a square of side 10cm as shown in the figure. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10cm.)



2.5 A point charge of $2.0\,\mu C$ is at the centre of a cubic Gaussian surface 9.0cm on edge. What is the net electric flux through the surface?

2.6 A point charge causes an electric flux of $-1.0 \times 10^3\,\mathrm{Nm^2/C}$ to pass through a spherical Gaussian surface of 10.0cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) what is the value of the point-charge?

2.7 A conducting sphere of radius 10cm has an unknown charge. If the electric field 20cm from the centre of the sphere is $1.5 \times 10^3\,\mathrm{N/C}$ and points radially inward, what is the net charge on the sphere?

2.8 A uniformly charged conducting sphere of 2.4m diameter has a surface charge density of $80.0\,\mu\mathrm{C/m^2}$. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

2.9 An infinite line charge produces a field of $9 \times 10^4\,\mathrm{N/C}$ at a distance of 2cm. Calculate the linear charge density.

2.10 Two large, thin metal plates are parallel and close to each other as shown in Fig. 2.8 in the text. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\,\mathrm{C/m^2}$. What is E (a) to the left of the plates, (b) to the right of the plates, and (c) between the plates?

2.11 Two charges 3×10^{-8}C and -2×10^{-8}C are located 15cm apart At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

2.12 A regular hexagon of side 10cm has a charge 5μC at each of its vertices Calculate the potential at the centre of the hexagon.

2.13 (a) Calculate the potential at a point P due to a charge of 4×10^{-7}C located 9cm away.

(b) Hence obtain the work done in bringing a charge of 2×10^{-9}C from infinity to the point P Does the answer depend on the path along which the charge is brought?

2.14 (a) Determine the electrostatic potential energy of a system containing two charges 7μC and -2μC separated by a distance of 18cm.

(b) How much work is required to separate the two charges infinitely away from each other?

2.15 Two charges 2μC and -2μC are placed at points A and B 6cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

2.16 A spherical conductor of radius 12cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18cm from the centre of the sphere?

2.17 A parallel-plate capacitor with air between the plates has a capacitance of 8pF (1pF = 10^{-12} F) What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

2.18 Three capacitors each of capacitance 9pF are connected in series

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120V supply?

2.19 Three capacitors of capacitances 2pF, 3pF and 4pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100V supply

2.20 In a parallel plate capacitor with air between the plates each plate has an area of 6×10^{-3}m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100V supply, what is the charge on each plate of the capacitor?

2.21 Explain what would happen if in the capacitor given in 2.20, a 3mm thick mica sheet (of dielectric constant = 6) were inserted between the plates

(a) while the voltage supply remained connected,

(b) after the supply was disconnected

2.22 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

2.23 A 600pF capacitor is charged by a 200V supply It is then disconnected from the supply and is connected to another uncharged 600pF capacitor How much electrostatic energy is lost in the process?

Additional Exercises

2.24 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9}C from a point P (0,0, 3cm) to a point Q (0, 4cm, 0) via a point R(0,6cm,9cm)

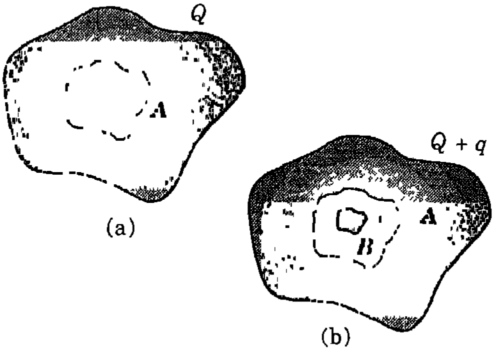
2.25 A cube of side b has a charge q at each of its vertices Determine the potential and electric field due to this charge array at the centre of the cube.

2.26 Two tiny spheres carrying charges 1.5μC and 2.5μC are located 30cm apart Find the potential and electric field:-

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10cm from this mid-point in a plane normal to the line and passing through the mid-point.

2.27 (a) A conductor A with a cavity as shown in Fig (a) is given a charge Q Show that the entire charge must appear on the outer surface of the conductor (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A Show that the total charge on the outside surface of A is $Q+q$ [Fig. (b)] (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment Suggest a possible way.



2.28 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero even if the shell is not spherical, but has any irregular shape? Explain

2.29 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$(\mathbf{E}_2 - \mathbf{E}_1)\ \hat{\mathbf{n}} = \frac{\sigma}{\epsilon_0}$$

Where $\hat{\mathbf{n}}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point (The direction of $\hat{n}$ is from side 1 to side 2) Hence show that just outside a conductor, the electric field is $\sigma\hat{n}/\epsilon_0$,

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. (Hint. For (a), use Gauss's law For (b), use the fact that work done by electrostatic field on a closed loop is zero.)

2.30 A hollow charged conductor has a tiny hole cut into its surface Show that the electric field in the hole is $(\sigma/2\epsilon_0)\,\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

2.31 A long charged cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

2.32 Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law [Hint: Use Coulomb's law directly and evaluate the necessary integral]

2.33 It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks A proton and a neutron consists of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge +(2/3)e, and the 'down' quark (denoted by d) of charge (-1/3)e, together with electrons build up ordinary matter. (quarks of other types have also been found which give rise to different unusual varieties of matter). Suggest a possible quark composition of a proton and neutron

2.34 In a hydrogen atom, the electron and proton are bound at a distance of about 0 53 Å

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1 06 Å separation?

2.35 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion (H_2^+). In the ground state of a H_2^+, the two protons are separated by roughly 1 5 Å, and the electron is roughly 1 Å from each proton Determine the potential energy of the system. Specify your choice of the zero of potential energy.

2.36 (a) Consider an arbitrary electrostatic field configuration A small test charge is placed at a null-point (i e where E=0) of the configuration Show that the equilibrium of the test charge is necessarily unstable.

(b)Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart

2.37 Two charged conducting spheres of radii a and b are connected to each other by a wire What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions

2.38 Answer carefully:

(a) A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity).

(b) Ordinary rubber is an insulator. But the special rubber tires of aircrafts are made slightly conducting Why is this necessary?

(c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?

(d) A bird perches on a bare high-power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

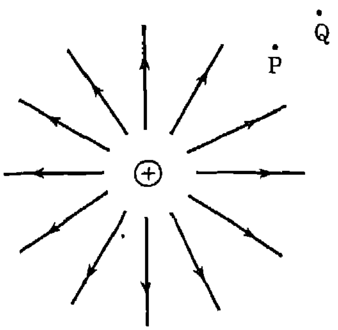
2.39 Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively, (a) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y.0)$?

(b) Obtain the dependence of potential on the distance r of the point from the origin when $r/a \gg 1$

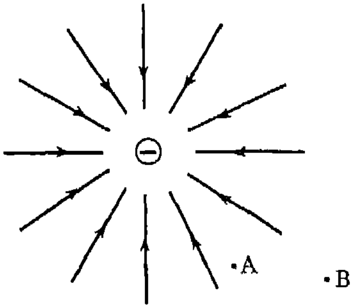
(c) How much work is done in moving a small test charge from the point (5,0,0) to (-7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

2.40 Figures (a) and (b) show the field lines of a single positive and negative charge respectively: (a) Give the signs of the potential difference

$$V_P - V_Q\ ;\ V_B - V_A$$



(a)



(b)

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P, A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

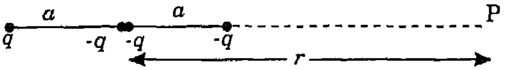
(d) Give the sign of the work done by an external agency in moving a small negative charge from B to A

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

2.41 A molecule of a substance has permanent electric dipole moment equal to

10^{-29}Cm A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude (10^6Vm^{-1}) The direction of the field is suddenly changed by an angle of 60° Estimate the heat released by the substance in aligning its dipoles along the new direction of the field For simplicity assume 100% polarization of the sample.

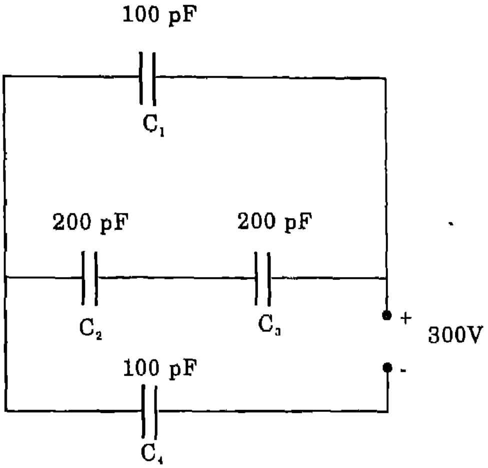
2.42 Figure below shows a charge array known as an 'electric quadrupole'. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a \gg 1$, and contrast your results with that due to an electric dipole, and an electric monopole(i.e. a single charge).



2.43 An electrical technician requires a capacitance of '2'μF in a circuit across a potential difference of 1kV. A large number of 1μF capacitor are available to him each of which can withstand a potential difference of not more than 400V. Suggest a possible arrangement that requires the minimum number of capacitors.

2.44 What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0 5cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less However, electrolytic capacitors do have a much larger capacitance (0.1F) because of very minute separation between the conductors]

2.45 Obtain the equivalent capacitance of the following network: For a 300V supply, determine the charge and voltage across each capacitor.



2.46 The plates of a parallel plate capacitor have an area of 90cm^2 each and are separated by 2 5mm The capacitor is charged by connecting it to a 400V supply (a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

2.47 A 4μF capacitor is charged by a 200V supply It is then disconnected from the supply, and is connected to another uncharged 2μF capacitor How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

2.48 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $1/2\ QE$, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates Explain the origin of the factor $1/2$.

2.49 A spherical capacitor has an inner sphere of radius 12cm and an outer sphere of radius 13cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. (a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12cm Explain why the latter is much smaller.

2.50 Answer carefully:

(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1Q_2/4\pi\epsilon_0 r^2$ where r is the distance between their centers?

(b) If coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(c) A small test charge is released at rest at a point in an electrostatic field configuration Will it travel along the line of force passing through that point?

(d) What is the work done by the field of a nucleus in a *complete* circular orbit of the electron? What if the orbit is elliptical?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(f) What meaning would you give to the capacity of a single conductor?

(g) Guess a possible reason why water has a much greater dielectric constant (=80) than say mica (=6).

2.51 A cylindrical capacitor has two coaxial cylinders of length 15cm and radii 1 5cm and 1.4cm The outer cylinder is earthed and the inner cylinder is given a charge of 3.5μC. Determine the capacitance of the system and the potential of the inner cylinders. Neglect end effects (i e bending of field lines at the ends).

2.52 A parallel plate capacitor is to be designed with a voltage rating 1kV, using a material of dielectric constant 3 and dielectric strength about 10^7Vm^{-1} (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e without starting to conduct electricity through partial ionization) For safety, we should like the field never to exceed say 10% of the dielectric strength What minimum area of the plates is required to have a capacitance of 50pF?

2.53 Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the z-direction,

(b) a field that uniformly increases in magnitude but remains in a constant (say z) direction,

(c) a single positive charge at the origin

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane

2.54 In a Van de Graaff type generator, a spherical metal shell is to be a 15×10^6V electrode The dielectric strength of the gas surrounding the electrode is $5 \times 10^7 \text{Vm}^{-1}$. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one can not build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential)

2.55 A small sphere of radius r and charge q_1 is enclosed by a spherical shell of radius r and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

2.56 Answer the following:

(a) The top of the atmosphere is at about 400kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100Vm^{-1} Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminum sheet of area 1m^2 Will he get an electric shock if he touches the metal sheet next morning?

(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

CHAPTER 3

Current Electricity

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3.1 Introduction

The phenomena discussed in the previous Chapters arise from the circumstance that positive and negative electric charges can be separated from each other by macroscopic distances (e.g. distances of order 10^{-3}m or more) whereas in atoms, the distance between elementary charges is of order 10^{-10}m This large separation of charges and clumping together of like ones (electrification) makes it possible to investigate the laws of force between them in the laboratory However the charges are at rest and in static equilibrium (*electrostatics*). The motion or *dynamics* of charges gives rise to new effects which we discuss in this and several subsequent Chapters.

We introduce in this Chapter the ideas of *electrical current* (or the rate of flow of electric charge) and *electrical resistance* or resistance to the flow of electrical current (Section 3 2). Some features of electrical resistance, an explanation of its origin, and systems with unusual resistance behaviour, are discussed in Section 3.3. Most electrical devices consist of a number of circuit elements through which electrical current flows, these are interconnected, often in complicated ways. The basic rules (Kirchhoff's rules) determining currents in such circuit elements are introduced in Section 3.4. Finally methods for measuring resistances, currents and voltages are discussed in Section 3 5.

3.2 Current, voltage and resistance

3.2.1 Current

We are all familiar with many examples of currents One such is the current of water in a flowing river. Similarly, if electric charge flows through some region, say a wire of copper, we say that there is an electric *current* in that region, e.g. through the copper wire. For example, when a battery torch is switched on, a current of electric charge circulates continuously through the battery bulb, heating it and producing light. What we see as lightning is the flow of electric charges from a cloud to the earth, or from one cloud to another In such and similar situations, effects occur which are due to the *motion* of charges, or due to currents. These effects are different from those due to charges at rest (electrostatics) which have been studied in the last two Chapters. In this Chapter we begin with the description of electric current, of resistance to its flow, and of how these are measured

How does one describe current? In the case of water flowing in a river, it is the amount of water passing across the river per unit time. The amount can be measured in kilograms or cubic metres, so water current can be described as so many cubic metres per second. (For example, the average current of water flowing through the Ganga is 38,000 m^3/s, exceeded only by two rivers, the Amazon with 180,000 m^3/s and the Congo with 40,000 m^3/s). Electric current, namely the flow of electric charge, can be described by the amount of charge flowing through a particular area in a unit time, or as the rate of flow of electric charge. Suppose a charge ΔQ passes through a given cross section or area in a short time Δt. The electric current I (or current) I is given by

$$I = \frac{\Delta Q}{\Delta t} \tag{3 1a}$$

More precisely, the current at a given time t is the ratio of charge ΔQ flowing to time interval Δt around t in the limit that the time interval becomes very small, i.e.

$$I = \lim_{\Delta t \to 0} \left(\frac{\Delta Q}{\Delta t} \right) = \frac{dQ}{dt} \tag{3.1b}$$

If the current is steady i.e. it does not change with time, then the charge Q flowing through is just proportional to time t, and we have

$$I = \frac{Q}{t} \text{ (for a steady current)} \tag{3.2}$$

The SI unit of current is the ampere, after Andre Marie Ampere, a French scientist who discovered many consequences of electric currents The ampere is one of the base SI units As we shall see in greater detail in Chapter 5, Section 5 3 current carrying conductors produce forces on each other and the ampere is defined in terms of this force The formal definition is:- *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length and of negligible cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length* This definition will become more understandable in Chapter 5 where magnetic effects of electric currents are discussed. The system of units is such that *an ampere is equal to one coulomb per second,* though since the ampere is the base unit, the coulomb should more appropriately be thought of as ampere second. (See Section 1 5 for a discussion of this). The symbol for ampere is A, e.g we say that the current through the electric motor is 2.5 A (or 2.5 amperes)

3.2.2 Voltage and electromotive force (emf)

What makes electrical current flow? No current flows in a copper wire, by itself, just as water in a horizontal tube does not flow. If one end of the tube is connected to a tank with water, such that there is a pressure difference between the two ends of the horizontal tube, water flows out of the other end at a steady rate (we have discussed this in Section 10.6 on viscosity, in the Class XI Physics textbook). The rate at which water flows out depends on the pressure difference, for a given tube. If the flow rate or current is to be kept constant, the water flowing out for instance has to be put back into the jar (see Fig 3.1) to maintain the pressure head This requires work to be done (at a certain rate) by another agency. The above analogy brings out several features of electrical current flow: An

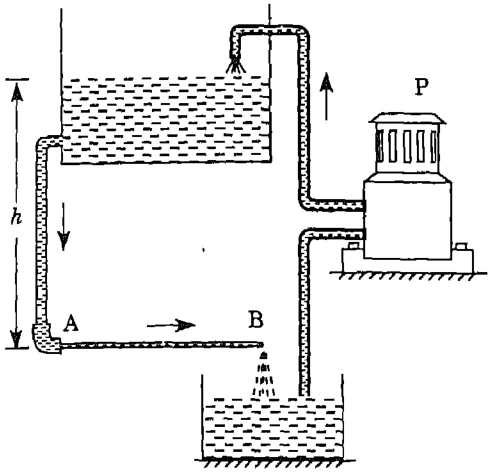


Figure 3.1: Water flows continuously out of a horizontal tube AB at a steady rate if a pressure head (pressure difference between its two ends) is maintained by the tank at a height h, and if a pump P pumps the water back to the tank at the same rate as it flows out at B

electric current flows across a conductor only if there is an electric potential difference between its two ends, just as water flows through the horizontal tube only if there is a pressure difference between the two ends. Secondly, to maintain a steady current flow, one needs an agency which does work on the charges. This agency is the seat of the electromotive force or emf., as explained further below In the case of water flow, the agency is the pump P which does work at a steady rate in putting the water back up into the tank. Finally just as for a given tube, the current of fluid flowing out depends on its *viscosity* (compare say, honey and water), similarly the electrical current flowing for a given potential difference depends on the electrical *resistance* (or simply the resistance) of the conductor. In Fig 3 2 we show a simple electrical or charge flow circuit similar to the water flow circuit of Fig. 3.1. A steady electric current i flows through the resistor R, from A to B. The potential drop between A and B is V. A source P of emf does work on the charges as they come through at B The charge is transferred from

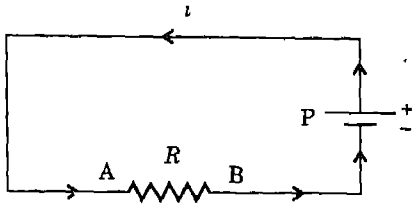


Figure 3.2: A charge flow circuit analogous to the water flow circuit of Fig 3 1. A steady electric current flows across the resistor AB if a source P of emf (e g , a cell) is connected across it.

one end of the source of emf to the other (from - to + in the figure) and a work qV is done on a charge q

The source of emf, by doing work on electric charges, maintains a potential difference V between its terminals or positive and negative ends. Such a device is called a seat of electromotive force or emf., If such a device is connected as shown in Fig. 3 2 across the resistance R, a steady current i flows through the circuit.

The seat of the emf could be a *battery* or an *electrochemical* cell In this, chemical energy is converted to electrical energy in such a way that a potential difference exists between two terminals of the cell. (See Section 4 4). This was historically the first source of emf and led to the establishment of steady electrical currents and discovery of its consequences. The most important or widespread seats of emf are *electric generators*. These could, for example, produce potential differences of the order of 11 to 15 thousand volts These voltages are further increased to larger values, and electricity is transmitted across large distances before the voltages are reduced in steps to about 220 V, the emf that you can normally measure with a suitable meter between the two holes of a socket in your house. (Chapters 7 and 8 discuss the principle of the generator). Some other seats of emf are *solar* or *photovoltaic* cells (solar energy converted to electrical energy) and *thermocouples* (Section 4 3).

The emf $\mathcal{E}$ *of the seat (or source) is defined as the work done per unit charge,* W/Q, *where W is the work done by the source on a charge Q, i e.*

$$\mathcal{E} = \frac{W}{Q}. \qquad (3.3)$$

The work is done in taking a positive charge through the seat of the emf from the low potential end to the high potential end. Clearly, the seat has emf whether current flows or not. The emf (when no current flows through the seat) is exactly equal to the potential difference between its ends. *The unit of emf is the same as that of electric potential, namely volt* (V) However, the phrase emf is always associated with a seat or source of potential difference, an active device which maintains a potential difference across its terminals. For example, when we say that the dry cell has an emf of 1.5 V, we mean that it maintains a potential difference of 1.5 V between its positive and negative terminals; it does a work of 1.5 J on one coulomb passing through it from the negative to the positive terminal, enabling the charge to pass through. Finally, the name electromotive force suggests an active device moving electrical charge (electro *motive* means *moving* electrical charges); however, the word force is somewhat misleading since the dimension of emf is energy per charge, whereas the dimension of force as you already know is energy per unit length (or distance).

3.2.3 Resistance

When a source of emf is connected to a wire or any other object (called conductor) allowing the passage of electrical charge, there is a voltage drop V across the conductor and a steady or direct current (say I) flows through it (and the source of emf). The ratio of the two, namely (V/I), is called the electrical resistance of the conductor Clearly if a small current I flows through the conductor for a given voltage V, its resistance is high The unit of emf is a volt (after A. Volta, who was one of the inventors of the electrochemical cell). This unit has been defined earlier, in Section 2.2. The ratio of one volt and one ampere (the SI unit of current) is the unit of electrical resistance, and is called ohm (symbol Ω)

3.3 Ohm's law and resistivity of materials

3.3.1 Ohm's law

In 1828, Georg Simon Ohm, a German physicist, found that for many substances, the electrical current I flowing through them is proportional to the voltage V across their ends. This empirical relation

$$V \propto I \qquad (3\,4)$$

is called Ohm's law The constant of proportionality, called the resistance and given the symbol R, is independent of V or I, but depends only on the substance and its shape and size Thus Ohm's law is

$$V = RI \qquad (3.5)$$

As mentioned above, the unit of resistance, (volt per ampere) is given a separate name, ohm and a symbol Ω.

3.3.2 Resistance and resistivity

The resistance of a material depends (as mentioned above) on the nature of a substance as well as on its shape and size. The latter dependence is easy to understand Consider a steady current I flowing through an electrically conducting wire of constant cross section A and length l, the potential drop across the conductor being V. Suppose now that the length of the wire is doubled to $2l$, the current flowing through remaining the same (Fig. 3.3a). Clearly the voltage drop across the second half, of length l, is the same as that across the first half, i e. V. The voltage across the wire of length $2l$ is $2V$ Thus the resistance doubles on doubling the length, if the cross section and material remain the same. We can then argue that generally,

$$R \propto l. \qquad (3\,6)$$

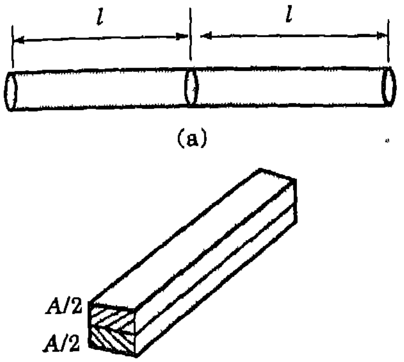


Figure 3.3: Illustration of factors affecting resistance and current flow (a) A wire of length $2l$ and constant cross section, divided into two halves of length l (b) a wire of square cross sectional area A, imagined to consist of two rectangular wires of cross sectional area $(A/2)$

How does resistance depend on the cross-sectional area A? To find out, again imagine a simple case, say of a conductor with a square cross section, and imagine it as being made of two rectangular parallel wires, each of cross sectional area $(A/2)$, touching each other throughout their length (Fig. 3.3b). Clearly both wires carry equal current, being identical. Since the total current is I, the current flowing in each half is $(I/2)$. Since the potential drop across either half is V, its resistance is $V/(I/2) = 2R$. Since the half wire slice and the full wire differ only in cross sectional area and that too by a factor of two, we conclude, on generalizing this example, that

$$R \propto (1/A) \tag{3.7}$$

Combining (3.6) and (3.7), we see that

$$R \propto (l/A) \tag{3.8a}$$

or

$$R = \rho(l/A) \tag{3.8b}$$

where the constant of proportionality ρ, called resistivity, depends neither on the length nor on the (uniform) cross-sectional area of the conductor. It is a property of the material; it also depends on physical conditions such as temperature and pressure. Resistivity is measured in units of ohm metre (Ωm) as one can see from Eq. (3.8b). If the resistivity of a substance is known, the resistance of a given piece of it can be calculated provided its length, cross-section etc. are known. Very often the inverses of R and ρ, called conductance and conductivity are used. Their commonly used symbols are G and σ; their units are $(\text{ohm})^{-1}$ or mho and $(\text{ohm m})^{-1}$ or mho m^{-1}. The unit siemens (symbol S) is also used for $(\text{ohm})^{-1}$

The electrical resistivity of substances varies over a very wide range as can be seen from Table 3.1. We notice that metals have low resistivities, in the range of 10^{-8} Ωm to 10^{-6} Ωm. They are good conductors of electricity. At the other end are substances like glass and rubber which are 10^{22} to 10^{24} times more resistive! These are insulators. In between, on a logarithmic scale, lie the resistivities of semiconductors like Si and Ge.

Different materials are useful for different electrical purposes. A copper or aluminium wire, having low resistance, is ideal for transporting electrical current without appreciable loss of energy, as we shall see in Chapter 4. In many electrical and electronic circuits, resistances of fairly large specified values, e.g. 1 kΩ, or 33.7 kΩ are needed. Such resistances are often made of some kind of semiconducting material, e.g. carbon. If electrical current is not to pass between two points, then a good insulator (e.g. mica, bakelite, glass) should intervene.

Resistances with a wide range of values are extensively used in electrical and electronic circuits. A colour code is used to indicate the resistance value and its percentage accuracy. The resistor has a set of coaxial rings on it with their significance as indicated in Table 3.2. The first two bands from the end indicate the first two significant figures of the resistance, in ohms. The third band indicates the decimal multiplier and the last band stands

Table 3.1: Electrical resistivities of some substances

Material	Resistivity (Ω m) at 0° C ρ	Temperature coefficient (° C)$^{-1}$ $\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$ at 0°C	Number of outer (valence) electrons per unit cell
A Conductors			
Silver	1.6×10^{-8}	0.0041	1
Copper	1.7×10^{-8}	0.0068	1
Aluminium	2.7×10^{-8}	0.0043	3
Tungsten	5.6×10^{-8}	0.0045	6
Iron	10×10^{-8}	0.0065	8
Platinum	11×10^{-8}	0.0039	10
Mercury	98×10^{-8}	0.0009	2
Nichrome (alloy of Ni, Fe, Cr)	$\sim 100\times10^{-8}$	0.0004	
B. Semiconductors			
Carbon (graphite)	3.5×10^{-5}	-0.0005	4
Germanium	0.46	-0.05	4
Silicon	2300	-0.07	4
C. Insulators			
Glass	10^{10} - 10^{14}		?
Hard Rubber	10^{13} - 10^{16}		?
NaCl	$\sim 10^{14}$		8

Table 3.2: Resistor colour code

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^{1}	
Red	2	10^{2}	
Orange	3	10^{3}	
Yellow	4	10^{4}	
Green	5	10^{5}	
Blue	6	10^{6}	
Violet	7	10^{7}	
Gray	8	10^{8}	
White	9	10^{9}	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

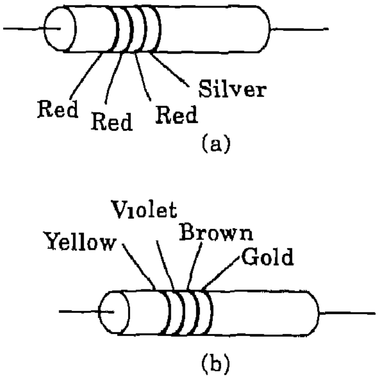


Figure 3.4: Colour coded resistors
(a) $(22\times10^{2}\Omega)\pm 10\%$ (b) $(47\times10\Omega)\pm 5\%$

for the tolerance or possible variation in per cent about the indicated value (See Fig. 3.4)

So far, we have not said anything about what flows when there is an electrical current, why current and voltage are related by Ohm's law, and what determines electrical resistivity. The remainder of this Section is concerned with such questions.

3.3.3 Carriers of current

In most substances, *electrons* flow when there is a voltage difference between two parts of it. In a few cases, e g. ionic crystals and some electrolytic liquids, e g. salt solution, positive and negative *ions* carry the electrical current (Chapter 4). How do we know that electrons carry the current in most conductors? An experiment directly suggesting this was performed by the American physicists Tolman and Stewart in 1917. They measured the angular momentum due to the charges flowing with a certain average drift speed in a circular loop carrying current This angular momentum can be shown to depend on the charge to mass ratio (e/m) of the charge carriers, and thus to identify them.

In metals or good electrical conductors, electrons are free to move inside the solid. In many cases each atom contributes all its electrons outside the closed shell (outer or valence electrons) so that the number of free electrons is equal to the number of atoms times the number of valence electrons per atom. However, these electrons do not spill out of the metal because they are attracted back to the ions in the solid, and also because of other surface forces. Therefore, energy called the work function is needed to remove an electron from inside the metal to the space outside When two metals are joined together, changes (must and do) take place at their interface such that no electrons flow freely from one metal to another if there is no potential difference between the two, and electrons flow only if the potential difference is nonzero

---

★ Why is it that electrical current is carried in most solid conductors by electrons? As you know (Chapter 9 of Class XI textbook) atoms are very close to each other in a solid This means that the outer or valence electrons of one atom are close to neighbouring atoms, experiencing forces due to them If these forces are strong enough an electron in a solid need not be attached to a particular atom, but can hop from one atom to the next, it moves around in the entire solid The electrons outside a closed shell are 'free' in this sense and can carry electrical current (see below for details of the mechanism) This picture is incomplete because it would imply that all solids containing atoms with electrons outside a closed shell ought to be good conductors of electricity. This is not true Many such solids, e g sulphur and diamond are insulators The reason that some solids are metals and others insulators has to do with the wave nature of electrons. It turns out that if the number of valence electrons per unit cell of the crystalline solid (See Section 10 1 of Class XI textbook for a discussion of the unit cell) is even, the solid *can* be an insulator *or* a metal It *is* a metal if this number is odd This rule is not easy to explain at this stage, but can be applied easily enough Consider some first group solids, namely Li, Na, K, Rb, Cs, Cu, Ag, Cu They all have one atom in a unit cell of the crystalline solid An atom of any one of these elements has one electron called the valence electron, outside the closed shell This is true of the noble metal atoms in their monovalent or 'ous' state, not in their divalent or 'ic' state Thus these solids with one outer electron per unit cell are expected to be metallic as indeed they are Now consider the fourth group elements C, Si, Ge, Sn, Pb They all again have one atom per unit cell in the crystalline solid state, and four valence electrons per atom According to the rule above, these solids with four outer electrons per unit cell can be insulating or metallic. Carbon in the form of diamond is an insulator, as are silicon and germanium Tin and lead are metallic The idea can be applied to compounds as well, e g. NaCl has 1 + 7 = 8 valence electrons per unit cell, and is an insulator

In some cases, this rule is violated, and solids with an odd number of electrons per unit cell are insulators. In other cases, e g for glassy and amorphous systems and liquids, a unit cell cannot be defined, many such insulators, e g glass, rubber, transformer oil, are well known We do not fully understand why such systems are insulators, this is an active area of research. ★

3.3.4 Nature of electrical resistance

We now try to understand Ohm's law, or the flow of electric charge in a conductor when a potential difference V exists across its ends, in terms of free electrons moving under the influence of the voltage V. The conductor is of length l, so that the electric field E inside it is (V/l), i.e.

$$E = (V/l). \tag{3.9}$$

The field points in the direction shown Fig. 3.5.

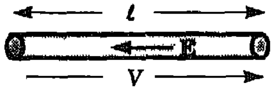


Figure 3.5: A conductor of uniform cross-section and length l. The electric potential difference across it is V. The electric field inside is **E** as shown, with a magnitude (V/l)

In the absence of an electric field, electrons in a conductor move with different velocities in all possible directions, no particular direction being preferred The distribution of speeds is shown in Fig. 3.6. The electric field causes an acceleration (eE/m) of each electron in its direction. This acceleration continues till it is disrupted by random collisions of electrons with vibrating atoms or with atoms in the wrong place. The path of an electron undergoing random collisions may look like Fig. 3.7. If the average time between collisions is τ, the average drift velocity of an electron is

$$v_d = (eE/m)\tau. \tag{3.10}$$

To calculate the resulting current, we count the number of electrons passing through the cross section A of the conductor in a short

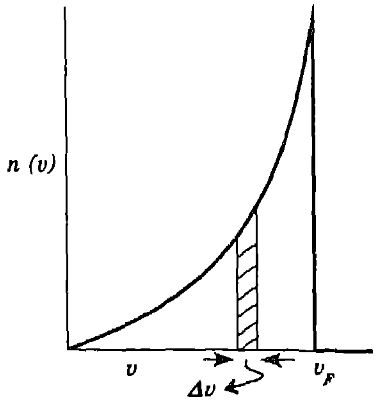


Figure 3.6: The distribution of speeds of free electrons in a metal at 0 K The number between v and $v + \Delta v$ is proportional to the shaded area. There are no electrons with speed larger than v_F typically of order 10^6m/s The curve is a parabola. The distribution of speeds at room temperature is nearly the same.

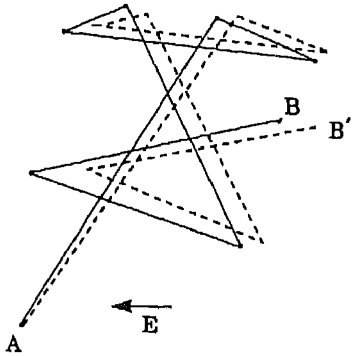


Figure 3.7: A schematic picture of an electron moving from a point A to another point B through repeated collisions and straight line travel between collisions(full lines) If an electric field is applied as shown, the electron ends up at point B′(dotted lines). A slight drift in a direction opposite the electric field is visible

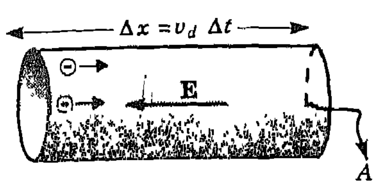


Figure 3.8: Current in a metallic conductor. The symbols are as discussed in the text

time Δt If the drift velocity is v_d all electrons within a distance $\Delta x = v_d\ \Delta t$ of the cross-section will pass through it within the time Δt (See Fig. 3.8). Now if the number of electrons per unit volume is n, the number in the volume $A\Delta x$ is $nA\Delta x$ These carry a charge $\Delta q = neA\Delta x$. Thus the current I through A is

$$I = \frac{\Delta q}{\Delta t} = neA\,\frac{\Delta x}{\Delta t} \qquad (3.11)$$

$$= \frac{neAv_d\Delta t}{\Delta t} = neAv_d \qquad (3.12)$$

The current I can also be thought of as follows· a current density $j = nev_d$ (current per unit area) flows through an area A so that the current $I = jA = nev_dA$ Now,

$$I = neAv_d = neA\frac{eE}{m}\tau$$

$$= neA\,\frac{eV}{lm}\tau$$

$$= \frac{ne^2\tau}{m}\frac{A}{l}V \qquad (3\ 13)$$

$$\text{or } V = RI = \frac{\rho l}{A}I \qquad (3\ 14)$$

$$\text{where } \rho = \frac{m}{ne^2\tau}. \qquad (3\ 15)$$

We have thus derived Ohm's law from first principles; what is more, we have been able, in Eq. (3.15), to relate the resistivity ρ to properties of the conductor such as electron density n and average time τ between collisions (called relaxation time). An interesting feature of Ohm's law is that a uniform force eE on electrons gives rise, *not* to a net acceleration (eE/m), but a drift *velocity* $v_d = (eE\tau/m)$, Eq. (3.10), on account of random, deflecting collisions which relax the electron distribution to equilibrium. The formula Eq. (3 10) is somewhat similar to the Stokes formula Eq (10 20) of the class XI text book, where it was shown that the *velocity* of a sphere in a viscous medium is proportional to the *force* acting on it In our case, the analogue of viscosity is the resistance to the flow of electrons caused by the collisions or deflections described above.

★ One is tempted to argue that if the uniform acceleration (eE/m) acts for a time τ, the average velocity is $(1/2)(eE\tau/m)$ i e half the value given in Eq (3.11) This argument is wrong, because collisions are random processes. The probability P that an electron does not collide in a time t is $P(t) = \frac{e^{-t/\tau}}{\tau}$ so that the average velocity gain is

$$v_d = \int_0^\infty v(t)P(t)\,dt$$

$$= \int_0^\infty (\frac{eE}{m}t)\ \frac{e^{-t/\tau}}{\tau}\ dt = \frac{eE\tau}{m}. \qquad (3.16)$$

★

We now discuss the relation Eq (3.15) for resistivity . the resistivity depends inversely on electron density n and the collision time τ. For a given metal (fixed n), the latter is a function, clearly, of what causes collisions. They may be due to atomic vibrations. Since the amplitude of vibrations increases with thermal energy or temperature, collisions become more effective and frequent as temperature increases Thus the resistivity of metals is expected to increase with temperature. Fig. 3.9a shows that for pure copper, this is indeed the case. Around room temperature and above, resistivity increases linearly with temperature In the temperature range in which the resistivity or resistance increases linearly with temperature, one can define a *temperature coefficient* (of resistivity or resistance) as the *fractional increase* in resistivity or resistance per unit increase in temperature It is given a symbol α and has dimensions of per degree celsius (or whatever the temperature unit). Typical values of α range for metals from 10^{-2} to $10^{-4}/(°C)$ At low temperatures, however, the resistivity increases as a higher power of temperature (see Fig. 3.9(a)). In a number of other metals, e.g. nichrome, which is an alloy of nickel with chromium, the resistivity is very large, and very weakly temperature dependent [Fig. 3.9(b)] This is because an electron in the alloy sees a very disordered arrangement of Ni^{2+} and Cr^{3+} ions and is randomly scattered by them, this scattering being very little affected by temperature. Thus there is a large resistivity even at the absolute zero (zero degree Kelvin), this is called residual resistivity. This fact can be and actually is used to check on the purity of elemental metals. If the metal is very pure, the residual resistivity is very small.

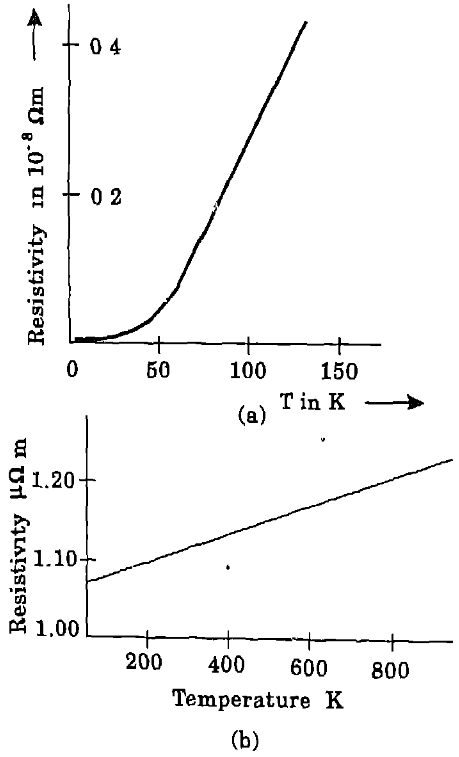


Figure 3.9: Resistivity $\rho(T)$ of (a) copper and of (b) nichrome as a function of absolute temperature T The resistivity is in units of $10^{-8}\Omega$m or $\mu\Omega$ cm, and the temperature is in kelvin

In many systems, the resistivity *increases* nearly exponentially as temperature *decreases*, becoming very large at low temperatures (infinite in the limit $T \to 0$). Such substances (i.e. those having infinite resistance at $T = 0$) are called insulators (or semiconductors, see below for an explanation of the difference). By contrast, metals have either zero electrical resistance (if pure and perfect) or finite resistance (if alloyed or disordered) as $T \to 0$. The temperature dependence of resistivity in insulating substances is often given by

$$\rho(T) = \rho_0(T) \ e^{(E_g/k_BT)} \tag{3.17}$$

where k_B is the Boltzmann constant and E_g is a positive energy. The pre-exponential factor $\rho_o(T)$ is weakly dependent on temperature and is typically of order one to a hundred microhm meter or so. The very large variation in the resistivity of substances is due to the fact that E_g can vary from nearly zero to several electron volts (eV). Now at room temperature $k_BT \simeq 0.03$eV so that the exponent can be as small as zero and as large as hundred or more. The division of nonconducting substances into semiconductors and insulators has to do with the size of the energy E_g. If $E_g \lesssim 1$eV or so, the room temperature resistivity is not very high and the substances are called semiconductors For $E_g \gtrsim 1$ eV, the resistivity is very high (greater than about 10^5 Ωm at room temperature) and the substance is called an insulator.

★ Eq (3.15) for electrical resistivity, in conjunction with the Eq (3.17) means that either the number density n of electrons or the relaxation time changes exponentially with temperature. It turns out that the former is correct; the electrical conductivity of an insulator is small because the number density of electrons carrying the current is exponentially small. The probability of a particle in thermodynamic equilibrium at a temperature T having an energy E is proportional to exp $(-E/k_BT)$. (This is known as the Boltzmann thermal probability distribution and k_B is therefore called the Boltzmann constant) Thus in insulators an energy E_g is required to 'free' an electron for transporting current. One has

$$n(T) = n_o exp\,(-E_g/k_BT) \tag{3.18}$$

where n_o is typically of order 10^{28} /m^3 ('metallic' electron density). The energy E_g is called an energy gap. The exponential increase in resistivity as temperature decreases is a characteristic electrical property of semiconductors and insulators.

★

Example 3.1 : (a) Estimate the average drift speed of conduction electrons in a copper wire of cross sectional area 1.0×10^{-7} m^2 carrying a current of 1.5A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/ m^3, and its atomic mass is 63.5.

(b) Compare the drift speed obtained above with (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speeds of electrons carrying the current and (iii) speed of propagation of electric field along the conductor which causes the drift motion.

Answer : (a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e. electrons drift in the direction of increasing potential. The drift speed v_d is given by

$$v_d = (I/neA) \tag{3.19}$$

Now, $e = 1.6 \times 10^{-19}$C, $A = 1.0 \times 10^{-7}$m^2, $I = 1.5$A. The density of conduction electrons, n is equal to the number of atoms per cubic metre (assuming one conduction electron per Cu atom as is reasonable from its valence electron count of one). A cubic metre of copper has a mass of 9.0×10^3kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g,

$$n = \frac{6.0 \times 10^{23}}{63.5} \times 9.0 \times 10^6$$
$$= 8.5 \times 10^{28}\text{m}^{-3}$$

which gives

$$v_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$
$$= 1.1 \times 10^{-3}\,\text{ms}^{-1}.$$

(b) (i) At a temperature T, the thermal speed of a copper atom of mass M is typically of the order of $\sqrt{k_BT/M}$, where k_B is the Boltzmann constant. For copper, at 300K, this is about 4×10^4 m/s. This figure indicates the random vibrational speeds of copper atoms in a conductor. Note, the drift speed of electrons is much smaller, about 10^{-7} times the typical thermal speed at ordinary temperatures.

(ii) The maximum kinetic energy 1/2 (mv_F^2) of electron in copper corresponds to a temperature $T_o \simeq 10^5$K, i.e. $(1/2)\ mv_F^2 = k_B T_o$ with $T_o \simeq 10^5$K. (This information is from a book on solid state physics) From this we estimate $v_F = 1.4 \times 10^6$m/s. Thus the drift velocity is nearly 10^{-9} times typical electron speeds!

(iii) An electric field travelling along the conductor has a speed of any electromagnetic wave, namely equal to $3.0 \times 10^8\,\text{ms}^{-1}$. The drift speed is, in comparison, extremely small; smaller by a factor of 10^{-11}.

★ The drift velocity is rather small, as Example 3.1 shows. How is it then that an electric bulb lights up as soon as we turn the switch on, i.e. connect it to the mains? To see what is involved, imagine a simple circuit with a battery, a switch, and a small battery lamp, connected as shown (Fig. 3.10a). When the switch is off (Fig. 3.10a), the voltage at the two ends of it is as shown, i.e. the difference $(V_1 - V_2)$ is equal to the emf of the cell. The moment the switch is turned on, the voltage difference across it is reduced to zero, and the distribution of electric potential is as shown in Fig. 3.10b. This is quite different from the final voltage distribution when a steady current flows through the circuit (Fig. 3.10c). The question then is, how long does it take to go from the state described in Fig. 3.10b to that in Fig. 3.10c? Clearly, the voltage or electric field in various parts of the circuit depends on time during this period. We shall see in Chapter 9 that a time varying electric field produces a magnetic field and in Chapter 7 that a time varying magnetic field produces an electric field. Further, we will see

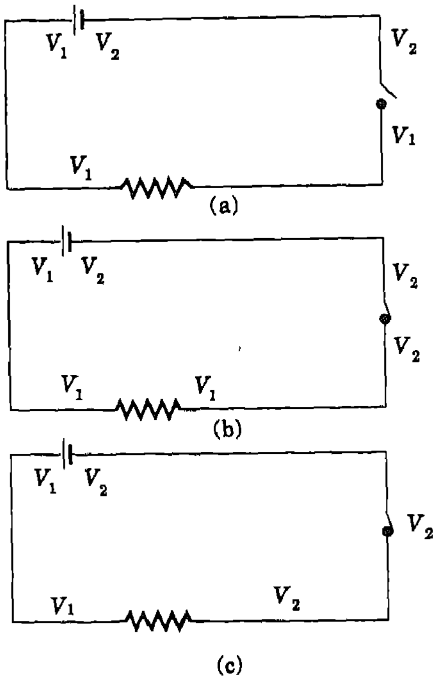


Figure 3.10: Three stages of completion of an electrical circuit (a) Open circuit (b) Just after the circuit is completed (c) Steady state voltage distribution

(Chapter 9) that these connected electrical and magnetic fields travel (propagate) with a speed $c \simeq 3 \times 10^8$ m/s in free space This is the speed of light, which is an electromagnetic wave. It turns out that in many situations the speed with which a steady state is attained, is close to (but less than) c The speed also depends on a number of circuit features, e g. capacitance, as we shall see for simple examples in Chapter 8 We note that a particular electron does not have to physically reach the electric bulb in order for the current to flow. An electric field has to be established in each region of the conductor such that an appropriate steady current flows in that part (in our simple case the same current flows in each part of the circuit) This process can occur with a speed comparable to that of light For example, when a message is tapped out from one end of a telegraph wire (i e, when the circuit is connected and disconnected in a particular way so as to produce voltage pulses with a certain pattern), it reaches the other end with a speed close to c. ★

3.3.5 Limitations of Ohm's Law

Ohm's law is not a fundamental law of nature, but a commonly found property of many substances under certain conditions. In many cases, therefore, the relation between voltage and current is different from that of Eq. (3.5). Some of these are specially worth mentioning.

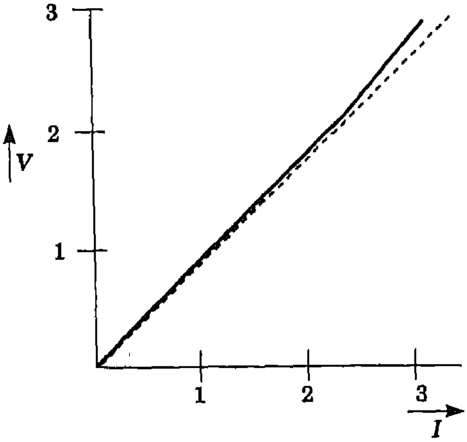


Figure 3.11: The dashed line represents the linear Ohm's law. The solid line is the voltage V vs current I for a good conductor

There are a number of commonly used circuit elements with one or more of the following properties:

(a) V depends on I nonlinearly.

(b) The relation between V and I depends on the sign of V for the same absolute value of V.

(c) The relation between V and I is non-unique, that is, for the same current I there is more than one value of voltage V.

We illustrate these with examples now If the current flowing through a conductor is increased, the conductor becomes hotter and its resistance increases. But even if the temperature is kept constant, some conductors show an increase in resistivity as current increases

(Fig. 3.11). This is genuine nonlinear behaviour, easiest to see in p-n junctions, briefly described below. A detailed discussion of their nature and charactersitics is given in Chapter 14.

A p-n junction diode rectifier combines properties (a) and (b). It consists of a junction of two kinds of semiconductors, p and n type germanium for example (Fig. 3.12). A voltage V is applied across the junction The resulting current is shown in Fig. 3.13a. We notice that V is not proportional to I. Further, hardly any current flows till a fairly large negative voltage (called negative or reverse bias) V is applied, whereas a sizeable current begins to flow for much smaller positive or forward bias. This property makes it useful as a circuit element which allows current to pass in only one direction; such a device is called a *rectifier*.

Fig. 3.13b shows the characteristics of a device known as the thyristor, which consists of four alternate layers of p- and n-type semi conductors. Notice again that V is not proportional to I. All the features (a), (b) and (c) can be seen in the graph. The region AB is interesting because here the current carried by the device increases as the voltage decreases!

A more subtle and fundamental breakdown of Ohm's law, or more precisely the relations Eqs (3 6) and (3.15) occurs in certain alloys at very low temperatures of order 4K or less. It is found for example that the resistance of a wire of constant cross section and the same material, with length $2l$ is *more* than twice the resistance of the wire with length l We have thought of electron flow as similar to flow of a viscous liquid in a tube under a pressure gradient (Poiseuille's experiment, Chapter 10, class XI text book). However, electrons have a wave character (Chapter 12). This leads to interference effects, increasing the resistance with respect to the value it would have if electrons flowed like water.

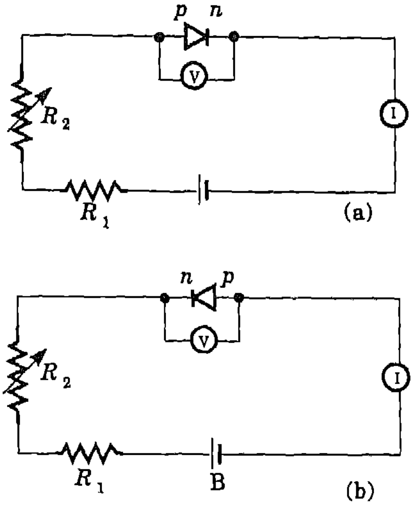


Figure 3.12: A p-n junction diode rectifier (shown as $-\triangleright|-$). (a) Diode in forward bias (b) Diode in reverse bias

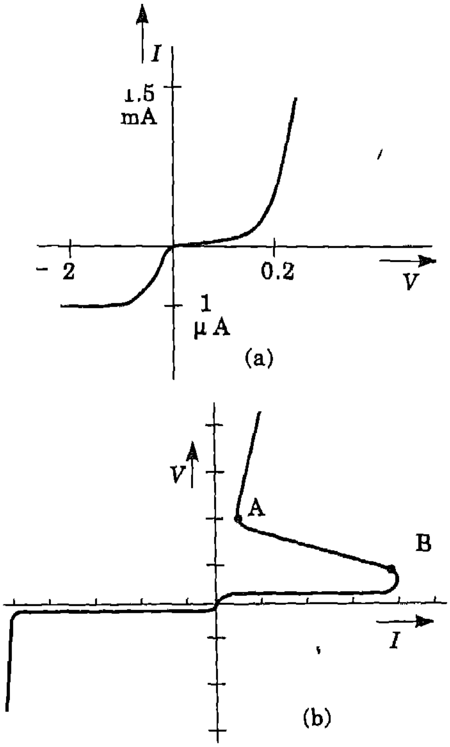


Figure 3.13: (a) Characterstics of a diode Note the different scales for negative and positive values of the voltage and current
(b) Characterstics of a thyristor

3.3.6 Superconductivity

Are there systems in which electrons can flow without any resistance whatsoever? As we all know through newspaper reports in the last few years (1987- to present), there are many such; namely the superconductors. Below a certain temperature (e.g. 10 K for Nb and 90 K for $YBa_2Cu_3O_7$), these substances lose their electrical resistivity *entirely*. This spectacular behaviour is certainly not because ionic vibrations that cause deflections cease. The reason is rather that electrons are not independent in a superconductor, but are mutually coherent. Ionic vibrations are unable to deflect this coherent or cooperative cloud of electrons. It is still not clear how exactly this coherence comes about in the recently discovered oxide superconductors, e g. $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_7$

Example 3.2 : **An electric toaster uses nichrome (an alloy of nickel and chromium) for its heating element When a negligibly small current passes through it, its resistance at room temperature (27.0°C) is found to be 75.3Ω. When the toaster is connected to a 230V supply the current settles after a few seconds to a steady value of 2 68A What is the steady temperature of the nichrome element? The temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} °C^{-1}.**

Answer : When the current through the element is very small, heating effects can be ignored and the temperature T_1 of the element is the same as room temperature. When the toaster is connected to the supply, its initial current will be slightly higher than its steady value of 2.68A. But due to heating effect of the current, the temperature will rise. This will cause an increase in resistance and a slight decrease in current. In a few seconds, a steady state will be reached when temperature will rise no further, and both the resistance of the element and the current drawn will achieve steady values The resistance R_2 at the steady temperature T_2 is

$$R_2 = \frac{230\text{V}}{2.68\text{A}} = 85.8\Omega$$

Using the relation

$$\begin{aligned} R_2 &= R_1\,[1 + \alpha\,(T_2 - T_1)] \\ \text{with } \alpha &= 1.70 \times 10^{-4}\ \text{C}^{-1} \text{ we get} \\ T_2 - T_1 &= \frac{(85.8 - 75.3)}{(75.3) \times 1.70 \times 10^{-4}} \\ \text{i.e. } T_2 &= 820 + 27\ 0 = 847°\text{C}. \end{aligned}$$

Thus the steady temperature of the heating element (when the heating effect due to the current equals heat loss to the surroundings) is 847° C

3.4 Electrical circuits and Kirchhoff's rules

In this Section, we introduce two simple and general ideas which can be applied to find out the electrical current flowing through or the voltage drop across resistances in complicated circuits Such circuits are extremely common in modern electrical and electronic systems. Fig. 3 14 shows a real life medium wave receiver circuit It contains several circuit elements which will be discussed later (see e.g. Chapters 7 and 14, and appendix C for explanation of symbols).

Here we make a beginning with resistances as the circuit elements, and show how Kirchhoff's rules, based on charge neutrality in a metal and conservation of energy, greatly help in calculating electrical properties. The rules are obviously applicable to more complicated circuit elements.

3.4.1 Electrical circuits

Internal resistance

Let us first consider a very simple circuit, i.e. one with a resistance R connected to a source of emf (labelled B in Fig. 3.15), with emf $\mathcal{E}$ Now most sources of emf have internal resistance, e.g. in a cell, the ions have to move against the background of other ions and of

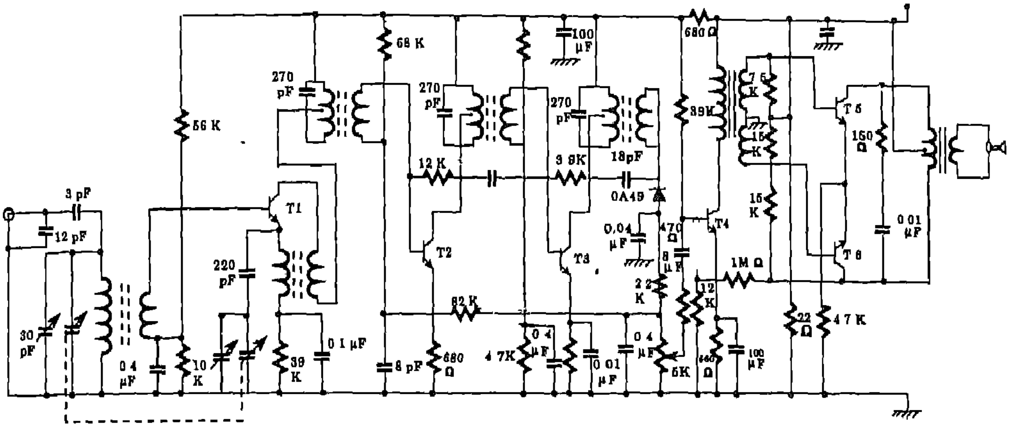


Figure 3.14: Medium wave receiver circuit

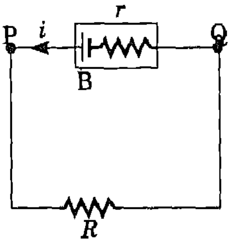


Figure 3.15: Source of emf connected to a resistance R The emf source has an internal resistance r, shown conventionally as in the figure. P and Q are the external terminals of the source of emf.

neutral atoms thus giving rise to electrical internal resistance.

The total work done by the source in taking a unit charge from the negative to the positive terminal is $\mathcal{E}$, as per the definition of emf given in Eq.(3.3). Because of internal resistance, part of this work (denoted by W$'$) is used up in the source. The remaining, namely $(\mathcal{E} - W')$ is used to maintain the potential difference V_{ext} between the terminals $V_{ext} = \mathcal{E} - W'$.

This is the voltage that would be measured across the terminals PQ of the emf source if current flows through the circuit. Now if Ohm's law is valid for current flow inside the cell B, we can characterize it by an internal resistance r. In a circuit, it can be represented as in Fig. 3.15 by a resistance r in series with the cell; P and Q are the external terminals of the cell, and r is actually the internal resistance of the cell If a current i flows through the circuit,

$$V_{ext} = \mathcal{E} - W' = \mathcal{E} - ir \tag{3.20}$$

but $V_{ext} = iR$ by Ohm's law. So, we have

$$\mathcal{E} = V_{ext} + ir = i(R + r) \tag{3.21}$$

Example 3.3: (a): A car has a fresh storage battery of emf 12 V and internal resistance $5.0 \times 10^{-2}\Omega$ If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on?

(b) After long use, the internal resistance of the storage battery increases to 500 Ω What maximum current can be drawn from the battery? Assume the emf of the battery to remain unchanged.

(c) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater or less than its emf 12 V?

Answer: (a) The voltage drop across the terminals of the battery (terminal voltage) V

is related to the open- circuit voltage drop (i.e emf of the battery) by the relation

$$V = \mathcal{E} - ir$$

$$\text{For } \mathcal{E} = 12\text{V}, \; i = 90\text{A and } r = 0\,05\Omega$$

$$V = 12 - 4.5 = 7\,5\text{V}$$

(b) The maximum current from a battery can be drawn by shorting it. Then $V = 0$ and

$$i_{max} = (\mathcal{E}/r) = 2\,4 \times 10^{-2} = 24\,\text{mA}.$$

Clearly, the battery is now useless for starting the car and must be charged again

(c) During charging of the accumulator, the current inside the cells (of the accumulator) is opposite to what it is when the accumulator discharges. That is, during charging, current flows from the +ve to -ve terminal inside the cells. Consequently, during charging

$$V = \mathcal{E} + Ir$$

Hence V must be greater than 12 V during charging

Resistances in series and in parallel

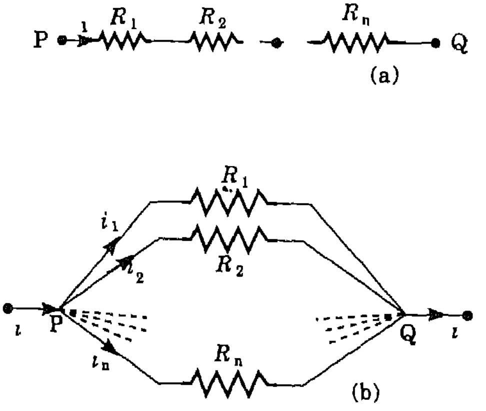


Figure 3.16: (a) Resistors in series (b) Resistors in parallel

(a) Resistances in series Imagine two or more resistors in series, i e connected one after another so that the same current flows through them (Fig 3 16a) As you know, the total resistance of the collection is the sum of individual resistances To see this, suppose a current i flows through the resistances. The potential difference V between the points P and Q is the sum of voltage differences across the sequence of resistors, i e., $i(R_1 + R_2 + \,..\, R_n)$, and the current flowing is i, so that the resistance is

$$(V/i) = R = R_1 + .. \; + R_n \qquad (3\,22)$$

This is the well known formula for the total resistance of n resistors in series

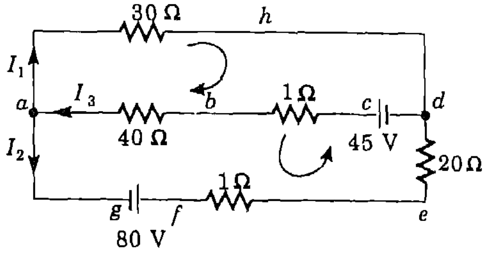


Figure 3.17: An electrical circuit

(b) Resistors in parallel: In Fig. 3.16b, we show n resistors connected 'in parallel' with one another. In this case, the current flowing into P is divided among the n resistors.

$$i = i_1 + i_2 + i_3 + \; i_n \qquad (3.23)$$

However, the potential difference across any resistor is the same, namely

$$i_1 R_1 = i_2 R_2 = \ldots = i_n R_n = V. \qquad (3\,24)$$

These equations can be thought of as determining the currents $i_1, i_2 \;.\;, i_n$.

Substituting in Eq. (3 23), we have

$$i = \frac{V}{R_1} + \frac{V}{R_2} + .. \; + \frac{V}{R_n} = \frac{V}{R} \qquad (3.25)$$

or

$$\frac{1}{R_1} + \frac{1}{R_2} + \quad + \frac{1}{R_n} = \frac{1}{R} \qquad (3.26)$$

This is the equally well known formula for the total resistance of n parallel resistors.

We notice that the Eq. (3 26) uses the fact that there is no net current at the junction P, i e. the current going in equals the current going out Also, the potential difference across any resistor is the same, or if we complete the circuit $PQ \rightarrow QP$ via a path involving any two resistors, the total potential change is zero These facts, called Kirchhoff's rules, are very useful for many electrical circuit problems; they are discussed in detail below, with a simple circuit (Fig. 3 17) as an example.

3.4.2 Kirchhoff's rules

(a) First or junction rule. This rule is based on the fact that charge cannot accumulate at any point in a conductor in a steady situation It states that *at any junction of several circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it* Unless this happens, net positive or negative charge will accumulate at the junction at a rate equal to the net electrical current at the junction. Consider for example, the circuit in Fig. 3.17. At the junction a, I_3 flows in and I_1 and I_2 flow out. We must then have

$$I_3 = I_1 + I_2 \qquad (3\ 27)$$

Applying this to another junction of this circuit leads to nothing new. For further progress in analysing this circuit, we need another rule, given below

(b) Second or loop rule· The algebraic sum of changes in potential around any closed resistor loop must be zero. Otherwise, one can continuously gain energy by circulating charge around a closed loop in a particular direction. So, this rule is based on energy conservation Now consider the loop *ahdcba* in Fig. 3 17

We have, from Kirchhoff's second law,

$$-30I_1 - 41I_3 + 45 = 0. \qquad (3.28a)$$

For the second loop, the circuit ahdefga is taken We have

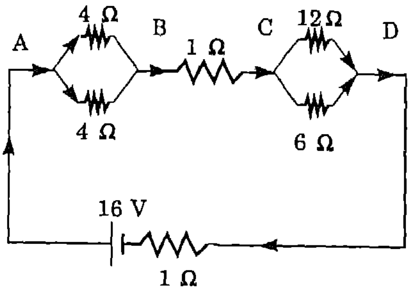
$$-30I_1 + 21I_2 - 80 = 0. \qquad (3\ 28b)$$

These three equations (Eqs 3 27-3.28) can be used to calculate I_1, I_2 and I_3 to be -0 86 A, 2.59 A and 1.73 A respectively

Thus the rules enable one to determine the currents and voltages in different parts of the circuit

---

Example 3.4: A network of resistances is connected to a 16 V battery with internal resistance of 1Ω as shown in the figure (a) Compute the equivalent resistance of the network (b) Obtain the current in each resistor (c) Obtain the voltage drops V_{AB} , V_{BC} and V_{CD}



Answer: (a) The network is a simple series and parallel combination of resistances. First, the two 4Ω resistors in parallel are equivalent to a resistance = $[(4 \times 4)/(4+4)] = 2\Omega$

In the same way, the 12Ω and 6Ω resistances in parallel are equivalent to a resistance of $[(12 \times 6)/(12+6)] = 4\Omega$.

The equivalent resistance R of the network is obtained by combining these resistances (2 Ω and 4 Ω) with 1 Ω in series, i.e $R = 2 + 4 + 1 = 7\Omega$.

(b) The total current I in the circuit is

$$I = \frac{\mathcal{E}}{R+r} = \frac{16\text{V}}{(7+1)\Omega} = 2\text{A}$$

Consider the resistors between A and B. If I_1 is the current in one of the 4 Ω resistors and I_2 the current in the other,

$$I_1 \times 4 = I_2 \times 4$$

i e $I_1 = I_2$, which is obvious otherwise from the symmetry of the two arms But $I_1 + I_2 = I = 2$A Thus $I_1 = I_2 = 1$A i.e current in each 4Ω resistor is 1A.

Next consider the resistances between C and D If I_3 is the current in the 12 Ω resistor, and I_4 in the 6 Ω resistor,

$$I_3 \times 12 = I_4 \times 6, \text{i e } I_4 = 2I_3.$$

$$\text{But } I_3 + I_4 = I = 2\text{A}.$$

$$\text{Thus } I_3 = \left(\frac{2}{3}\right) \text{A}, I_4 = \left(\frac{4}{3}\right) \text{A}$$

i e the current in the 12 Ω resistor is (2/3)A, while the current in the 6 Ω resistor is (4/3)A.

(c) The voltage drop across AB is

$$V_{AB} = I_1 \times 4 = 1 \times 4 = 4\text{V}$$

which is also obtained by multiplying the total current between A and B by the equivalent resistance between A and B, i e.

$$V_{AB} = 2 \times 2 = 4\text{V}$$

The voltage drop across BC is

$$V_{BC} = 2 \times 1 = 2\text{V}$$

Finally, the voltage drop across CD is

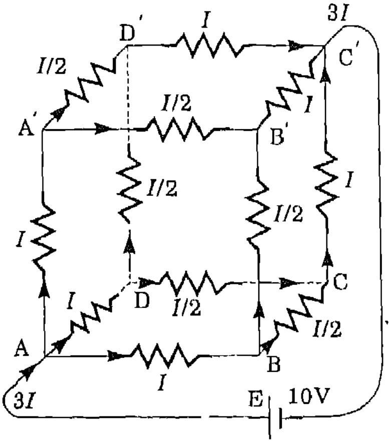
$$V_{CD} = 12 \times I_3 = 12 \times \left(\frac{2}{3}\right) = 8\text{V}$$

which is also obtained by multiplying total current between C and D by the equivalent resistance between C and D, i e.

$$V_{CD} = 2 \times 4 = 8\text{V}$$

Note that the total voltage drop across AD is 4V + 2V + 8V = 14 V Thus the terminal voltage of the battery is 14 V, while its emf is 16V. The loss of the voltage (= 2V) is accounted for by the internal resistance 1Ω of the battery 2A × 1Ω = 2V

Example 3.5: A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1Ω Determine the equivalent resistance of the network and the current along each edge of the cube



Answer: The network is not reducible to a simple series and parallel combinations of resistances. There is, however, a clear symmetry in the problem which we can exploit to obtain the equivalent resistance of the network

The paths AA', AD and AB are obviously symmetrically placed in the network Thus the current in each must be the same, say I. Further, at the corners A', B and D, the incoming current I must split equally into the two outgoing branches In this manner, the current in all the 12 edges of the cube are easily written down in terms of I, using Kirchhoff's first rule, and the symmetry in the problem.

Next take a closed loop say ABCC'EA, and apply Kirchhoff's second rule:

$$-IR - (1/2)IR - IR + \mathcal{E} = 0$$

where R is the resistance of each edge and $\mathcal{E}$ the emf of the battery. Thus,

$$\mathcal{E} = \frac{5}{2} I R$$

The equivalent resistance of the network R_{eq} is:

$$R_{eq} = \frac{\mathcal{E}}{I} = \frac{5}{6} R$$

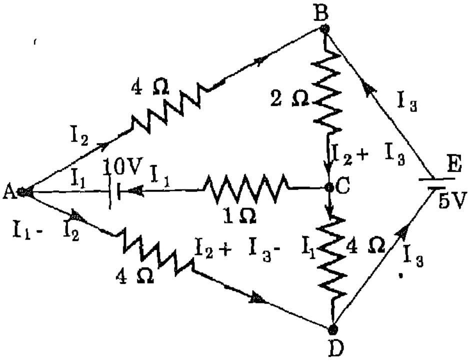
For $R = 1\Omega$, $R_{eq} = (5/6)\Omega$ and for $E = 10V$, the total current $(= 3I)$ in the network is

$$3I = \frac{10}{5/6} = 12\text{A i.e. } I = 4\text{A}$$

The current flowing in each edge can now be read off from the figure

It should be noted that because of the symmetry of the network, the great power of Kirchhoff's rules has not been very apparent in example 3.5. In a general network, there will be no such simplification due to symmetry, and only by application of Kirchhoff's rules to junctions and closed loops (as many as necessary to solve the unknowns in the network) can we handle the problem. This will be illustrated in Example 3.6

Example 3.6: Determine the current in each branch of the network shown in the following figure



Answer: Each branch of the network is assigned an unknown current to be determined by the application of Kirchhoff's rules. To reduce the number of unknowns at the outset, the first rule of Kirchhoff is used at every junction to assign the unknown current in each branch. We then have three unknowns I_1, I_2 and I_3 which can be found by applying the second rule of Kirchhoff to three different closed loops. Kirchhoff's second rule for the closed loop ADCA gives,

$$10 - 4(I_1 - I_2) + 2(I_2 + I_3 - I_1) - I_1 = 0 \qquad (1)$$

i.e., $\quad 7I_1 - 6I_2 - 2I_3 = 10$

For the closed loop ABCA, we get

$$10 - 4I_2 - 2(I_2 + I_3) - I_1 = 0$$

i.e. $\quad I_1 + 6I_2 + 2I_3 = 10 \qquad (2)$

For the closed loop BCDEB, we get

$$5 - 2(I_2 + I_3) - 2(I_2 + I_3 - I_1) = 0$$

i.e., $\quad 2I_1 - 4I_2 - 4I_3 = -5 \qquad (3)$

Eqs (1), (2) and (3) are three simultaneous equations in three unknowns. They can be solved by the usual method to give

$$I_1 = 2.5\text{A}, I_2 = \frac{5}{8}\text{A}, I_3 = 1\frac{7}{8}\text{A}$$

The currents in the various branches of the network are

$$AB : \frac{5}{8}\text{A}, \quad CA : 2\frac{1}{2}\text{A}, \quad DEB \; 1\frac{7}{8}\text{A}$$

$$AD \; 1\frac{7}{8}\text{A}, \quad CD : 0\text{A}, \quad BC : 2\frac{1}{2}\text{A}.$$

It is easily verified that Kirchhoff's second rule applied to the remaining closed loops does not provide any additional independent equation, that is, the above values of currents satisfy the second rule for every closed loop of the network. For example, the total voltage drop over the closed loop BADEB is

$$5 + \left(\frac{5}{8} \times 4\right) - \left(\frac{15}{8} \times 4\right)$$

equal to zero, as required by Kirchhoff's second rule.

3.5 Measurement of voltages, currents and resistances

Though this Chapter is about currents, voltages and resistances, we have not yet said anything about how these are measured One basic measuring instrument, whose working will be discussed in Chapter 5, is a galvanometer. It produces, finally, a deflection proportional to the electric current flowing through it Using a galvanometer, one can make devices for measuring voltages and currents called voltmeters and ammeters respectively (see below) For accurate comparison of voltages, one uses a potentiometer, whereas resistances are accurately measured using a resistance bridge. These instruments are discussed below·

3.5.1 Voltmeter and ammeter

A common requirement for the two is that the current flow through these should not disturb the electrical circuit, i.e. should not change the currents flowing in or voltages across various parts of the circuit Their design should meet this condition.

A voltmeter is connected across the two points A and B between which the potential difference is to be measured. If this potential difference is fixed, say by an outside source of emf, then connecting a voltmeter will reduce the total current flowing through the other circuits connected across AB (see Fig 3 18a) We clearly want this reduction to be very small (ideally zero). This can be achieved if the voltmeter has a high resistance (ideally infinite), since then, for a given voltage across AB, the current through the voltmeter is small. Thus, typically, a voltmeter consists of a high resistance R and a galvanometer in series (Fig. 3.18b). The value of the resistance R determines the 'range' of the voltmeter (as also how much the rest of the circuit is disturbed by it) For example, suppose a full scale deflection of the galvanometer occurs for a current i_o If the voltmeter has to have a range V_o , the resistance R (plus the galvanometer resistance, which is usually negligible in comparison) should be (V_o/i_o). Typical voltmeters have several high resistances of different values in parallel One or the other can be put in series with the galvanometer by turning a 'range' switch

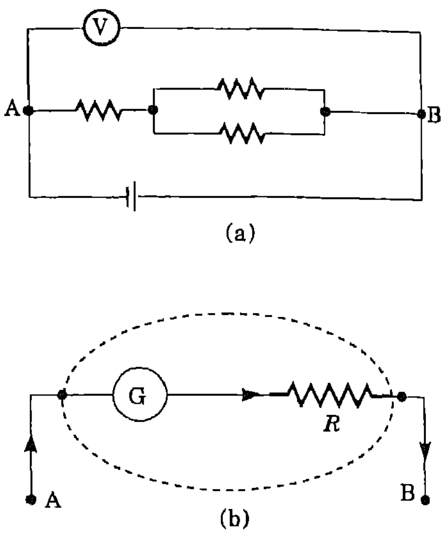


Figure 3.18: (a) A voltmeter connected across a circuit AB
(b) It consists of a high resistance R in series with a galvanometer G

An ammeter, since it is to measure the current flowing through a circuit element, is connected in series with it Clearly, if the current flowing through is not to change on inserting the ammeter, the ammeter should have zero resistance! Ammeters are therefore designed to have very small effective resistances. This is done by connecting a low resistance R in parallel to a galvanometer of resistance R_G (Fig 3.19) The effective resistance of the ammeter is

$$R_{eff} = \frac{RR_G}{R+R_G} \simeq R \text{ (if } R \ll R_G) \qquad (3.29)$$

Example 3.7: A galvanometer coil has a resistance of 15Ω and the meter shows full scale deflection for a current of 2.0 mA How will you convert the meter into

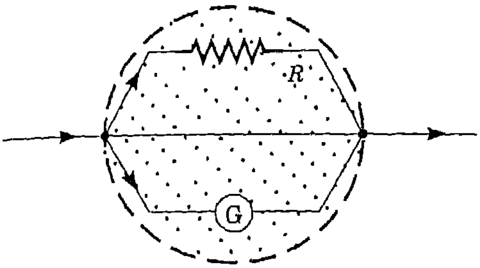


Figure 3.19: An ammeter consists of a galvanometer (G) and a low resistance R in parallel

(a) an ammeter of range 0 to 5.0 A

(b) a voltmeter of range 0 to 15.0 V?

Also determine the net resistance of the meter in each case.

Answer: (a) If the incoming current is 5.0 A, the meter is expected to show full scale deflection. It is to be shunted with a resistance R_s such that only 2 mA of current goes through the meter and the remaining current of 5-0.002 = 4.998A passes through the shunt. Since the voltage across the meter is equal to the voltage across the shunt, we get

$$15 \times 2 \times 10^{-3} = R_s \times 4.998$$

which gives $R_s \simeq 6.00 \times 10^{-3}\Omega$

The net resistance of the meter and the shunt is given by

$$\frac{1}{R} = \frac{1}{15} + \frac{1}{6 \times 10^{-3}}$$

i.e. $R \simeq 0.006\Omega$. The low resistance shunt serves the dual purpose of increasing the range of the meter and reducing its net resistance very substantially. The shunted meter when placed in series in a circuit does not materially affect the current in the circuit to be measured because of its very low resistance.

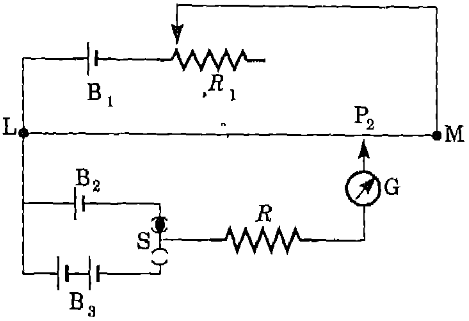


Figure 3.20: The circuit diagram of a potentiometer

(b) If the voltage drop is 15.0 V, the meter will show full scale deflection when a resistance R′ is connected in series with the meter such that the current passing through the meter (and the resistance R′) is 2.0 mA. The total voltage drop is then shared by the meter and the resistance R' i.e.

$$15 = 15 \times 2 \times 10^{-3} + 2 \times 10^{-3} \times R'$$

$$\text{i.e. } R' = \frac{15 - 0.03}{2 \times 10^{-3}} = 7.485k\Omega = 7485\Omega$$

The net resistance R of the meter then is

$$R = 7485 + 15 = 7500\Omega$$

The high resistance R' in series with the meter serves the dual purpose of increasing the range of the meter and also increasing its net resistance very substantially. Because of its high resistance, the voltmeter when placed in parallel with the main circuit draws a very small current from the main circuit, and therefore does not materially affect the voltage drop in the main circuit which it sets out to measure.

3.5.2 Potentiometer

This is a commonly used device for comparing potentials. It is shown schematically in Fig 3.20. A source of emf B_1 is connected to a variable resistance and a long uniform conducting wire LM in series. The batteries B_2 and B_3 (or other sources of potential difference) whose emf's are to be compared, are connected as shown, S being a two way switch used to bring B_2 or B_3 into the electrical circuit. The galvanometer G monitors current flow, and a high resistance R can be used to prevent too much current from flowing through G. The other end of G is connected

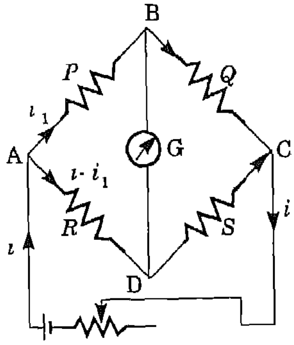


Figure 3.21: Circuit diagram of the Wheatstone Bridge

to a moving contact maker or divider which moves along LM

One of the batteries, say B$_2$ is brought into the circuit, and the divider moved till no current flows through the galvanometer G. Suppose this happens at P$_2$. Similarly, suppose the null point (condition of no current) for B$_3$ is at the point P$_3$. Then, the emf's $\mathcal{E}_2$ and $\mathcal{E}_3$ of the batteries B$_2$ and B$_3$ are in the ratio

$$\frac{\mathcal{E}_2}{\mathcal{E}_3} = \frac{L_2}{L_3}. \qquad (3\ 30)$$

Where LP_2 has the length L_2, and $LP_3 = L_3$.

To see why this is so, consider the circuit LP_2B_2L If a current flows through this circuit, the potential drop V_{LP_2} across LP_2 is $\mathcal{E}_2 - iR_t$, where R_t is the resistance of the part P_2B_2L of the circuit (R_t is R plus the internal resistance of the cell B$_2$ plus that of the galvanometer G) Now if no current flows through the circuit, i.e

$$i = 0,\ V_{LP_2} = \mathcal{E}_2 \qquad (3\ 31)$$

Since the wire LM is uniform, the potential decrease across it is proportional to the length along LM. The constant of proportionality depends on the resistance R_{LM} of the full wire LM, and on the current i_o flowing through LM, i e.

$$V_{LP_2} = (i_oR_{LM})\frac{LP_2}{LM} \qquad (3.32)$$

At the null point, current flows only in the circuit LMB_1L and so i_o depends only on its parameters which do not change in a particular experiment. Using Eqs (3 31) and (3.32), and a similar equation for V_{LP_3}, we find that Eq (3 30) follows

Notice that since the method of measurement involves a condition of no current flow, emf's are compared *independent* of the internal resistance of the sources If one of the emf's has a standard voltage, this device can be used to find absolute emf's also.

3.5.3 Wheatstone bridge

The Wheatstone bridge is an arrangement of four resistances used for measuring one of them in terms of the other three It was first suggested by C F Wheatstone, a British physicist, in the 19th century. It consists of four resistances connected as shown in Fig. 3.21. Between the points A and C, a source of emf is connected

A galvanometer is connected between B and D The unknown resistance is one of the four 'arms' P, Q, R and S of the bridge, say R

The experiment consists of changing the other resistances, say P till no current flows between B and D (there is no deflection of the galvanometer) When this happens,

$$\frac{P}{Q} = \frac{R}{S} \qquad (3.33a)$$

or

$$R = (P/Q)S \qquad (3\ 33b)$$

Thus by changing S or Q, many experimental determinations of R can be made

The result Eq (3 33) follows simply on using Kirchhoff's rules Consider the junction A, and suppose a current i enters it. If a current i_1 flows through AB, then from Kirchhoff's first rule, $(i - i_1\ )$ flows through AD. Now when no current flows through BD, applying Kirchhoff's second rule to ABDA, we have

$$i_1P = (i - i_1)R. \qquad (3.34)$$

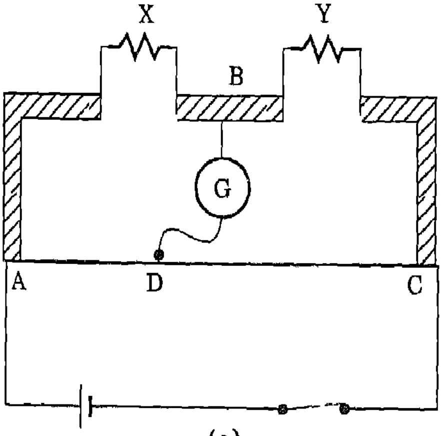
Similarly, applying the rule to BCDB,

$$i_1Q = (i - i_1)\ S \qquad (3\ 35)$$

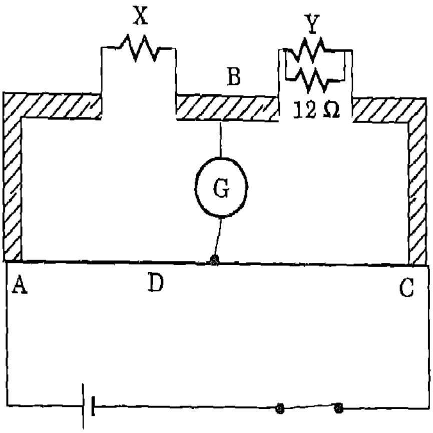
Dividing Eq. (3.34) by Eq. (3.35), we have the desired Eq. (3.33).

The Wheatstone bridge, being a null method, is not affected by internal resistances, resistance of ammeters and voltmeters etc.. It is the simplest example of an arrangement, variants of which are used for a large number of electrical measurements. It can be made very accurate with proper choice of standard and variable resistances.

Example 3.8: The figure (Figure (a)) shows a metre bridge (which is nothing but a practical Wheatstone bridge) consisting of two resistors X and Y together in parallel with a metre long constantan wire of uniform cross section. With the help of a movable contact D, one can change the ratio of the resistances of the two segments of the wire, until a sensitive galvanometer G connected across B and D shows no deflection. The null point is found to be at a distance of 33.7 cm from the end A. The resistance Y is shunted by a resistance Y' of 12.0Ω (Figure (b)) and the null point is found to shift by a distance of 18.2 cm. Determine the resistance of X and Y.



(a)



(b)

Answer: Since the wire is of uniform cross section, the resistances of the two segments of the wire AD and DC are in the ratio of the lengths of AD and DC. Using the null-point conditions of a Wheatstone bridge we have

$$(X/Y) = (33.7/66.3) \qquad (1)$$

When Y is shunted by a resistance of 12.0, its net resistance changes to

$$Y' = 12Y/(Y + 12)$$

Since Y' is less than Y, the ratio X/Y' is greater than X/Y. Thus the null point must shift towards the end C, i.e.,

$$(X/Y') = (51.9/48.1) \text{ or}$$
$$X(Y + 12)/12Y = (51.9/48.1)$$

i.e.

$$\frac{Y+12}{12} = \left(\frac{51.9}{48.1}\right) \times \frac{66.3}{33.7}$$

which gives $Y = 13.5\Omega$, and $X = 6.86\Omega$, using (1).

Summary

1. Current through a given area of a conductor is the net charge passing per unit time through the area.

2. To maintain a steady current we must have a closed circuit, in which an external agency moves electric charge from lower to higher potential energy. The work done per unit charge by the source in taking the charge from lower to higher potential energy (i.e. from one terminal of the source to the other) is called the electromotive force, or emf, of the source. Note, the emf is not a force, it is the voltage difference between the two terminals of a source in open circuit.

3. **Ohm's law:** The electric current I flowing through a substance is proportional to the voltage V across its ends, i.e. $V \propto I$ or $V = RI$, where R is called the resistance of the substance. The unit of resistance is ohm: $1\ \Omega = 1\ \text{VA}^{-1}$.

4. The resistance R of a conductor depends on its length l and constant cross-sectional area A through the relation,

$$R = \frac{\rho l}{A}$$

where ρ, called resistivity is a property of the material and depends on temperature and pressure.

5. Electrical resistivity of substances varies over a very wide range. Metals have low resistivity, in the range of $10^{-8}\,\Omega\text{m}$ to $10^{-6}\,\Omega\text{m}$. Insulators like glass and rubber have 10^{22} to 10^{24} times greater resistivity. Semiconductors like Si and Ge lie roughly in the middle range of resistivity, on a logarithmic scale.

6. In most substances, the carriers of current are electrons, in some cases, e.g. ionic crystals and electrolytic liquids, positive and negative ions carry the electric current.

7. Current density **j** gives the amount of charge flowing per second per unit area normal to the flow,

$$\mathbf{j} = nq\mathbf{v}$$

where n is the number density (number per unit volume) of charge carriers each of charge q, and **v** is the drift velocity of the charge carriers. For electrons $q = -e$. If **j** is normal to a cross-sectional area A and is constant over the area, the magnitude of the current I through the area is ($nevA$).

8. Using $E = V/l$, $I = nevA$, and Ohm's law one obtains

$$\frac{eE}{m} = \rho\frac{ne^2}{m}v_d$$

The proportionality between the *force* eE on electrons in a metal due to the external field E and the *drift velocity* v_d (not acceleration) can be understood, if we assume that the electrons suffer collisions with ions in the metal, which deflect them randomly. If such collisions occur on the average at a time interval τ,

$$v_d = a\tau = eE\tau/m$$

where a is the acceleration of the electron This gives

$$\rho = \frac{m}{ne^2\tau}$$

9 In the temperature range in which resistivity increases linearly with temperature, the temperature co-efficient of resistivity α is defined as the fractional increase in resistivity per unit increase in temperature.

10 **Ohm's law** is obeyed by many substances, but it is not a fundamental law of nature It fails if

(a) V depends on I non-linearly

(b) The relation between V and I depends on the sign of V for the same absolute value of V

(c) The relation between V and I is non-unique
An example of (a) is when ρ increases with I (even if temperature is kept fixed) A rectifier combines features (a) and (b) A thyristor shows all the features (a), (b) and (c)

11 When a source of emf $\mathcal{E}$ is connected to an external resistance R, the voltage V_{ext} across R is given by

$$V_{ext} = IR = \frac{\mathcal{E}}{R+r}R$$

where r is the internal resistance of the source

12 (a) Total resistance R of n resistors connected in *series* is given by

$$R = R_1 + R_2 \;\ldots\ldots + R_n$$

(b) Total resistance R of n resistors connected in *parallel* is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$$

13. **Kirchhoff's Rules**

(a) *First Rule (Junction Rule)* At any junction of circuit elements, the sum of currents entering the junction must equal the sum of current's leaving it

(b) *Second Rule (Loop Rule)* The algebraic sum of changes in potential around any closed loop must be zero

14 A *voltmeter* consists of a galvanometer (of resistance R_G) in series with a high resistance R It is put in parallel to the circuit element across which the voltage is to be measured Because of its high resistance ($R + R_G \simeq R$), it draws a very small current and thus does not disturb the circuit. If the full scale deflection of the galvanometer occurs for a current I_0 and the voltmeter is to have a range V_0, we have

$$V_0 = I_0(R + R_G) \text{ i.e.; } R = \frac{V_0}{I_0} - R_G \simeq \frac{V_0}{I_0}$$

15. An *ammeter* consists of a galvanometer (of resistance R_G) and low resistance R in parallel. The effective resistance of the ammeter is

$$\frac{RR_G}{R+R_G} \simeq R \text{ for } R \ll R_G$$

Because of its very low resistance, the ammeter placed in series in a circuit does not materially change the current in the circuit to be measured If the full scale deflection of the galvanometer occurs for current I_0 and the ammeter is to have a range I_{max}, we have

$$R = \frac{I_0 R_G}{I_{max} - I_0}$$

16 The *potentiometer* is a device to compare potentials Since the method involves a condition of no current flow, the device can be used to compare emf's of two sources.

17. The *Wheatstone* bridge is an arrangement of four resistances P, Q, R, S as shown in the text The null-point condition is given by

$$\frac{P}{Q} = \frac{R}{S}$$

using which the value of one resistance can be determined, knowing the other three resistances.

Exercises

3.1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4Ω, what is the maximum current that can be drawn from the battery?

3.2 A battery of emf 10 V and internal resistance 3Ω is connected to a resistor. If the current in the circuit is 0.5A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

3.3 (a) Three resistors 1Ω, 2Ω, and 3Ω are combined in series. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

3.4 (a) Three resistors 2Ω, 4Ω and 5Ω are combined in parallel. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

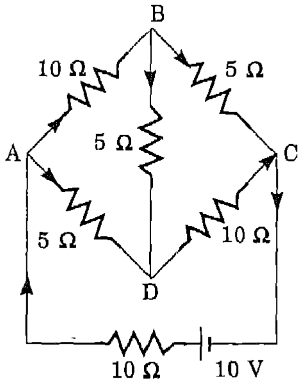
3.5 At room temperature (27.0°C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}.

3.6 A negligibly small current is passed through a wire of length 15 m and uniform cross section 6.0×10^{-7} m^2, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

3.7 A silver wire has a resistance of 2.1Ω at 27.5° C, and a resistance of 2.7Ω at 100° C. Determine the temperature coefficient of resistivity of silver.

3.8 A heating element using nichrome connected to a 230V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0° C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} °C^{-1}.

3.9 Determine the current in each branch of the following network.



3.10 (a) In a metre bridge (see for example figure of Example 3.8a), the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or metre bridge made of thick copper strips?

(b) Determine the balance point of the bridge above if X and Y are interchanged

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

3.11 A storage battery of emf 8 0 V and internal resistance 0 5 Ω is being charged by a 120 V d.c supply using a series resistor of 15 5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

3.12 A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA How will you convert the meter into a voltmeter of range 0 to 18 V?

3.13 A galvanometer coil has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A?

3.14 In a potentiometer arrangement a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire If the cell is replaced by another cell and the balance point shifts to 63 0 cm, what is the emf of the second cell?

3.15 Clarify your elementary notions about current in a metallic conductor by answering the following queries

(a) In Example 3 1, the electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few A How then is current established almost the instant a circuit is closed?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?

(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amount of current in a conductor?

(d) When electrons drift in a metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction?

(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field? (Do check your ideas with the answers at the end).

3.16 The number density of conduction electrons in a copper conductor estimated in Example 1 is $8.5 \times 10^{28} m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross section of the wire is $2.0 \times 10^{-6} m^2$ and it is carrying a current of 3.0 A.

Additional Exercises

3.17 Name the carriers of electric current in

(a) a bar made of silver

(b) a hydrogen discharge tube

(c) a voltaic cell

(d) lead accumulator being charged by an external supply

(e) a germanium semiconductor

(f) a wire made of the alloy nichrome

(g) a superconductor

In each case relate the motion of the carrier with the direction of current

3.18 The earth's surface has a negative surface charge density of 10^{-9} C m^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe). (Radius of earth = 6.37×10^6m)

3.19 (a) Six lead-acid type of secondary cells each of emf 2.0V and internal resistance 0.015Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

(b) A secondary cell after long use has an emf of 1.9V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

3.20 (a) Make a small survey of the types of primary cells ('dry' batteries) and secondary cells (accumulators) available in the market and tabulate their important characteristics and uses

(b) Besides its emf you might notice another characteristic marked on a storage battery. its 'capacity'. (Do not confuse this term with the capacity of a condenser or capacitor). A storage battery is marked as having a capacity of 3.5 Ah at 1h discharge rate. What does this signify? Would the cell provide 14A for 15 min?

(c) Which type of cell would you want to use if your device required

(i) a current of 100A for 20 s

(ii) a current of 10mA occasionally?

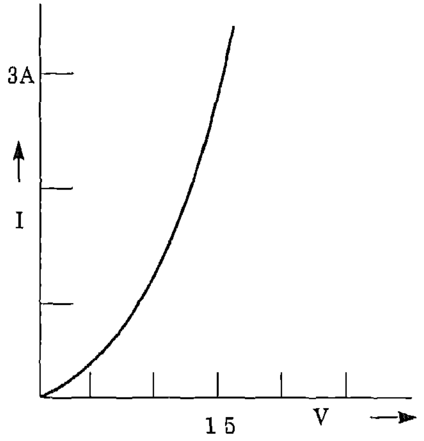
3.21 In a discharge tube, the number of hydrogen ions (i.e. protons) drifting across a cross section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across another cross section is 2.7×10^{18} per second. If the supply voltage is 230 V, what is the effective resistance of the tube?

3.22 Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63 \times 10^{-8} \Omega$ m, $\rho_{Cu} = 1.72 \times 10^{-8} \Omega$ m. Relative density of Al=2.7, of Cu=8.9).

3.23 What conclusion can you draw from the following observations on a resistor made of alloy manganin:

Current A	Voltage V	Current A	Voltage V
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

3.24 A cell of emf 1.5V and internal resistance 0.5 Ω is connected to a (non-



linear) conductor whose $V - I$ graph is shown in the figure. Obtain graphically the current drawn from the cell and its terminal voltage.

3.25 Answer carefully.

(a) A simple voltaic cell has an emf equal to 1.0V. When the circuit is open, is there a net field, which would give rise to a force on a test charge, (i) inside the electrolyte of the cell, (ii) outside the cell?

(b) Is there a net field inside the cell when the circuit is closed and a steady current passes through? Explain.

(c) Consider a voltaic cell in open circuit with the Cu plate at higher potential with respect to the Zn plate. Since the electrolyte separating them is a conducting medium, why are the positive and negative charges on the electrodes not neutralized immediately? (This question is related to (a) above)

(d) When a secondary cell of emf 2 0V is being charged by an external supply, is the terminal voltage of the secondary cell greater or less than 2 0 V?

(e) Does emf represent a force or potential energy or work done per unit charge or potential difference? Does emf have an electrostatic origin?

3.26 Answer the following questions.

(a) A steady current flows in a metallic conductor of non-uniform cross section. Say which of these quantities is constant along the conductor: current, current density, electric field, drift speed?

(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.

(c) It is easier to confine electric currents to definite paths (by the use of electric insulators) than to direct heat flow along definite routes using heat insulators. Why?

(d) It is easier to start a car engine on a warm day than on a chilly day. Why?

(e) In which respect does a nearly discharged lead-acid secondary cell differ mainly from a freshly charged cell? In its emf or in its resistance?

(f) A low voltage supply from which one needs high currents must have very low internal resistance. Why?

(g) A high tension (HT) supply of say 6 kV must have a very large internal resistance. Why?

3.27 Choose the correct alternative.

(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

(c) Doping a semiconductor (with small traces of impurity atoms) reduces/increases its resistivity.

(d) The resistance of graphite and most non-metals increases/decreases with increase in temperature.

(e) The resistivity of a semiconductor increases/decreases rapidly with increasing temperature

(f) The resistivity of the alloy manganin (is nearly independent of/ increases rapidly with) increase of temperature.

(g) The resistivity of a typical insulator (e.g. amber) is greater than that of a metal by a factor of the order of $(10^{22}/10^{3})$

3.28 Two identical cells of emf 1.5V each joined in parallel provide supply to an external circuit consisting of two resistors of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1.4 V. What is the internal resistance of each cell?

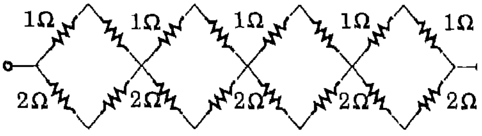
3.29 (a) Three cells of emf 2.0V, 1.8V, and 1.5V are connected in series. Their internal resistances are 0.05 Ω, 0.7Ω, and 1Ω respectively. If the battery is connected to an external resistor of 4Ω via a very low resistance ammeter, what would be the reading in the ammeter?

(b) If the three cells above were joined in parallel, would they be characterized by a definite emf and internal resistance (independent of external circuit)? If not, how will you obtain currents in different branches of the circuit?

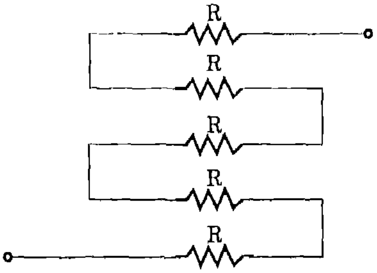
3.30 (a) Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

(b) Given the resistances of 1 Ω, 2Ω, 3Ω, how will you combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5)Ω, (iii) 6Ω, (iv) (6/11)Ω?

(c) Determine the equivalent resistance of the following networks:

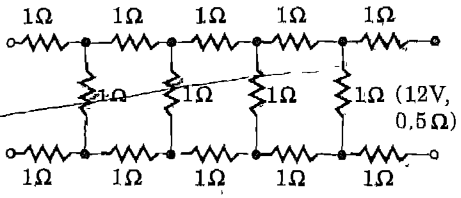


(i)



(ii)

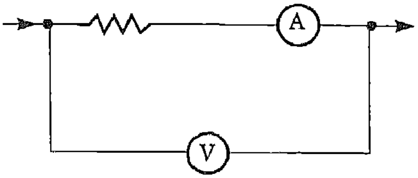
3.31 Determine the current drawn from a 12 V supply with internal resistance 0.5Ω by the following infinite network: Each resistor has 1Ω resistance.



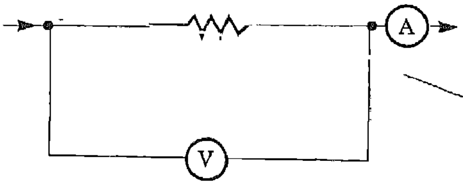
3.32 A galvanometer with a coil of resistance 12.0 Ω shows a full scale deflection for a current of 2.5 mA. How will you convert the galvanometer into (a) an ammeter of range 0 to 7.5 A, (b) a voltmeter of range 0 to 10.0 V.

Determine the net resistance of the meter in each case. When an ammeter is put in a circuit, does it read (slightly) less or more than the actual current in the original circuit? When a voltmeter is put across a part of the circuit, does it read (slightly) less or more than the original voltage drop? Explain.

3.33 You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5Ω and a voltmeter of resistance 20 k Ω. It is known that X is in the range of a few ohms, while Y is in the range of several thousand ohms. In each case, which of the following two connections would you choose for resistance measurement? Justify your answer quantitatively. (Hint: For each connection, determine the error in resistance measurement. The connection that corresponds to a smaller error (for a given range of resistance) is to be preferred.)



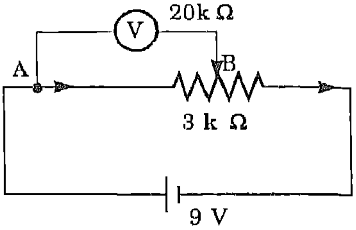
(a)



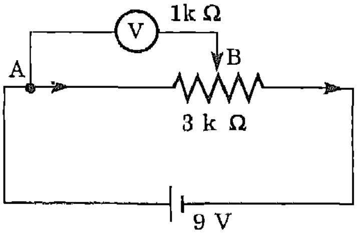
(b)

3.34 (a) A battery of emf 9V and negligible internal resistance is connected to a 3kΩ resistor. The potential drop across a part of the resistor (between points A and B in the figure) is measured by (i) a 20 kΩ voltmeter, (ii) a 1 kΩ voltmeter. In (iii) both the voltmeters are connected across AB. In which case would you get the (1) highest, (2) lowest reading?

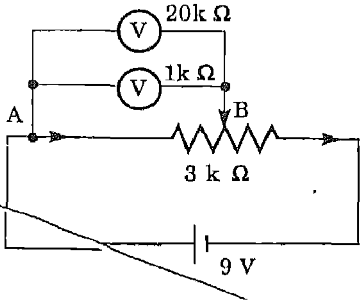
(b) Do your answers to this problem alter if the potential drop across the entire resistor is measured? What if the battery has non-negligible internal resistance?



(i)



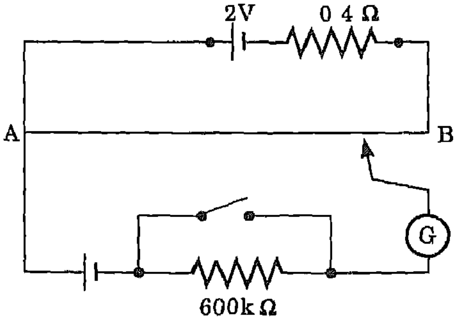
(ii)



(iii)

3.35 A voltmeter reads 5.0V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000Ω/V How will you convert it into a voltmeter that reads 20V at full scale deflection? Will it still be graded as 5000Ω/V? Will you prefer this voltmeter to one that is graded as 2000 Ω/V?

3.36 The figure below shows a potentiometer with a cell of 2 0V and internal resistance 0.40Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1 02V (for very moderate currents upto a few μA) gives a balance point at 67.3 cm length of the wire To ensure very low currents drawn from the standard cell, a very high resistance of 600kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf $\mathcal{E}$ and the balance point found similarly turns out to be at 82.3 cm length of the wire (a) What is the value of $\mathcal{E}$?



(b) What purpose does the high resistance of 600kΩ have?

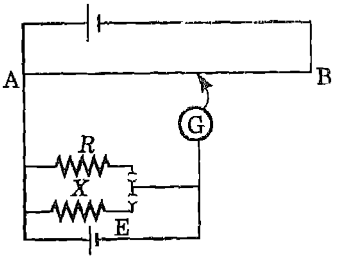
(c) Is the balance point affects by this high resistance?

(d) Is the balance point affected by the internal resistance of the driver cell?

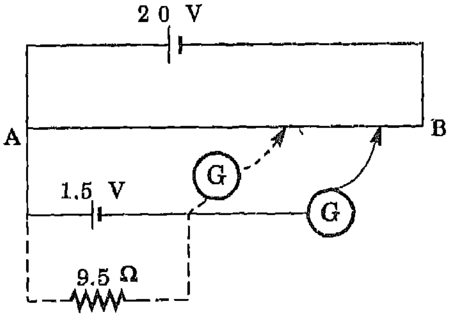
(e) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1 0V instead of 2.0V?

(f) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?

3.37 The figure below shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R=10.0Ω is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm Determine the value of X. What might you do if you failed to find a balance point with the given cell E?



3.38 The figure below shows a 2.0V potentiometer used for the determination



of internal resistance of a 1.5V cell. The balance point of the cell in open circuit is 6.3 cm. When a resistor of 9.5Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

3.39 Given a 3.0V potentiometer, design circuits for calibrating

(a) a voltmeter in the range of 0.5 to 2.5V

(b) an ammeter in the range of 0.1 to 0.5A

3.40 A d.c supply of 120V is connected to a large resistance X. A voltmeter of resistance 10kΩ placed in series in the circuit reads 4V. What is the value of X? What do you think is the purpose in using a voltmeter, instead of an ammeter, to determine the large resistance X?

3.41 Consider a parallel-plate capacitor (of plate area A) *during* its charging by an external source:

(a) Show that the conduction current I flowing from (to) the source to (from) the positive (negative) plate equals ϵ_0 (dE/dt)A where E is the electric field between the plates.

(b) Is Kirchhoff's first rule valid in this circuit? If not, can you suggest a way to generalize the rule so that it is valid here too?

3.42 Answer the following questions

(a) What is the order of magnitude of the resistance of a (dry) human body?

(b) You may be surprised by the answer to (a) above. If the resistance of our body is so large, why does one experience a strong shock (sometimes even fatal) when one accidentally touches the live wire of say 240V supply?

(c) There is an impression among many people that a person touching a high power line gets 'stuck' with the line. Is that true? Explain.

(d) Currents of the order of 0.1A through the human body are fatal. What causes the death: heating of the body due to electric current or something else?

(e) A nerve fiber contains a membrane separating two conducting 'fluids' maintained at a potential difference. What is the order of this potential difference?

(Caution: Do check your ideas on the above questions with the answers given at the end. In any case, remember that our body is sensitive to extremely minute currents and even small voltages can be risky.)

CHAPTER 4

Thermal and Chemical Effects of Currents

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4.1 Introduction

The idea of an electrical current or flow of charge was introduced and some basic physical quantities connected with it were discussed in the last Chapter (Chapter 3). Some obvious questions that come up are: How can charges be caused to flow? What are the physical consequences of such electrical currents? How are they related to other forms of energy? Living in an electrical (and increasingly electronic) age we are familiar in daily life with many practical answers to these questions, such as light bulbs, batteries, electroplating, electric fans, dynamos and radios In this and several later chapters we shall be concerned with some of the causes and consequences of electric currents. This chapter is concerned mainly with the connection between electricity, and thermal and chemical energy. We shall see in Section 4.2 that when a steady current flows through a conductor, heat or thermal energy is always produced. In a conductor with free electrons, if one part is hotter than the other, the electrons there have higher kinetic energy and tend to flow to the colder part, causing a current and a voltage difference between the hot and the cold parts. This is an example of *thermoelectricity* discussed in Section 4.3.

We have seen in the chapter on molecules (Chapter 9 of the Class XI Physics text book) that chemical binding is due to electrical forces between electrons and nuclei, and between electrons. It should therefore be possible to convert differences in chemical binding (as happens in chemical reactions) to electrical energy (electrochemical cells) Conversely electrical energy can cause chemical changes (e.g. electrolysis). This large field is called *electrochemistry* and is properly studied as part of chemistry In Section 4.4, we touch upon a few essential aspects, namely electrolysis and electrochemical cells (normally called just cells or batteries).

Much of the remainder of the book is about the relation between electricity and other forces. We shall see that steady electrical currents produce steady *magnetic* fields (Chapter 5). We will then learn that magnetic fields changing in time produce electric fields (Chapter 7) The reverse is also true, time dependent electric fields produce magnetic fields (Chapter 9) One consequence of this intimate connection between electricity and magnetism (*electromagnetism*) is the propagation of electric and magnetic fields together as waves (Chapter 9). These waves travelling with the speed of light, are known to us as radio waves, visible light, or X-rays depending on the wavelength. From this electromagnetic nature of light and other radiation follows the possibility of producing them purely electrically (Chapters 12 and 13) In Chapter 12, we discuss the reverse, namely the *photoelectric* effect where light energy is converted to electricity, e.g. to a flow of electrons or an electrical voltage difference.

4.2 Heating effects

Consider a steady electric current I flowing through a circuit. Suppose the voltage difference between the two ends of the circuit is V. The potential decreases in the direction of current flow A charge I flows *per second* down the potential drop of V. Thus, in such a downhill flow, an energy IV is lost per second by the drifting charges. Where does this energy go? This question has already been discussed earlier (Section 3 3) The argument may be summarized as follows. If a voltage is applied between the ends of a conductor, the resulting electric field accelerates the free electrons during the intervals between collisions with the ions or atoms of the lattice The electrons gain energy on account of this acceleration The energy gained is transferred to the lattice ions or atoms, since after colliding with them the electrons return to their equilibrium distribution of velocities. Thus the lattice ions or atoms receive energy in random bursts at the average rate of IV per unit time. This clearly increases their disorderly motion, i.e. appears as heat energy. We therefore conclude that if a current I flows through a potential drop V, the rate of production of heat

$$P = I\,V. \tag{4.1}$$

This relation was first recognized and verified experimentally by Joule (see Chapter 11, p.272 of the Class XI Physics text book). For this reason the heat produced when a current flows is called Joule heating or Joule heat.

Equation (4 1) can be expressed also in terms of the resistance of the conductor, or in general, in terms of the relation between V and I. For example, if the relation between V and I is given by Ohm's law, with $V = RI$, then

$$P = IV = I^2R = (V^2/R). \tag{4.2}$$

Thus if a circuit element of resistance R ohms carries a current I amperes, the power lost as heat is I^2R watts. If the voltage across the resistor is V volts the power lost is (V^2/R) watts. We notice also that, *independent* of the current direction (independent of the sign of I), power is always lost (unless the resistance is negative). The circuit element is said to 'consume' a power P or to have a 'power rating' of P. Since the energy lost by the flowing charges ends up as disorderly thermal motion in the resistor, the phrase 'ohmic dissipation' or ohmic loss is also used to describe it.

In many cases, ohmic dissipation is an unwanted (and undesirable, but inevitable) side effect of current flow. For example, in electronic circuits, this heat, produced in a small region, can increase the temperature of the components so much that their properties change This is clearly undesirable for reliable functioning of the circuit; means are found to carry away the heat. In some other cases, the purpose of the circuit element is to produce heat, as in an electric heater, oven, or an electric iron, or even an electric bulb. The main question then is the choice of the right kind of resistor, and proper design of the appliance or device for delivering heat (or light) in the way needed For example, in a light bulb, the resistor filament must retain as much of the heat generated as possible so that it gets very hot, and begins to emit light. This heat must not melt the resistor. This influences the choice of tungsten (high melting point, high resistivity, strong metal) and the thermal isolation of the filament (insulating support, evacuated bulb, ..).

Example 4.1: (a) A small heating element made of nichrome resistor connected to a 30V d.c. supply draws a current of 10 A. How much electric power is supplied to the heater? How much heat is produced in 2 h? (b) An electric motor operating on a 30V supply draws a current of 10 A and yields mechanical power of 120 W. What is the efficiency of the motor? How much energy is lost as heat in 2 h? (c) Compare and contrast the situations in (a) and (b) above.

Answer: (a) Power supplied to the heater = 30V × 10 A = 300 W. Since the portion of the circuit to which this electric power is supplied is a (passive) resistor, the entire electrical energy is converted into heat energy. Therefore,

Heat produced in 2h

$$= 300\,\mathrm{W} \times 2 \times 3600\,\mathrm{s}$$

$$= 2160000\,\mathrm{J} = 0.6\,\mathrm{kWh}.$$

(b) Power supplied to the motor = 30V × 10A = 300 W. The electric motor is not a (passive) resistor. Thus all of the power supplied to it does not appear as heat. Only part of the total power supplied is lost as heat, the rest appears as useful mechanical power. The efficiency of the electric motor is the ratio of this mechanical power to the total power input. Thus efficiency of the motor = 120 W/300 W = 0.4 or 40%

Power lost as heat

$$= 300\mathrm{W} - 120\mathrm{W} = 180\mathrm{W}$$

Energy lost as heat in 2 h

$$= 180\mathrm{W} \times 2 \times 3600\mathrm{s}$$

$$= 1296000\mathrm{J} = 0.36\mathrm{kWh}$$

(c) Electric power supplied in each case is given by VI. The important thing to note is that only for a (passive) resistor, $V = IR$ (Ohm's law) and therefore power supplied is $VI = I^2R$. For the case (a), $V = IR$ so that $R = 3\Omega$, and $VI = I^2R = 300\mathrm{W}$. For case (b), it is wrong to write $VI = I^2R$. For this case, while VI represents power input to the motor, I^2R represents the power lost (as heat) due to the resistance of the element (windings of the motor). The difference $VI - I^2R$ appears as mechanical power yielded by the motor. To get R for this case, equate I^2R to the power loss:

$$I^2R = 180\mathrm{W}$$

$$\text{i.e.,} R = \frac{180}{10 \times 10} = 1.8\Omega$$

Why is Ohm's law $V = IR$ not directly applicable here? The reason is that when the motor is running, a back emf is developed in the circuit. The current I is then given by (Ohm's law again!)

$$I = (V - V_{back})/R$$

or

$$VI = V_{back}I + I^2R$$

The first term on the right hand side (often abbreviated rhs) represents the mechanical power output of the motor, while the second represents power loss due to heat.

Example 4.2: A series battery of 6 lead accumulators each of emf 2.0V and internal resistance 0.25 Ω is charged by a 230V d.c. mains. To limit the charging current, a series resistance of 53Ω is used in the charging circuit. What is (a) the power supplied by the mains, and (b) the power dissipated as heat? Account for the difference in the two answers.

Answer: emf of the battery = 6 × 2.0 = 12V.

Internal resistance of the battery

$$= 6 \times 0.25 = 1.5\Omega$$

Charging current

$$= \frac{230 - 12}{53 + 1.5} = 4.0\mathrm{A}$$

Power supplied by the mains

$$= 230\mathrm{V} \times 4.0\mathrm{A} = 920\mathrm{W}$$

Power dissipated as heat

$$= 4 \times 4 \times (53 + 1.5) = 872\text{W}.$$

The difference (920 − 872) W = 48 W is the power stored in the accumulator in the form of chemical energy of its contents. (Note: You should compare the situation here with that for an electric motor dealt with in example 4.1. The role of 'back emf' there is taken up here by the emf of the battery. The difference between power input and power dissipated equals mechanical power for a motor, while here this equals the chemical power stored in the battery).

4.3 Thermoelectricity

We are by now familiar with the fact that when there is an electric potential difference between the ends of a conductor, an electric current flows in it. Now suppose there is a temperature difference between its ends. We know that heat or thermal energy flows then from the hotter to the colder end. This flow is also described as heat current or energy current. Clearly, if some of this energy is carried by the free electrons of the conductor, there will also be an *electrical* current. At the hotter end, electrons have a slightly higher average kinetic energy (they are moving faster). So one expects a net flow of electrons towards the lower temperature end, i.e., an *electric* current due to a *thermal* gradient. This is qualitatively the reason for various *thermoelectric* effects which we now describe. They are all very closely related to each other.

4.3.1 Seebeck Effect

Thomas Johann Seebeck, a German physicist, discovered in 1821 that a magnetic compass needle held close to a circuit made of two different conductors was deflected when the two junctions in the circuit were held at different temperatures. The deflection means that under the conditions described above, a voltage, called the thermoelectric emf (or Seebeck emf or Seebeck voltage) develops, causing a current to flow in the circuit. The magnetic field due to this current deflects the needle. The two junction circuit, often called a thermocouple, is schematically shown in Fig. 4.1. The voltage developed is rather small, of the order of a few microvolts per degree Celsius difference in temperature between the junctions. The sign and size of this *thermoelectric emf* depends on the materials involved as well on the temperatures T_1 and T_2. Fig. 4.2a shows a particular example, where the conductors are copper and constantan, and $T_1 > T_2$. The direction of the emf is shown. The effect is completely reversible, i.e., if the hot and cold junctions are interchanged, the emf changes sign, or the circulating current reverses direction.

Experiments on many different pairs of junctions show that the emf V_{AB} for two materials A and B is the difference between two terms V_A and V_B, characteristic of the materials A and B respectively, i.e.

$$V_{AB} = V_A - V_B. \qquad (4.3)$$

Thermoelectric emf (often called thermo emf) is thus a property of each material, though it is easily measured for a junction of two dissimilar materials at different temperatures. The thermoelectric emf of a number of thermocouples (i.e. conductor junctions) is given by a simple form

$$V_{AB} = \alpha\theta + \frac{\beta\theta^2}{2} \qquad (4.4)$$

where θ is the temperature difference between the junctions, and α, β are material parameters. The relation is valid over a limited range of temperatures. In Table 4.1, we give values of α and β in the temperature range 200 K to 400 K, for a number of substances with lead as the second junction. Lead is chosen because it has a small thermoelectric emf, and so is a convenient standard. Since the thermo emf of a thermocouple is the difference between that for individual materials, the thermo emf for *any* pair AC can be obtained if that for A and for C with a common material B is known. One has

$$V_{AC} = V_A - V_C$$

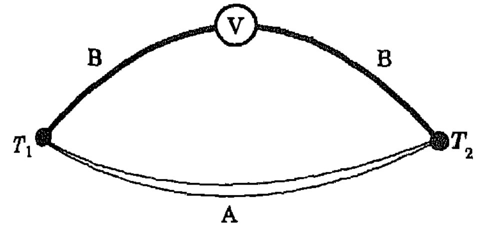


Figure 4.1: A schematic circuit for Seebeck effect. Two different conductors, A and B are joined together at two points or junctions, these are at different temperatures T_1 and T_2. The thermo emf can be measured via a voltmeter inserted as shown.

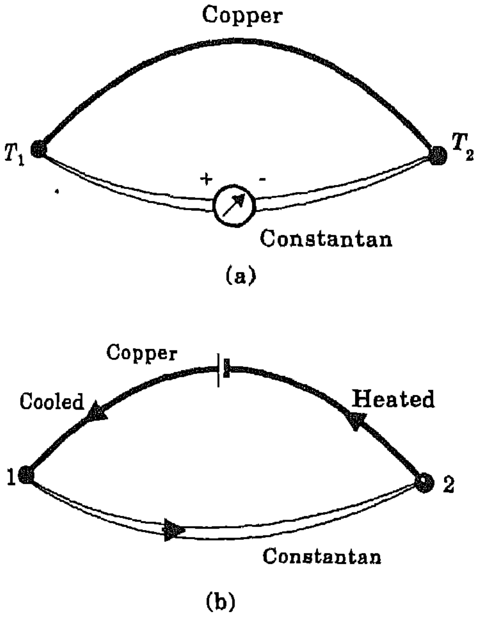


Figure 4.2: (a) The copper constantan thermocouple, with $T_1 > T_2$. The direction of emf is from copper to constantan at the hot end (T_1) (Seebeck Effect). (b) In the copper constantan thermocouple of Fig. 4.2a, if an electric current flows as shown, the junction 1 gets cooled and the junction 2 gets heated (Peltier Effect).

Table 4.1: Values of thermoelectric coefficients α and β

Substance	α (μV/°C)	β μV/(°C)^2
Aluminium	−0.76	+0.0039
Bismuth (commercial)	−43.7	−0.465
Copper	+1.34	+0.0094
Constantan (60% Cu 40% Ni)	−38.1	−0.089
Gold	+2.80	+0.010
Iron	+17.2	−0.048
Palladium	−7.4	−0.039
Platinum	−3.04	−0.033
(90% Platinum, 10% Rhodium)	+7.0	+0.0064

$$= (V_A - V_B) - (V_C - V_B)$$

$$= V_{AB} - V_{CB}. \tag{4.5}$$

Thus, for example, from the Table 4.1, the emf of the copper-constantan thermocouple can be found by subtracting the constantan thermo emf from that of copper. We see that it is positive, so that the direction of current flow is as shown in Fig. 4.2a. The rate of change of the thermoelectric emf with temperature, i.e. ($\mathrm{d}V/\mathrm{d}T$), is called the *thermopower* or the Seebeck coefficient S. One has

$$S = (\mathrm{d}V/\mathrm{d}T). \tag{4.6}$$

We see from Table 4.1 that good (highly conducting) metals like aluminium and gold have a small thermoelectric emf. Magnetic metals (e.g. Fe), alloys with magnetic elements (e.g. constantan) and semiconductors or semimetals have large thermoelectric emf's.

Example 4.3: For a copper–iron, and a chromel-alumel thermocouple, plots between thermoelectric emf and the temperature θ of the hot junction (when the cold junction is at 0°C) are found to satisfy approximately the parabola equation

$$V = \alpha\theta + \frac{1}{2}\beta\theta^2$$

with

$$\alpha = 14\mu V^\circ C^{-1}$$

(copper-iron)

$$\beta = -0.04\mu V^\circ C^{-2}$$

$$\alpha = 41\mu V^\circ C^{-1}$$

(chromel-alumel)

$$\beta = +0.002\mu V^\circ C^{-2}$$

Which of the two thermocouples would you use to measure temperature in the range of about 500°C to 600°C?

Answer: The temperature θ_0 (neutral temperature) corresponding to maximum emf is given by

$$\left(\frac{dV}{d\theta}\right) = 0 \text{ i.e. } \alpha + \beta\theta = 0$$

which gives

$$\theta_0 = -\left(\frac{\alpha}{\beta}\right)$$

For copper-iron thermocouple $\theta_0 = 14/0.04 = 350°C$. (Actually, the coefficients α and β vary slightly and the null temperature for Cu-Fe thermocouple is about 250°C). Beyond the null temperature, a thermocouple cannot be used because a given emf corresponds to two different values of θ. Thus for measuring temperatures beyond about 250°C, a Cu-Fe thermocouple is unsuitable. The chromel-alumel thermocouple, with the given values of α and β, shows no maximum for $\theta > 0$. Thus it can be used for measuring any high temperature and is suitable for the required range (500°C-600°C) in the question. In practice, its use is limited to about 1300°C.

4.3.2 Peltier Effect

An effect exactly converse to that discovered by Seebeck was found by Jean Charles-Athanase Peltier, a French watchmaker turned scientist, in 1834. He found, on passing an electric current through a junction of two dissimilar conductors, that heat was either absorbed or released at the junction, depending on the direction of the current. This is known after the discoverer's name as the Peltier effect. More precisely, if the Seebeck emf is from A to B at the hot junction an external emf applied in this direction will produce cooling at this junction and heating at the opposite junction. This effect is also entirely reversible. Fig. 4.2b shows, for the copper-constantan junction pair of Fig. 4.2a, that if an electric current is passed as shown, the junction 1 (hot junction of Fig. 4.2a) is cooled and junction 2 (cold junction of Fig. 4.2a) gets heated. We shall see later below that this cooling/heating can actually be used for refrigeration.

The Peltier coefficient π is defined as follows. The heat absorbed when a charge q coulombs passes from metal A to metal B is $q\pi$ joules. Again, the Peltier coefficient of a junction AB is the difference of the Peltier coefficients of A and B.

4.3.3 Thomson Effect

A third thermoelectric effect, harder to observe than the first two, was discovered by Thomson (known later as Lord Kelvin, see p 248 of class XI Physics text book). Thomson also showed that the three effects are all consequences of one basic phenomenon, and can therefore be quantitatively related to each other, as we shall describe below. The Thomson effect is related to the emf that develops between two parts of a *single* metal when they are at different temperatures. Specifically, if two points in a metal A have a small temperature difference dT, the electric potential difference is proportional to dT. The Thomson coefficient is the coefficient of proportionality, i.e.,

$$dV = \sigma dT \tag{4.7}$$

the units being volts for the potential difference and kelvin for temperature difference.

4.3.4 Relation between Thermoelectric Effects

The connection between the three thermoelectric phenomena namely the Seebeck, Peltier and Thomson effects, was first established by Thomson in 1857, using thermodynamic arguments.

The three effects, namely Seebeck, Peltier and Thomson discussed above are quantitatively defined in terms of the three coefficients V, π and σ (the thermoelectric emf, the Peltier heat and the Thomson emf respectively). It turns out that the basic quantity is the thermopower S, defined in Eq. (4.6) as the rate of change of the thermoelectric emf with temperature. It can be shown to be the ratio of the electric field developed across a conductor to the small temperature gradient between its ends, when no electric current flows in the system The Peltier coefficient π is related to S by the equation

$$\pi = TS = T\left(\frac{\mathrm{d}V}{\mathrm{d}T}\right). \tag{4.8}$$

The Thomson coefficient σ is related to S as

$$\sigma = -T\left(\frac{\mathrm{d}S}{\mathrm{d}T}\right) \tag{4.9}$$

Eqs (4.6), (4.8) and (4.9) complete the connection between different thermoelectric effects.

4.3.5 Applications

The thermoelectric effects have two important kinds of applications, one for measuring temperature, and the other in devices such as refrigerators and generators. We first discuss the former A thermocouple consisting of two dissimilar metals, for example, copper and constantan, has one junction of copper and constantan in contact with the region whose temperature is to be measured. The constantan wire is led to a vessel kept at a standard constant temperature, e g. ice at its triple point. Here it is joined to a copper wire. This copper wire, and that from the unknown temperature junction, are connected to a voltmeter. A commonly used voltmeter these days is an electronic digital voltmeter. Typically, this instrument has a least count of 10^{-7}V or less and a high internal or input resistance of 100MΩ to 1 Giga ohm. Since a copper constantan thermocouple develops a voltage of the order of 40μV per degree celsius, and has an internal resistance of 50-100Ω (depending on the length and thickness of the constantan, and copper wires), it is clear that the internal resistance has a negligible effect on measured values of thermoelectric emf. Temperature differences of order 0.0025°C are easily within reach. Different materials are used in different temperature ranges, since the voltages developed change in size and even sign with temperature. Copper constantan thermocouples are most suitable in the range 50K - 400K. For higher temperatures (upto 1500K or 2000K) junctions of platinum with platinum -rhodium alloys are specially convenient At cryogenic temperatures (1K - 50K) a junction of copper with gold-iron alloy wire is preferred since its thermoelectric emf is large Another common combination is chromel-alumel. The thermocouple is the preferred way of measuring temperatures in the industry and the laboratory, because of its accuracy and convenience The accuracy is high because a voltage or electrical measurement is involved. Since the junction is small, it absorbs very little heat, and quickly attains the temperature being measured. It is thus quite convenient

The other application of thermoelectricity is to devices. We have already seen that thermoelectric effects are reversible, so they can be used as heat engines, i.e , to absorb net heat and do useful work, or as heat pumps i.e , to remove heat if work is done on the working substance (see chapter 11 of class IX textbook) In this case, the work is done electrically, i.e., charges gain or lose electrical potential energy by moving through potential differences. For example, the Peltier effect can be the basis of a thermoelectric refrigerator, since on passing a current through a circuit with two junctions which are at different temperatures, one finds that heat is removed at one junction and liberated at the other junction. If on the whole, heat is removed from the working volume, one has a refrigerator. Such a refrigerator has, clearly no motors or moving parts, the work is all done electrically. The efficiency of a thermoelectric refrigerator is rather low in comparison with conventional ones. However, the former can be small in size without loss of efficiency, something which is not true of refrigerators with motors described in the class XI textbook. Thus when the region to be cooled is small or noise is not acceptable, the thermoelectric refrigerator is the device of choice, if a suitable thermoelectric material can be found. In general, one needs high thermopower, low resistivity (to minimise Joule losses) and low thermal conductivity (to enable large temperature differences to be maintained). Extensive research, especially in the former Soviet Union, has shown that perhaps the best materials from these points of view are based on the semiconductor Bi_2Te_3. The thermoelectric effect (Seebeck effect) can be used to generate electrical power. In remote areas, kerosene oil lamps which heat up one junction, with the cold junction exposed to air, have been used to power radio receivers and even transmitters!

4.4 Chemical effects

4.4.1 Introduction

The fact that chemical changes produce electrical effects was discovered accidentally in 1791, by Luigi Galvani, professor of anatomy at the University of Bologna, Italy. He found that an electrical current flowed across two dissimilar metals between which was a moist substance In his case, the moist substance was a frog and the passage of electrical current was detected by the twitching of its leg. Galvani thought that this was a manifestation of animal electricity while Volta, a professor of natural philosophy at the nearby University of Pavia, successfully reproduced some of Galvani's results with inanimate things. He assembled a series of silver and zinc discs in pairs (Fig. 4.3), separating each pair with a sheet of pasteboard soaked in a conducting liquid (salt water!). When the topmost disc of silver was connected by a wire with the lowest zinc disk, a steady current of electricity was produced. This is the voltaic pile. The basic reason for this electrochemical effect became clear rather slowly. Scientific investigation in this field actually began with the converse phenomenon, namely chemical effects of electrical currents. Michael Faraday, the great British experimental physicist, began in 1834 his experiments on the passage of electricity through liquids. Two metallic rods or plates, connected to two ends of a battery, are immersed in the liquid being investigated. Faraday found that a significant electrical current flows through some liquids, and not through others This flow of electric charge leads to interesting chemical changes which were quantified by him. Some examples of such electrochemical changes and

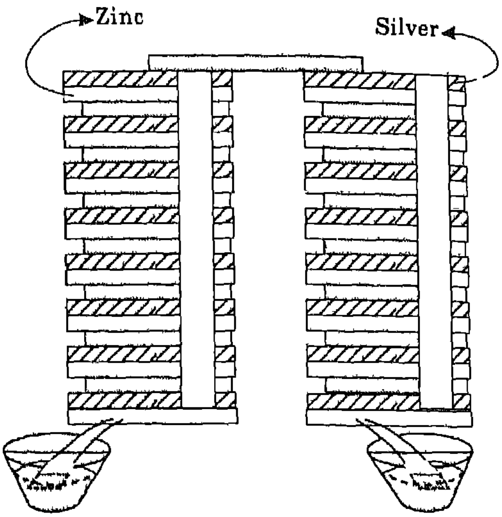


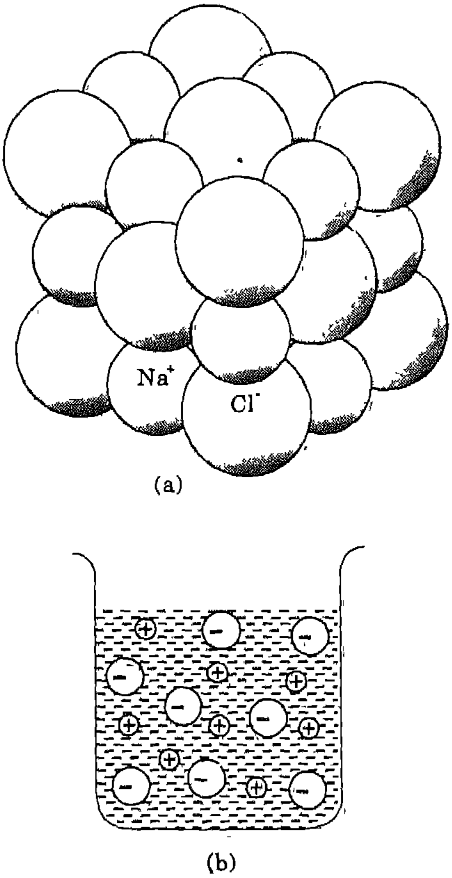
Figure 4.3: A voltaic pile more or less as Volta made it. The shaded slabs are silver, the blank ones are zinc and the salt soaked paste board is dark. The emf develops between silver and zinc in each pair of slabs and adds up so that there is considerable voltage difference between the top and the bottom (silver and zinc).

then the quantitative relations (laws of electrolysis, electrochemical series) are discussed below. We first introduce some terms which make it easier to talk about the phenomena, and then look into conduction of electricity in ionic liquids.

4.4.2 Electrolytic conduction

The conducting pieces (rods, plates or other objects connected to the sources of emf and immersed in the liquid or solid medium), are called electrodes. Positively charged ions are called *cations* and negatively charged ions are called *anions* If they exist in an electrolyte, the former migrate towards the *cathode* and the latter towards the *anode* These are the two electrodes. If the circuit is completed from outside the electrolyte (say by joining the anode and the cathode by a conductor), external current flows from the cathode to the anode (electrons flow in the conductor from the anode to the cathode).

Consider the medium in which the electrodes are immersed. How does it conduct electricity? Let us start with a simple example of such a medium, namely common salt dissolved in water. We first describe each of these separately in molecular terms, and then try to understand what happens when they are brought together. Common salt or rock salt, i e., crystalline NaCl, consists of Na^+ and Cl^- ions attracting each other, and arranged as shown in Fig. 4.4a. To dissociate NaCl, i.e., to remove Na^+ and Cl^- ions from each other, costs an energy of 7.9 eV per molecule. This is much larger than the average thermal agitation energy of 0.03 eV which each molecule has at room temperature. Now consider water. Because the H-O-H atoms do not lie in a straight line, clearly a water molecule has an electric dipole moment (the measured value is 1.9×10^{-29} coulomb metre). Water molecules tend to stick to each other (associate) because of electrical forces specific to the distribution of electrons in H_2O (this is the well-known hydrogen bonding). Due to these two reasons, i e., dipole moment and association, water molecules are easily aligned in an external electric field. We say that water is highly polarizable It has therefore a very large dielectric constant, nearly 81 at room temperature (see chapter 2 for a discussion of polarizability and dielectric constant)



(a)

(b)

Figure 4.4: (a) Arrangement of Na^+ and Cl^- ions in a rock salt crystal (b) Free Na^+ and Cl^- ions in solution.

What happens when NaCl, (or more descriptively, Na^+Cl^-) is dissolved in water? At distances large compared to the size of Na^+ and Cl^- ions, as well as of H_2O molecules, the former can be approximated as point charges, and water as a continuous medium. For such distances, the attractive electrostatic potential energy is reduced by a factor of 81(!)

compared to that in free space. At shorter distances, the reduction is not so large: but the electric fields due to Na^+ and Cl^- ions still polarize the H_2O dipoles, and the water molecules around a particular ion are attracted to each other or associated. A sizable polarized cluster of water molecules thus forms around each ion. (This is known as hydration in chemistry). The attraction between Na^+ and Cl^- ions is therefore reduced a great deal in water by the same processes that make its dielectric constant so large. The dissociation energy of an Na^+Cl^- molecule in water is so much less than in rock salt that the energy of thermal agitation at room temperature is enough to dissociate a dilute solution of NaCl molecules completely into free Na^+ and Cl^- ions (Fig. 4.4). These are free to move under the action of an external electric field. Substances like NaCl in water which conduct electricity because a fraction of their molecules dissociate into positive and negative parts are called electrolytes. The most common electrolytes are acids, bases or salts which 'ionize' or partially break up into anion and cation parts when dissolved in water, or say, alcohol. Many salts such as NaCl or KCl are electrolytes in their molten state, without any solvent. There are also solids such as AgI in which ions (in this case Ag^+ ions) are mobile. These are the solid state electrolytes.

Electrolytes conduct electricity, but not as well as metals. Their conductivity at room temperature is about 10^{-5} to 10^{-6} times that of good metals. The reasons for low conductivity are many viz., the small number density of free ions in comparison to that of free electrons, the relative difficulty with which they move in the solution which is dense and disordered, and the large mass of ions which makes them drift much more slowly than electrons in a given electric field.

Water, when pure, has a small concentration of dissociated parts, namely H^+ ions (protons, actually H_3O^+ or hydronium ion because of hydration, or attachment of the proton to water) and OH^- (hydroxyl) ions, about one part in 10^7. The logarithm of this, to the base ten, is given the symbol pH; neutral, pure water has a pH of 7. This leads to a conductivity of 10^{-4} mho/m at room temperature. By adding acid or alkali to water, its pH (hydrogen ion concentration) can be increased or decreased. This leads to very large changes in its conductivity; such changes are important for many natural (biochemical, bioelectrical), laboratory and industrial processes.

4.4.3 Electrolysis

With the above background, we are ready to describe the phenomena of electrolysis in modern terms, and to understand **Faraday's laws of electrolysis. These laws are as follows:**

(a) The mass of a substance produced by a cathode or anode reaction in electrolysis is directly proportional to the quantity of electricity (i.e., charge) passed through the cell.

(b) The masses of different substances produced by the same quantity of electricity are proportional to the equivalent weights of the two substances.

We first consider two examples of electrolysis, namely electrodeposition and electrolysis of water, and try to show how these are consistent with Faraday's laws of electrolysis. To understand these facts, let us trace the flow of charge, starting from the negative end C′ of the battery (Fig.4.5). Electrons (free electrons of the metallic wire) drift from C′ into the cathode C. In the electrolyte, positive and negative ions carry the current. In the present case, i.e. $CuSO_4$ in water,the possible ions are Cu^{++}, SO_4^{--} and H^+, OH^-. (The relative concentration of H^+ ions depends on the acidity of the medium and can be substantial for an acidic electrolyte. This improves the conductivity of the electrolyte and does not adversely affect the reactions we describe below, involving Cu^{++} ions.) The Cu^{++} ions drift to the cathode C, and are neutralized by the elec-

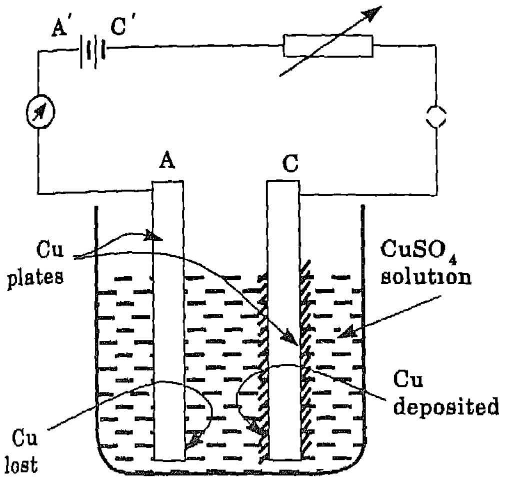


Figure 4.5: Electrolysis of copper. A steady voltage is applied between two copper plates immersed in $CuSO_4$ solution (electrolyte) Cu^{++} ions drift inside the electrolyte from the anode to the cathode.

trons of the metal, flowing in from C′. The reaction at the cathode is

$$Cu^{++} + 2e^- \rightarrow Cu. \tag{4.10}$$

(This, i.e., addition of electrons is called *reduction* in chemistry). The flow of current in the electrolyte is maintained by Cu from the anode dissolving into the solution, i.e., via the *oxidation* reaction at the anode

$$Cu \rightarrow Cu^{++} + 2e^- \tag{4.11}$$

with the two electrons flowing via AA′ to the positive pole of the battery. (Flow of appropriate ions inside the electrochemical cell or battery from C′ to A′ completes the circuit) The concentration gradient set up by the variation in Cu^{++} concentration from anode (higher) to cathode (lower) sets up a potential difference, which alters the concentration of SO_4^{--} ions.

The result of current flow in the circuit is thus deposition of Cu at the cathode and its 'plating out' from the anode. In the presence of free electrons from the metal wires and with a steady source of emf, a steady current flows through the circuit Any accumulation of charge anywhere is not possible, as this is a higher energy arrangement (due to Coulomb repulsion among the accumulated charges) which is unstable because of the flow of charges attracted or repelled from this arrangement. Because the flow of charge is at a steady rate, it is clear from Eqs (4.10) and (4 11) that (i) one atom of Cu is deposited at the cathode for each pair of electrons flowing through the wires, and (ii) the numbers of Cu atoms deposited at the cathode and 'plated out' from the anode are the same.

From (i) above, we conclude that the amount of copper deposited is proportional to the charge flowing through the circuit. Since the latter is proportional to the time for which the current flows in case of a steady current, the amount of copper deposited is proportonal to the time for which the current flows. We have thus been able to make sense of the basic observed facts.

As you probably know, the phenomenon of electro- deposition has many applications. One of them is electroplating. The metallic material to be electroplated is made the cathode of an electrolytic cell, the anode being the metal whose coating is desired. On passing a current through the circuit with a suitable electrolyte, metal deposits on the cathode (Fig. 4.6). There are certain requirements on battery voltage and the types of the two metals, if the reverse reaction, namely electrolysis of the object to be electroplated and deposition of its ions on the anode metal, is to be avoided. Other applications are reduction of ores to metals (e.g. Al_2O_3 or alumina to aluminium) and the purification of metals.

Electrolysis of water: The chemical reaction

$$2H_2O = 2H_2 + O_2 \tag{4.12}$$

requires 296 kJ per mole of water, or 2 9 eV per molecule. This energy can be supplied electrically, by an appropriate potential difference. The process requires water as an electrolyte, in an electrolytic cell, between the electrodes of which a potential difference is

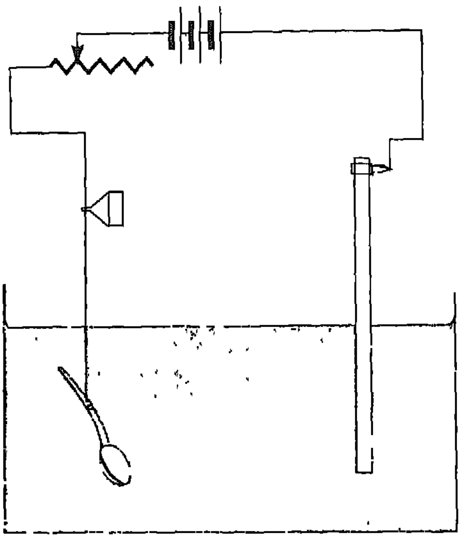


Figure 4.6: Schematic diagram showing electro-deposition

maintained.

The set up for this electrochemical reaction is shown in Fig. 4.7. It consists of two platinum electrodes, these being chosen because platinum does not dissolve into the electrolyte and does not react with the products of electrolysis. The electrodes are immersed in water, with a small amount of sulphuric acid added to it. The latter makes the water more conducting by increasing the number of free ions in the solution. The evolved gases are collected as shown in Fig. 4.7.

The reactions leading to electrolysis are as follows: At the cathode, the electrons flowing in from the negative terminal of the battery reduce water, i.e.

$$4H_2O + 4e^- \longrightarrow 4OH^- + 2H_2. \qquad (4.13)$$

Thus hydrogen gas is evolved, and the OH^- ion concentration near the cathode rises. At the anode, since the OH^- concentration is low, and SO_4^{--} is difficult to oxidize, direct oxidation of H_2O is the major reaction, i.e.

$$2H_2O - 4e^- \longrightarrow 4H^+ + O_2 \qquad (4.14)$$

The electrons released flow from the anode to the positive terminal of the battery. The oxygen gas can be collected and the H^+ concentration rises. In the electrolyte H^+ and OH^- diffuse away, move towards the opposite electrodes and neutralize:

$$4H^+ + 4OH^- \longrightarrow 4H_2O \qquad (4.15)$$

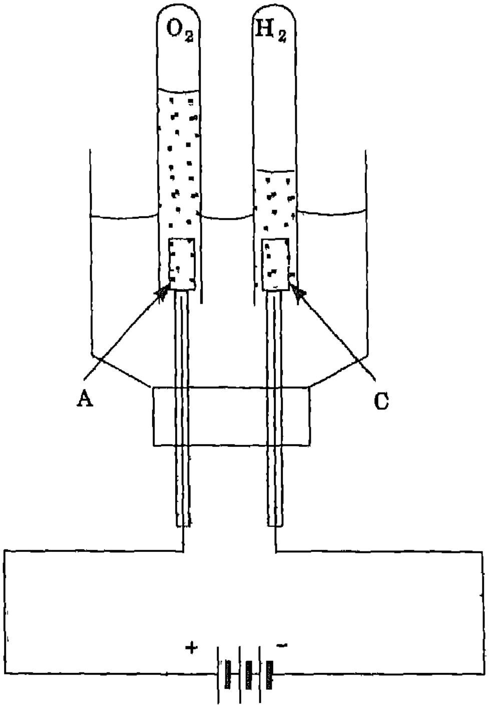


Figure 4.7: Schematic figure of an experimental arrangement for electrolysis of water. The electrodes, the external circuit, the electrolyte and the collection of H_2 and O_2 are shown.

Adding Eq. (4.13) to Eq. (4.14) and using Eq. (4.15), we have, finally,

$$2H_2O = 2H_2 + O_2 \qquad (4.16)$$

with four electrons moving through the circuit per two molecules of water hydrolyzed. Since the energy W required to break up a molecule of H_2O is about 2.9 eV and two electrons circulate in the process, the voltage V supplied to one electron is given by $W/2 = eV$ or $V = (W/2e)$ where e is the electron charge.

This voltage is therefore $2.9/2 = 1.45$ V. The experimental number is 1.23 V.

Faraday's laws of electrolysis: We have seen earlier from the example of copper electrodeposition that the mass of copper deposited is proportional to the charge Q flowing through. If the current I through the circuit is constant then $Q = It$ where t is the time for which the current flows. Thus

$$m \propto It \tag{4.17a}$$

or

$$m = zIt \tag{4.17b}$$

where z is a constant of proportionality. This is true in general, for any electrolytically deposited material, since it depends only on the fact that depositing an atom requires a fixed (integral) number of electrons to flow through. This is *Faraday's first law of electrolysis*.

What is the constant of proportionality in Eq. (4.17b)? Consider a mole, i.e., an Avogadro number N_A of the atoms deposited on an electrode. If the valence of the atom in the particular electrolyte or electrode is p, then for each atom deposited, a charge pe must flow through the circuit where e is the electron charge, so that for a mole, the charge flowing through is $N_A p|e|$. We have used the magnitude symbol $|e|$ because the sign depends on the direction of current, and the ions deposited. Now consider Eq (4.17b) again. We have, for m = one mole = M, $It = N_A p|e|$, so that

$$z = \frac{1}{N_A|e|}\frac{M}{p}; \text{ i.e. } z \propto \frac{M}{p}. \tag{4.18}$$

The quantity (mole divided by valence) is called the equivalent mass or *chemical equivalent*, and the fundamental constant $N_A|e|$ is called a *Faraday* and given the symbol F. It has the value 96,487 ($\simeq$ 96,500) C/mol. Equation (4.18), which says that z is proportional to mole divided by valence, is *Faraday's second law of electrolysis*

Eq (4.18) suggests several very important things, some qualitative and one quantitative. The fact that for the same charge passing through, the amount deposited is proportional to molar mass (mole) divided by valence, clearly implies that valence and electrical charge are connected. Also, since the gram atom or molecule (mole) contains a fixed (Avogadro) number of atoms by definition, the fact that to deposit it requires the same charge (times valence) means that there is an elementary unit of charge, common to all matter; and that all charges are integral multiples of this unit. These implications concerning the 'molecule of electricity' were already clear to physicists like Maxwell by 1870 or so, though they were not easy to reconcile with the prevailing ideas of *continuous* charge distributions. Maxwell, for example, used the hypothesis of a 'molecule of electricity' to explain electrolysis, but remarked that 'it is out of harmony with the rest of this treatise (on Electricity and Magnetism)'. The quantitative point of course is that if the value of a Faraday is measured accurately by experiment and the Avogadro number N_A is known, e.g. from Brownian motion, the elementary electronic charge e can be found. Notice that neither of these are measurements on an individual electron, but are results of large scale experiments! From the value 96,500 C/mol for a Faraday, and 6.02×10^{23}/mol for the Avogadro number, we estimate $e \simeq 1.6 \times 10^{-19}$C.

Example 4.4: It is desired to deposit 0.54 kg of silver per hour on the cathode of a silver voltameter whose resistance is 0.72 mΩ. How much potential difference must be maintained between the plates of the voltameter during electrolysis. Use the known value of Faraday's constant: F = 96,500 C mol^{-1}. Relative atomic mass of silver is 108.

Answer: Silver is monovalent. Its ion Ag$^+$ carries one unit of electronic charge (e). Since $F = N_A e$ (where N_A is Avogadro's number), 1 mole of silver ions carries a charge 96,500 C. That is, to deposit 108 g of silver, the charge required is 96,500 C. Therefore the charge required to deposit 540 g of silver is 96,500 × 5

= 482,500 C The required current then is

$$I = \frac{482,500C}{3,600s} = \frac{4825}{36} \simeq 134A.$$

The steady potential difference required across the plates during electrolysis is

$$\frac{4825}{36} \times 0\ 72 \times 10^{-3} = 96\ 5mV.$$

Note that we have used Ohm's law above This is known experimentally to be valid for voltameters with soluble electrodes (such as silver or copper voltameters) But for a water voltameter, for instance, a 'back emf' of about 1.7V needs to be taken into account before applying Ohm's law

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Example 4.5 : A steady current is passed for a certain time through three voltameters connected in series a copper voltameter (Cu electrodes in $CuSO_4$), a silver voltameter (Ag electrodes in $AgNO_3$) and an iron voltameter (Fe electrodes in $FeCl_3$) The mass of copper dissolved off the anode of the first voltameter is found to be 473 1 g Predict the masses of silver and iron deposited on the respective cathodes of the other two voltameters. Relative atomic masses of Cu, Ag and Fe are 63 54, 107.9 and 55.85 respectively.

Answer : The valences of Cu (in $CuSO_4$), Ag (in Ag NO_3) and Fe (in $FeCl_3$) are 2, 1 and 3 respectively The ratio of relative atomic mass to valency is 31.77 for Cu, 107 9 for Ag, and 18 62 for Fe. According to Faraday's second law of electrolysis, for a given amount of charge (current × time) passed, the masses deposited on the cathodes (or dissolved off from the anodes) are in proportion to the ratios of relative atomic mass divided by valency Therefore, mass of silver deposited

$$m_{Ag} = \frac{473.1 \times 107.9}{31\ 77} = 1606.8\ g$$

mass of iron deposited

$$m_{Fe} = \frac{473\ 1 \times 18.62}{31.77} = 277.2g$$

4.4.4 Electrochemical cells

Many chemical reactions take place with release of energy. If this happens in an electrolyte, with one or more of the ionic species in it as participants, it is then possible that the energy released directly and solely increases the electrical potential energy of the ions The *chemical* reaction is thus a source of *electrical* energy The system can be used as a source of electrical power if the chemical reaction proceeds at a steady rate This is what one tries to achieve in an electrochemical cell, or battery Now in a chemical reaction, the energy released can typically be about 2 $\times 10^5$ joule per mole. This corresponds to moving a mole of ions, each of charge say $2|e|$ (the ionic charge on one of the reactants or products, say) up by a potential of about one volt!* We see that the emf of a cell cannot be very large. Known emf's are in the range 1 to 1.5V, the highest being 2.5V or so Since the emf of a cell is derived from a chemical reaction, the total energy it can provide is limited by the amount of reactants, unless the reactants can be continuously supplied and the products removed. The latter is what is done in a fuel cell.

In spite of the above facts, namely the small emf and limited energy capacity of a cell, and despite unfavourable economics (primary electrochemical power is ten to hundred times more expensive than electromagnetic power we get from the mains), cells or batteries are still used because of their compactness and convenience. They have no moving parts, are pollution–free and are portable. Flashlights, transistor radios, electronic watches, hearing aids, car batteries, all use cells of various kinds. Also, increasing awareness of decreasing energy resources has led to a great deal of interest in electrochemical methods of energy conversion. We discuss below a few types of cells, mostly those in common use.

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*The energy released per molecule is $(2 \times 10^5/6 \times 10^{23})$J We equate this to the electrical potential energy $2e \times V$ where V is the voltage and $2|e|$ is the ionic charge 3.2×10^{-19} C, e being the charge on an electron We then find $V \simeq 1$ V.

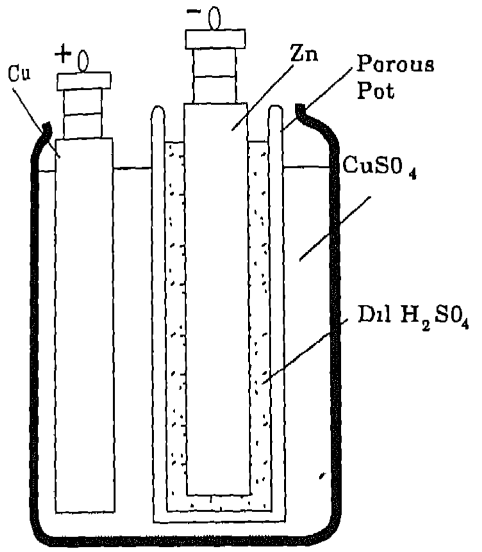


Figure 4.8: A Daniel cell showing the negative and positive terminals (Zn and Cu), the electrolytic solution and a porous pot separating them

Daniel cell. This is the simplest cell to talk about, though it is not much in use because of its inconvenience. The cell consists of a zinc electrode immersed in dilute sulphuric acid (or acidulated zinc sulphate solution) and a copper electrode in copper sulphate solution, with a membrane (a fired clay or porcelain pot) through which ions can pass from one solution to the other (Fig. 4.8). At the former electrode, zinc ions pass into solution, and at the positive end of the cell, Cu ions are deposited. The reactions are as follows:

$$(i) \quad Zn \longrightarrow Zn^{++} + 2e^- \qquad (4.19a)$$

at the Zn electrode,

$$(ii) \quad Zn^{++} + CuSO_4 \longrightarrow ZnSO_4 + Cu^{++} \qquad (4.19b)$$

(in solution) and

$$Cu^{++} \longrightarrow Cu - 2e^- \qquad (4.19c)$$

at the Cu electrode

Since inside the cell, the positive ion motion is from Zn to Cu, the former is called the anode and the latter is called the cathode If the circuit is completed externally by connecting the Zn and Cu electrodes, electrons flow out of the former and into the latter. Thus Zn is the external negative end and Cu is the external positive end The emf of this cell is 1.1V, the energy 2eV where $V = 1.1$ volt is roughly equal to the energy produced per Zn atom in the reaction

$$Zn + CuSO_4 \longrightarrow ZnSO_4 + Cu \qquad (4.20)$$

which is the total reaction represented by Eqs (4.19a-c). The emf of the cell can also be understood as the sum of the energies released in the half reactions (4.19a) and (4.19c) at the cathode and anode respectively The reactions involve removal of electrons or oxidation (4.19a) and addition of electrons or reduction (4.19c) The energies are described in terms of oxidation potentials, the oxidation potentials for the two reactions

$$Zn \text{ (solid)} \longrightarrow Zn^{++} + 2e^- \qquad (4.21)$$

and

$$Cu \text{ (solid)} \longrightarrow Cu^{++} + 2e^- \qquad (4.22)$$

are 0.76V and -0.34V respectively. Therefore the emf associated with the full electrochemical reaction in which Zn dissolves into solution and Cu out of it, is $0.76 + 0.34 = 1.1$ V.

The cell is not very useful because hydrogen is also produced by electrolysis in the cell, and migrates to the Cu electrode. There it discharges, bubbles of hydrogen cover the electrode, the electrical resistance (internal resistance) of the cell increases, the current decreases and finally stops. This problem, known as polarization, is common to many electrochemical cells, and is overcome for example by adding a substance which oxidizes H_2. This is done in the Leclanche dry cell as described below

Leclanche or dry cell: The carbon zinc dry cell is the most widely used primary cell. A cut-away diagram of this cell is shown in Fig. 4.9. The electrolyte is a moist paste of ammonium chloride containing some zinc chloride. One electrode is a carbon rod surrounded by a mixture of manganese dioxide and powdered carbon The MnO_2 prevents the formation of gaseous hydrogen which would polarize the cell, and the powdered carbon reduces the internal resistance of the cell The other electrode, which is consumed as the cell is used, consists of a zinc cup. The cup serves as the cell container

When an external circuit is connected across the cell, zinc atoms in contact with the electrolyte ionize, losing two electrons per atom. The electrons flow into the metal wire circuit, and Zn^{++} ions pass into solution. The zinc is thus the anode.

$$Zn \longrightarrow Zn^{++} + 2e^- \qquad (4.23)$$

The ammonium ions of the electrolyte remove electrons from the carbon cathode to which electrons flow in from the external circuit, one thus has

$$2NH_4^+ + 2e^- \longrightarrow 2NH_3 + H_2 \qquad (4.24)$$

at the cathode The hydrogen is neutralized by MnO_2 here. In the body of the electrolyte, the zinc ion Zn^{++} migrating towards the carbon cathode combines with the Cl^- ions to form zinc chloride, so that the overall chemical reaction is

$$Zn + 2NH_4Cl + 2MnO_2 \longrightarrow ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3 + Q'. \qquad (4.25)$$

Q' being the energy released per formula unit of reactants This is finally the chemical reaction which releases energy (and dissolves ammonia as well as water into the electrolyte). The reaction involves migration of ions into and out of the electrolyte, and thus the energy of the reaction can be converted to increase their electrostatic potential energy. Some energy is lost, of course in making the current flow inside the cell (its internal resistance), but this can be reduced by proper choice of materials as well as the size and shape for the cell The emf of a dry cell is about 1.5 V; it can supply continuously, an electric current of 0.25 A. Its internal resistance can vary from about 0.1Ω to 10Ω. A commonly used cell with an alkaline electrolyte is the mercury cell which has mercuric oxide (mixed with graphite) as the cathode, potassium hydroxide as electrolyte, and a zinc anode. The cell is very durable, has a high energy density (or amount of electro-chemical energy per unit volume) and large currents can be drawn. Under the common name 'button' cell, this is used in watches, cameras, hearing aids, etc..

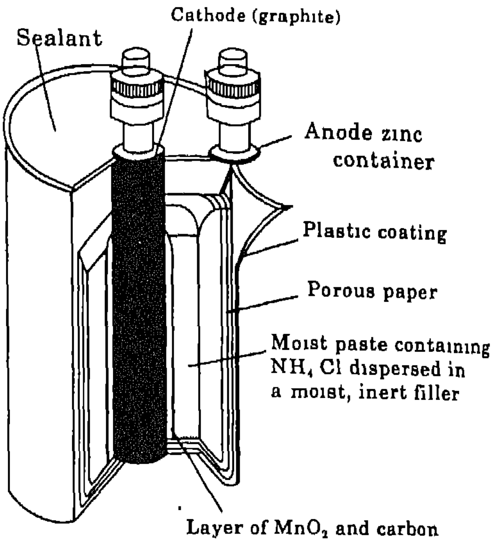

Cathode (graphite)
Sealant
Anode zinc container
Plastic coating
Porous paper
Moist paste containing NH_4Cl dispersed in a moist, inert filler
Layer of MnO_2 and carbon

Figure 4.9: The Leclanche or dry cell

Secondary cell or accumulator: Some cells can be recharged by passing a current through them in the reverse direction The chemical processes that occur at the electrodes during discharge are then reversed, and the cell recovers its original state, except that some energy is lost during the charge discharge cycle. Such cells are called secondary cells or accumulators. A common example is the lead sulphuric acid cell, invented by Gaston Plante, a French physicist, in 1859. The electrodes consist of alternating parallel plates of lead dioxide (positive external electrode or cathode) and spongy lead (Pb, negative electrode or anode) insulated from each other by porous separators made of wood, rubber, plastic or glass fibre This arrangement is immersed in an electrolyte of dilute sulphuric acid contained in a glass or rubber composition tank

(see Fig 4.10)

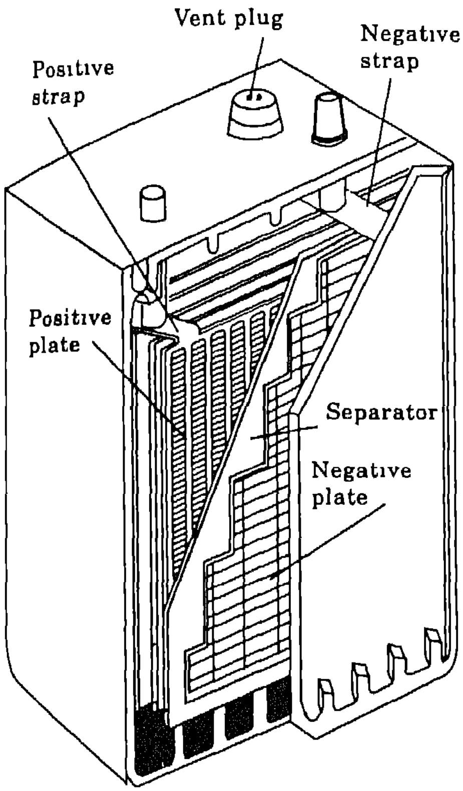


Figure 4.10: The lead storage battery, of the sort used in cars The cut-away view shows one of the positive plate grills, a negative plate grill, and separators

When the cell operates, the reactions are as follows:

Anode:

$$Pb(s) + HSO_4^- + H_2O \longrightarrow PbSO_4(s) + 2e^- + H_3O^+ \qquad (4.26a)$$

Cathode:

$$PbO_2(s) + HSO_4^- + 3H_3O^+ + 2e^- \longrightarrow PbSO_4(s) + 5H_2O \qquad (4.26b)$$

Overall reaction:

$$Pb(s) + 2HSO_4^- + 2H_3O^+ + PbO_2(s) \longrightarrow 2PbSO_4(s) + H_2O. \qquad (4.26c)$$

We notice that as the reaction proceeds lead sulphate is formed at each electrode, and both Pb *and* PbO_2 are depleted Also, the concentration of H_2SO_4 is diminished This is the *discharging* of the accumulator As the electrodes get converted to $PbSO_4$, the voltage falls. The accumulator is *charged* by passing a current in the opposite direction so as to reverse the reactions (4.26a-c), and thus to restore the cell to its original condition Normally several cells, each with several plate pairs connected in parallel, are used. Each cell produces a voltage of 2.05 V, and typically six are connected in series for a car battery to produce with an emf close to 12 V.

The state of charging is simply monitored by measuring the specific gravity of the electrolyte It varies from 1.28 when fully charged (sulphuric acid + water) to 1.12 when discharged (mostly water) This secondary cell has a low internal resistance (i.e. it can deliver a high current if needed), and can be recharged a very large number of times without deterioration in properties.

Fuel cell. The fuel cell is an electrochemical cell in which the chemical energy of fuels continuously supplied from the outside is converted into electrical energy. Their advantages are that there is no need to replace reactants, as in a primary cell, or to recharge, as in a secondary cell. These cells are in the process of development for large scale use An example is the hydrogen-oxygen fuel cell. The overall reaction is

$$2H_2(g) + O_2(g) \longrightarrow 2H_2O \qquad (4.27a)$$

The half-reactions at the anode and cathode respectively, and in an alkaline (basic) electrolyte solution, are as follows: (Anode):

$$H_2(g) + 2OH^- \longrightarrow 2H_2O + 2e^- \qquad (4.27b)$$

(Cathode):

$$O_2(g) + 2H_2O + 4e^- \longrightarrow 4OH^- \qquad (4.27c)$$

The basic solution is used in preference to the acidic because with the former problems of corrosion are less, and the reaction rate at the electrodes improves

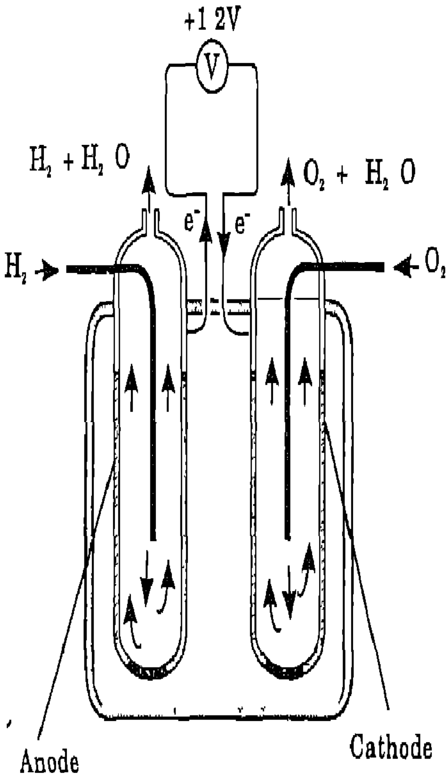


Figure 4.11: A hydrogen-oxygen fuel cell. Hydrogen gas is passed through a porous graphite anode impregnated with a catalyst and oxygen is diffused through a similarly constructed cathode. The electrolyte is a concentrated potassium hydroxide solution. The cell has an emf of +1.2 volts.

A schematic design of the hydrogen-oxygen fuel cell is shown in Fig.4.11. Two chambers constructed of porous carbon dip into an aqueous solution of KOH. Hydrogen gas is pumped into one chamber where it is oxidized according to the anode reaction (4.27b). Simultaneously, oxygen gas is pumped into the other chamber where it is reduced according to reaction (4.27c). Because H_2 and O_2 react very slowly at room temperature, catalysts are needed, and are mixed in with the carbon of the porous pots. Some catalysts at the anode are finely divided platinum or palladium, and at the cathode cobalt oxide, platinum or silver.

Fuel cells are attractive because they are very efficient; chemical energy can be *completely* converted to electricity. They (electrochemical cells in general) are *not* heat engines, so their efficiency is not limited by working temperatures etc. (Chapter 11, Class XI Physics textbook). In practice, the efficiency is in the range of 70%. The main problems at present are the cost of materials, lack of long term reliability, and uncertainties about initial fuels.

Summary

1. If a current I flows through a potential drop V, the energy lost per second by the drifting charges is IV. In a resistor of resistance R, this loss of energy appears as heat. The rate of heat production P is

   $$P = IV = I^2R = V^2/R,$$

   and is independent of the direction of current.

2. (a) **Seebeck effect.** If two junctions of dissimilar conductors in a circuit are held at different temperatures, an emf develops causing a current to flow in the circuit The thermoelectric emf V_{AB} for two materials A and B.is $(V_A - V_B)$, where V_A and V_B are characterstic of the materials A and B respectively This implies $V_{AC} = V_{AB} - V_{CB}$. The thermoelectric emf of a number of thermocouples (i e. conductor junctions) is given by a simple form

   $$V_{AB} = \alpha\theta + \frac{1}{2}\beta\theta^2$$

   where θ is the temperature difference between the junctions and α, β are material parameters. A thermocouple provides an accurate and convenient device for measuring temperature The copper-constantan thermocouple is very suitable in the range 50 K - 400 K. It develops a voltage of the order of 40 μV per degree Celsius

   (b) **Peltier effect.** When an electric current is passed through a junction of two dissimilar conductors, heat is either absorbed or released at the junction, depending on the direction of the current The effect finds application in thermoelectric refrigerators.

   (c) **Thomson effect** This refers to the emf that develops between two parts of a single metal when they are at different temperatures.

   All the three thermoelectric effects can be related to each other quantitatively using thermodynamics.

3. The water molecule has an electric dipole moment, and water molecules tend to 'associate'. These two facts make water a highly polarizable medium with a dielectric constant of about 81 at room temperature

   Crystalline NaCl consists of Na^+ and Cl^- ions bound by electrostatic attraction When it is dissolved in water, this electrostatic potential energy is reduced because of the high dielectric constant of water, and hydration. The thermal agitation at room temperature is then enough to dissociate a dilute solution of NaCl molecules into free Na^+ and Cl^- ions.

4. **Electrolysis of $CuSO_4$**

   At the cathode

   Cu^{++} ions drift to the cathode and are neutralized by the electrons flowing in from the negative terminal of the external source

   $$Cu^{++} + 2e^- \longrightarrow Cu$$

At the anode

Cu dissolves into the solution producing Cu^{++} and $2e^{-}$, the latter flowing to the positive terminal of the source

$$Cu \longrightarrow Cu^{++} + 2e^{-}$$

In effect, copper is dissolved off the anode and deposited at the cathode

5. **Electrolysis of water**

At the cathode:

$$4H_2O + 4e^{-} \longrightarrow 4OH^{-} + 2H_2$$

At the anode:

$$2H_2O - 4e^{-} \longrightarrow 4H^{+} + O_2.$$

In the electrolyte, H^{+} and OH^{-} diffuse away towards the opposite electrodes and neutralize.

$$4H^{+} + 4OH^{-} \longrightarrow 4H_2O$$

The net reaction is

$$2H_2O \longrightarrow 2H_2 + O_2$$

which requires 2 9 eV per molecule.

6 **Faraday's laws of electrolysis**

(a) The mass of substance (m) deposited at the cathode (or dissolved off the anode) in electrolysis is proportional to the total charge Q passing through the circuit If the current I is constant and flows for a time t, $Q = It$ and

$$m = zIt$$

where z is a constant of proportionality

(b) The masses of different substances produced in electrolysis by the same quantity of electricity (i e. total charge) are proportional to their equivalent masses

These laws are easy to understand in the atomic view For each atom of valency p to be deposited, a charge pe must pass through the circuit. Therefore, for a mole of the substance deposited i e. for $m = M$, where M is the relative atomic mass, the total charge flowing through is $N_A pe$, where N_A is the Avogadro's number Thus

$$M = zN_A pe \text{ or } z = M/N_A ep$$

where M/p is the equivalent mass of the substance Thus,

$$m = \frac{MIt}{N_A ep} \quad \text{i e} \quad m \propto \frac{M}{p}$$

for a given amount of charge It, which is the second law of electrolysis.

7 Faraday's second law of electrolysis has important implications. It suggests that valency and electrical charge are connected, and that there is an elementary unit of charge e common to all matter so that all charges are integral multiples of e The fundamental constant $N_A e$ is called Faraday and denoted by F From the measured values of $F \simeq$ 96500 C/mol (from experiments on electrolysis) and of $N_A = 6\,02 \times 10^{23}$/mol (from experiments on Brownian motion), e is estimated to be $1\,6 \times 10^{-19}$C

8 In ***electrochemical cells***, chemical reactions are a source of electrical energy Examples of primary cells are the Daniel cell and the carbon-zinc dry cell In a secondary cell or accumulator such as the lead sulphuric acid cell, chemical processes that occurred at the electrodes during discharge are reversed by passing a current through it in the reverse direction, and the cell gets charged i e. recovers its original state In a fuel cell, chemical energy of fuels continuously supplied from the outside is converted into electrical energy.

Exercises

4.1 A current of 4 0 A flows through a 12Ω resistor. What is the rate at which heat energy is produced in the resistor?

4.2 An electric motor operates on a 50V supply and draws a current of 12 A. If the motor yields a mechanical power of 150 W, what is the percentage efficiency of the motor?

4.3 A 10 V battery of negligible internal resistance is charged by a 200 V d.c supply If the resistance in the charging circuit is 38Ω, what is the value of the charging current?

4.4 A heating element is marked 210 V, 630 W What is the current drawn by the element when connected to a 210 V d c. mains? What is the resistance of the element?

4.5 A 10 V storage battery of negligible internal resistance is connected across a 50Ω resistor made of alloy manganin. How much heat energy is produced in the resistor in 1 h? What is the source of this energy?

4.6 An electric motor operating on a 50 V d.c supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor

4.7 An electric bulb is marked 100 W, 230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the bulb in 20 minutes?

4.8 The maximum power rating of a 20Ω resistor is 2 0 kW. (That is, this is the maximum power the resistor can dissipate (as heat) without melting or changing in some other undesirable way). Would you connect this resistor directly across a 300 V d c. source of negligible internal resistance? Explain your answer.

4.9 Two heaters are marked 200 V, 300 W and 200 V, 600 W. If the heaters are combined in series and the combination connected to a 200 V d.c. supply, which heater will produce more heat?

4.10 An electric power station (100 MW) transmits fixed power to a distant load through long and thin cables Which of the two modes of transmission would result in lesser power wastage power transmission at (i) 20,000 V or (ii) 200 V?

4.11 Give the direction of thermoelectric current (i) at the cold junction of copper-bismuth, (ii) at the hot junction of iron-copper and (iii) at the cold junction of platinum-lead.

Additional Exercises

4.12 A dry cell of emf 1 5V and internal resistance 0 10 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0A What is the steady (a) rate of chemical energy consumption of the cell, (b) rate of energy dissipation inside the cell, (c) rate of energy dissipation inside the resistor, and (d) power output of the source?

4.13 A series battery of 6 lead accumulators each of emf 2 0V and internal resistance 0.50Ω is charged by a 100V d.c. supply. What series resistance should be used in the charging circuit in order to limit the current to 8.0A? Using the required resistor, obtain (a) the power supplied by the d.c. source, (b) the power dissipated as heat and (c) the chemical energy stored in the battery in 15 min.

4.14 (a) A battery of emf $\mathcal{E}$ and internal resistance r is connected across a pure resistive device (e.g. an electric heater or an electric bulb) of resistance R. Show that the power output of the device is maximum when there is a perfect 'matching' between the external resistance and the source resistance (i.e. when $R = r$). Determine this maximum power output.

(b) What is the power output of the source above if the battery is shorted? What is the power dissipation inside the battery in that case?

4.15 A 24V battery of internal resistance 4.0Ω is connected to a variable resistor. At what value of the current drawn from the battery is the rate of heat produced in the resistor maximum?

4.16 (a) An electric motor runs on a d.c source of emf $\mathcal{E}$ and internal resistance r. Show that the power output of the source is maximum when the current drawn by the motor is $\mathcal{E}/2r$

(b) Show that the power output of an electric motor is maximum when the back emf is one half the source emf provided the resistance of the windings of the motor is negligible.

(c) Compare and contrast carefully the situation in this exercise with that in 4.14 above.

4.17 Power from a 64V d.c. supply goes to charge a battery of 8 lead accumulators each of emf 2.0V and internal resistance (1/8)Ω. The charging current also runs an electric motor placed in series with the battery. If the resistance of the windings of the motor is 7.0 Ω and the steady supply current is 3.5 A, obtain

(a) the mechanical energy yielded by the motor in 1h, and

(b) the chemical energy stored in the battery during charging in 1h.

4.18 Two ribbons A and B are given with the following particulars: For a fixed

	A	B
Alloy	Constantan	Nichrome
Length (m)	8.456	4.235
Width (mm)	1.0	2.0
Thickness (mm)	0.03	0.06
Temperature co-efficient of resistivity	Negligible	Negligible
Resistivity in $10^{-7}\Omega$m	4.9	11

voltage supply, which of the two ribbons gives rise to a greater rate of heat production?

4.19 Two wires made of tinned copper having identical cross section ($=10^{-6}$m^2) and lengths 10 cm and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.

4.20 A fuse with a circular cross sectional radius of 0.15 mm blows at 15A. What should be the radius of cross section of a fuse made of the same material which will blow at 30A?

4.21 (a) A nichrome heating element across 230V supply consumes 1.5 kW of power and heats upto a temperature of 750°C. A tungsten bulb across the same supply operates at a much higher temperature of 1600°C, in order to be able to emit light. Does it mean the tungsten bulb necessarily consumes greater power?

(b) Which of the two has greater resistance: a 1 kW heater or a 100W tungsten bulb, both marked for 230 V?

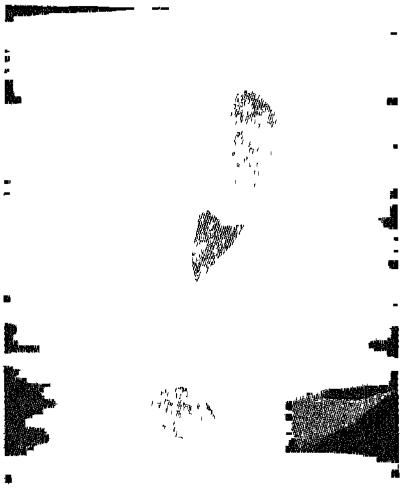
4.22 The thermoelectric emf $\mathcal{E}$ of a copper constantan thermocouple, and the temperature θ of the hot junction (with cold junction at 0°C) are found to satisfy approximately the following relation. $\mathcal{E} = a\theta + b\theta^2$ where $\mathcal{E}$ is in μV, θ in °C and $a = 41\mu\text{V}\ {}^\circ\text{C}^{-1}$ and $b = 0.041\mu\text{V}\ {}^\circ\text{C}^{-2}$. What is the temperature of the hot junction when the thermoelectric emf is measured to be 5.5mV?

4.23 A sensitive microphone cannot withstand currents greater than 0.05A. When connected across a thermocouple of emf 8.5 mV, the current in a very low resistance ammeter placed in series in the circuit reads 34 mA. What is the resistance of the microphone?

4.24 Heat is produced at a junction of two metals when a current passes through. When the direction of current is reversed, heat is absorbed at the junction (i.e. the junction gets cooler). Is the usual formula (I^2R=power dissipated as heat) applicable for this situation? If not, why?

Michael Faraday (1791-1867)

Perhaps the greatest experimental genius the world has known, he always believed that certain fundamental relations were waiting to be found, and was not dismayed by many failures from persisting till basic discoveries were established. He made fundamental contributions to physics and chemistry. He was the discoverer of electromagnetic induction, of the laws of electrolysis, and of basic relations between light and magnetism. He was the originator of the idea of the electro-magnetic field and lines of force.



CHAPTER 5

Magnetic Effects of Currents

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5.1 Introduction and Oersted's discovery

We begin to look into the connection between electricity and magnetism in this chapter. For several centuries, electric and magnetic forces were thought to be completely different, and somewhat uncommon For example, the electrical force exists between bodies which can be 'electrified', e g by rubbing. The conditions under which this happens, the size and sign of electrical forces, etc have been discussed in Chapter 1. Magnetic forces seem even more obscure, they are observed only between suitably prepared pieces of iron, as well as a few other materials (see Chapter 6) We shall see beginning with this chapter that firstly, electricity and magnetism are very closely related to each other. Secondly and even more importantly, electrical and magnetic forces are not obscure but pervade *all* matter, determining the structure and properties of atoms, molecules and collections of them

In a rough sense, electricity and magnetism give rise to each other As we shall see in this chapter, a steady electric current (i.e. a steady flow of electric charge) produces a magnetic field This was discovered early in the nineteenth century. About ten years or so later, in the early 1830's, Faraday in England and Henry in the U S.A. found that a magnetic field depending on time produces an electric field, i.e an electromotive force (Chapter 7) Forty years later (in 1873) Maxwell argued that the converse relation, namely a time dependent electric field producing a magnetic field, is also necessary. One of the consequences of this mutual relation is the electromagnetic wave, already familiar to you in the form of radio and light waves (Chapters

9 and 10) The way electrical forces determine the nature and structure of atoms is discussed in Chapters 12 and 13

The first practical step in this long journey was the invention of the cell or the electrochemical cell around 1800 (Chapters 3 and 4) With its help, a steady current of electric charge could be made to flow through a conductor, the power needed for this being supplied by suitable chemical reactions Very soon, many experiments were being done using currents As early as 1802, Gian Domenico Romagnosi, an Italian lawyer and judge, found that an *electrical current* flowing in a wire affected a *magnetic* needle placed near it, and published his observations in a local newspaper (called *Gazetta di Trentino*). However, this observation was ignored, and the astounding effect was rediscovered in 1820 by Hans Christian Oersted, a Danish physicist. He noted during a lecture demonstration that a magnetic compass needle, brought close to a straight wire carrying an electric current, aligned itself perpendicular to the wire. More precisely, the alignment is tangential to a circle which has the wire as centre, and which has its plane perpendicular to the wire (Fig. 5.1). Oersted also noticed that reversing the direction of current nearly reversed the direction in which the needle pointed i e., a needle pointing N-S turned to S-N From such observations he concluded that a magnetic field is associated with an electric current, and that it is this magnetic field which tends to align a magnetic needle much as the earth's magnetic field does.

The news of this stunning discovery spread rapidly, and the effects of a steady current flow were quantitatively understood and described essentially within the year 1820, mainly by four French physicists. Francois Arago showed that a current acts like an ordinary magnet also in its ability to attract iron filings and to induce permanent magnetism in iron needles Andre-Marie Ampere found that currents exert force on each other and determined the laws governing them, namely their size as well as direction Jean–Baptiste Biot and Felix Savart determined from experiments the dependence of the magnetic field produced by a current-carrying wire on the distance from it, the direction of this magnetic field, etc. In the next few sections, we describe these basic consequences of current flow

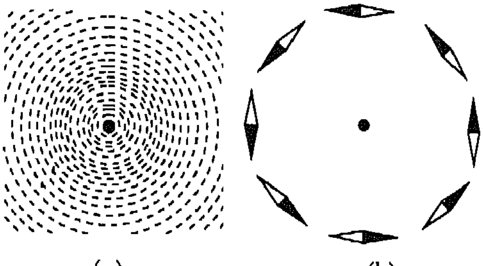
(a) (b)

Figure 5.1: Magnetic field due to a straight, long current-carrying wire The wire is perpendicular to the plane of the paper, and comes out of it (a) Iron filings around such a wire (b) An array of compass needles surrounding the wire. The dark ends of the needles mark their north poles

5.2 Magnetic field due to a current (Biot–Savart law)

5.2.1 Biot–Savart law

We have seen in Chapter 1 that the electrical force between two point electric charges q_1 and q_2 separated by a distance $\mathbf{r}_{12}$ is the simplest such force to describe Coulomb's law (Eq. (1.2)) for the electric force states its observed dependence on q_1, q_2 and $\mathbf{r}_{12}$ We also found (Section 1.5) that this force can be thought of as being due to an electric *field* $\mathbf{E}_1$ caused throughout space by the charge q_1 and acting on the charge q_2 at its location (or vice versa) The electric forces and fields due to more general and complicated arrangements of charges can be obtained by putting together the Coulomb's law forces and fields in a straightforward way. The similar obvious questions for magnetic forces are then the following: What are the known simple sources for magnetic forces? Can magnetic forces be conveniently described in terms of magnetic fields? What are the force and field laws for magnetism analogous to Coulomb's laws for electrostatics? How can magnetic forces and fields due to a general distribution of sources be described?

It turns out that there are no sources of magnetic force similar in nature to point electric *charges* giving rise to electric fields. Such *magnetic monopoles* have been looked for extensively, but have not been found so far. The known sources are electric currents (or moving electric charges, as we shall see later) or permanent magnets. The elementary source of magnetic force is a small length dl (or a length element) of a conductor carrying a current I. The force on another similar conductor can be expressed conveniently in terms of a magnetic field **dB** due to the first (Section 5.3). The dependence of such a magnetic field on the current I, on the size and orientation of the line element **dl**, and on the distance **r** from it was guessed at by Biot and Savart from measurements on finite (not infinitesimal) current-carrying wires with simple shapes, e.g. lines, rectangles and circles. These measurements were of forces as well as of the direction in which a magnetic needle comes to equilibrium near such wires. This direction turns out to be that of the total resultant magnetic field, i.e., the vector sum of the magnetic fields due to the current-carrying wire and the earth.

In this chapter, we shall first describe the magnetic field due to a small piece of current-carrying conductor (this section) and then the force due to this field (Section 5.3). This is the reverse of the 'natural' order used earlier in electrostatics, where the Coulomb force between charges was discussed first and the electric field described later. This reversal of order is essentially for the sake of convenience and simplicity[1].

The magnitude of the magnetic field dB at a distance r from a current element dl carrying a current *I* is found to be proportional to *I*, to the length dl and inversely to the square of the distance | r |. The direction of the magnetic field is perpendicular to the line element dl as well as the radius r

These features of the field dB can be written compactly[2] as

$$\begin{aligned}\mathbf{dB} &= \left(\frac{\mu_0}{4\pi}\right) I \frac{\mathbf{dl} \times \hat{\mathbf{r}}}{r^2} \\ &= \left(\frac{\mu_0}{4\pi}\right) I \frac{\mathbf{dl} \times \mathbf{r}}{r^3} \qquad (5.1)\end{aligned}$$

Here, $(\mu_0/4\pi)$ is a constant of proportionality discussed below.

The Biot-Savart relation (Eq. (5.1)) between current element and magnetic field is illustrated in Fig. 5.2. The current element **dl** is in the plane of the paper, as is the radius vector **r** connecting the line element midpoint O to the field point P at which the magnetic field is to be found. From Eq. (5.1), we expect the field to be perpendicular to both **dl** and **r**, i.e., perpendicular to the plane of the paper. From the definition of the cross product, and the corkscrew rule illustration, the field at P points into the paper. Such a field direction is shown by a circled cross (⊗), according to convention. (The conventional sign for a field coming *out* of the plane, and perpendicular to it, is a dot, namely •). One way of remembering the sign convention is to imagine an arrow pointing in the direction of the vector.

---

[1] The question of whether forces between charges, between currents, as also between masses should be considered as 'action at a distance' or as due to *fields* produced by one 'source' and acting on the other, is a basic one in physics. We shall take a practical attitude here, adopting the field point of view because of its usefulness and power.

[2] In Eq. (5.1) we use the symbol **a** × **b** for the 'vector product' **c** of two vectors **a** and **b**. Such vector products have been used earlier, e.g. p. 140 and 160 of the Class XI physics text-book. We briefly recall the results here. The vector product **c** = **a** × **b** of two vectors **a** and **b** has a magnitude |**a**||**b**| sin θ where θ is the angle between **a** and **b** (going from **a** to **b**). The direction is perpendicular to the plane containing **a** and **b**. More specifically, imagine a right-handed corkscrew, whose handle turns from **a** to **b**. The cross product **c** is in the direction of advance of the corkscrew. The cartesian components are $c_x = (a_y b_z - a_z b_y)$; $c_y = (a_z b_x - a_x b_z)$, $c_z = (a_x b_y - a_y b_x)$. A vector of unit length (called unit vector) in the direction of a vector **v** is denoted by $\hat{\mathbf{v}}$, e.g. $\hat{\mathbf{r}}$ is a unit vector along **r** (Eq. (5.1)). Unit vectors along the cartesian axes x, y and z are given by symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

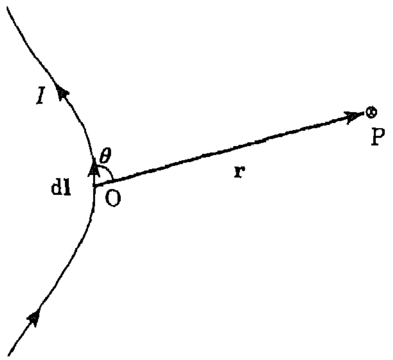

Figure 5.2: A small segment or line element dl of a wire carrying an electric current I. The quantity of interest is the magnetic field dB at a point P which is a distance r away The angle between dl and OP (or r) is θ The field at P is perpendicular to the plane of the paper, pointing into it.

If the vector points out of the paper, we see the head of the arrow namely the •. If the vector points into the paper, we see the tail of the arrow, namely the ⊗. The magnitude dB of the field is

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{I|\mathbf{dl}|\sin\theta}{r^2} \qquad (5.2)$$

where θ is the angle between the current element dl (whose length is often denoted as dl) and the radius vector **r** (Fig. 5.2) It is worthwhile to note the similarities and differences between the Biot–Savart law for the magnetic field (Eq (5.1)) and the Coulomb law for the electric field (Eq. (1.13)).

(a) Both magnetic and electric fields depend inversely on the square of the distance between the source and the field point Both are therefore of long range.

(b) The electric field is produced by a scalar source, i.e., the electric charge, which is specified completely by a number, positive or negative, in some units. The magnetic field is produced by a vector source, i.e., by a current element which has a magnitude $I|\mathbf{dl}|$ and a direction along the line element **dl**

(c) The electric field is along the radius vector joining the source and field point, while the magnetic field is perpendicular to the radius vector (and to the current element vector) We shall indicate later how the differences continue to be reflected in electric and magnetic forces (Section 5.3). Both electric and magnetic fields are proportional to the source strength, namely charge q and current element Idl, respectively. This circumstance (linearity) makes it simple to find the fields due to more complicated distributions of charge and current by *superposing*, i e., adding (as vectors) those due to elementary charges and currents. We have seen and used this for charges in Chapters 1 and 2. We shall show here in simple cases how to find the magnetic field due to finite current-carrying conductors.

We now briefly describe various kinds of sources of magnetic fields Since an electric current is due to a flow of electric charge at a certain rate, moving charges produce magnetic fields (Section 5.3). In addition, many elementary particles (both charged and neutral) have intrinsic magnetic moments These also produce magnetic fields (see Section 5 5 for a discussion of magnetic moments and fields due to them).. The magnetic field at a point P in space due to a general, finite (not infinitesimal) current-carrying conductor is obtained by imagining the latter to be made up of a large number of small current-carrying length elements, finding the field at P due to each of these elements using the Biot–Savart law, and adding the fields. (In the limit that the length of each element becomes infinitesimally small, this sum becomes an integral.) The process involved is sketched roughly in Fig. 5.3, which shows a loop of wire through which a steady electric current I flows. The magnetic field at a particular point P is the sum of the fields due to a large number of small segments. Some of these (labelled 1 to 7) are explicitly shown in the figure. We imagine these segments becoming smaller and more numerous, so that the actual wire is, in the limit, continuously and exactly reproduced by this collection The total field at P in this limit is that measured there. We shall show how to calculate B for some com-

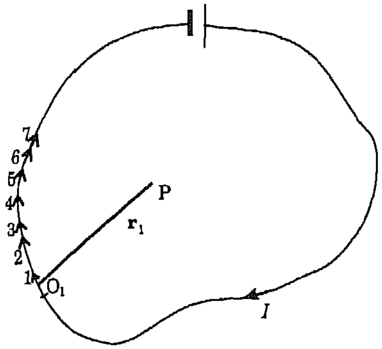

Figure 5.3: The magnetic field at a point (marked P) due to a current I flowing through a given circuit can be obtained by imagining the circuit to be made up of a large number of small current segments, finding the field due to each using the Biot–Savart law (Eq. (5.1)), and adding the fields.

monly encountered examples, such as a long straight current-carrying wire and a circular loop. Magnetic fields are also produced by permanent magnets, such as pieces of iron, which are described in the next chapter. Their magnetism is finally due to the intrinsic magnetic moment of an electron and also to the magnetism connected with circulating electrons in atoms.

The SI unit of magnetic field is called 'tesla' after the great Yugoslav inventor and scientist Nikola Tesla (1856–1943; see biography) The tesla is defined by the Eq (5.1), where the constant of proportionality $(\mu_0/4\pi)$ has the exact value 10^{-7} tesla metre/ampere, abbreviated 10^{-7} Tm/A This means the following: A small current element dl metres long, carrying a current of one ampere (one coulomb per second in terms of quantities we know, but for the basic definition of ampere, see Section 5.3) produces at a distance one metre from the element and perpendicular to it (i e., in Fig. 5.2, $\theta = 90°$, $|\mathbf{r}| = 1\text{m}$) a magnetic field of magnitude $10^{-7}|\mathbf{dl}|$ tesla The direction is perpendicular to the plane containing $\mathbf{dl}$ and $\mathbf{r}$. The tesla is defined in terms of electric current in amperes, and distance in metres. The relation between $(\mu_0/4\pi)$, a constant with physical dimension, and $(4\pi\epsilon_0)$, another constant with physical dimension used in electrostatics, will be discussed in Chapter 9 where it will be pointed out that $(\mu_0\epsilon_0)^{-1} = c^2, c$ being the velocity of light in vacuum ($\simeq 3 \times 10^8$ m/s). Thus this product has both a fixed size and a fixed dimension. Therefore if one of the two constants of proportionality is chosen according to convenience, the other is fixed automatically. In the SI units $(\mu_0/4\pi)$ is assigned the *exact* numerical value 10^{-7} (see Appendix D on units in electricity and magnetism for further discussion of this).

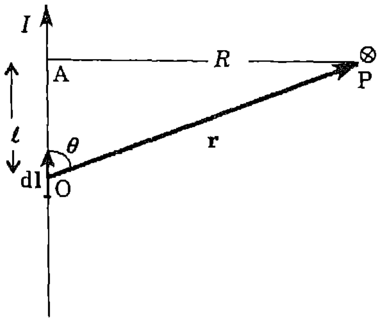

Figure 5.4: A straight very long wire carrying a steady current I The symbols are as explained in the text The field P is perpendicular to the plane of the paper and into it, as shown by the symbol ⊗

5.2.2 Magnetic field due to a straight (infinitely) long current-carrying wire

Consider a straight infinitely long wire carrying a steady current I We are interested in the magnetic field at a point P distant R from the wire (Fig 5.4) The line AP is perpendicular to the wire, and is of length R From the Biot–Savart law, (Eq (5.1)), the magnetic field $\mathbf{dB}$ due to a small element $\mathbf{dl}$ of the wire near the point O at a distance $|\mathbf{r}| = r$ from P $(\mathbf{OP} = \mathbf{r})$ is

$$\mathbf{dB} = \left(\frac{\mu_0}{4\pi}\right) I \frac{\mathbf{dl} \times \mathbf{r}}{r^3} \qquad (5.3)$$

Since the current element $\mathbf{dl}$ and the vector $\mathbf{r}$ make an angle θ with each other, the

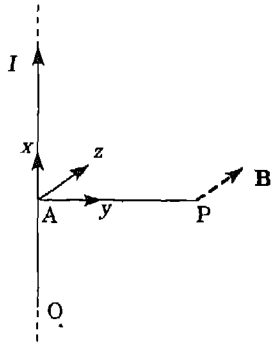

Figure 5.5: A set of coordinate axes useful for describing the field at a point P (which lies on the y axis) due to a long straight current-carrying wire. The wire passes through the origin A and lies along the x axis OA The magnetic field at P is along the dotted line (parallel to the z axis)

product $\mathbf{dl} \times \mathbf{r}$ has a magnitude $dlr \sin\theta$ It is directed perpendicular to both dl (along x axis as shown in Fig. 5.5) and $\mathbf{r}$ (which is in the xy plane). This is the direction perpendicular to the plane of the paper and going into it, as is clear from the right handed corkscrew rule (direction of advance of a right handed corkscrew turning from **dl** to **r**) Figure 5.4 shows this direction by a circled cross According to the choice of coordinate axes in Fig. 5.5, this is the z axis. One can therefore write

$$\mathbf{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2} \hat{\mathbf{k}} \tag{5 4}$$

where $\hat{\mathbf{k}}$ is a vector of unit length along the z axis Now we express $\sin\theta$ and r in terms of R which is a fixed distance for any given point P in space, and l which describes where the current element dl is, on the straight wire We notice that $\sin\theta = [R/(R^2 + l^2)^{1/2}]$ and $r = (R^2 + l^2)^{1/2}$. Thus, Eq. (5 4) becomes

$$\mathbf{dB} = \frac{\mu_0}{4\pi} I \frac{dl\, R}{(R^2 + l^2)^{3/2}} \hat{\mathbf{k}} \tag{5.5}$$

To find the field due to the entire straight current carrying wire, we have to add such contributions coming from different current

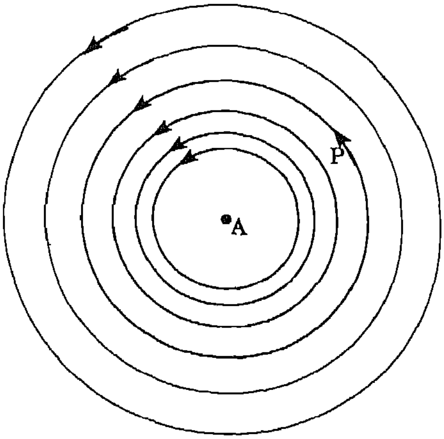

Figure 5.6: Some magnetic field lines showing the direction of the magnetic field for a straight wire carrying current The wire is shown coming out of the plane of the paper at A The field point P, and the nearest point A on the wire (Figs. 5 4 and 5 5) are shown The direction of the magnetic field at any point on the curve is along the tangent to the curve at that point The field lines are closed concentric circles

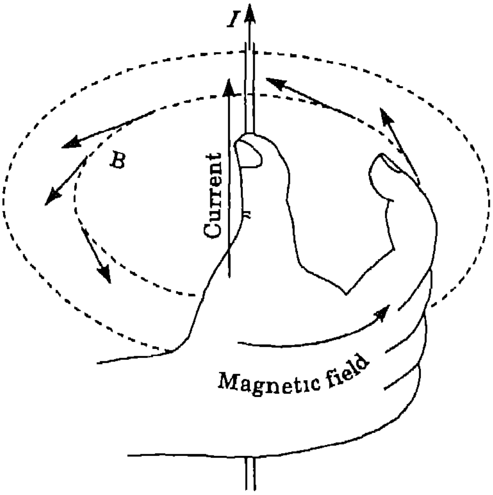

Figure 5.7: Right hand rule for the direction of B due to a straight current-carrying conductor

elements on the wire. We first notice that the *direction* of the magnetic field due to any such element is the same, namely along the z direction. Thus all we have to do is to add the magnitudes of the fields due to each element. This sum becomes an integral in the limit that each element dl becomes very small (infinitesmal) We then have

$$\mathbf{B} = \int \mathrm{d}\mathbf{B} \qquad (5.6a)$$

$$= \frac{\mu_0}{4\pi} I R \hat{\mathbf{k}} \int_{-\infty}^{+\infty} \frac{\mathrm{d}l}{(R^2 + l^2)^{3/2}} \qquad (5.6b)$$

We can actually find the dependence of the magnetic field on the distance R without knowing the integral in Eq. (5.6b)

Since R is the only scale of length in the problem, the running length l can be expressed in terms of it, by writing l as $(l/R)R = xR$ Now x is dimensionless, and varies from $-\infty$ to $+\infty$ as l varies from $-\infty$ to $+\infty$ Thus Eq. (5 6b) for $\mathbf{B}$ can be rewritten as follows·

$$\mathbf{B} = \frac{\mu_0}{4\pi} I R \hat{\mathbf{k}} \int_{-\infty}^{+\infty} \frac{R}{R^3} \frac{\mathrm{d}\left(\frac{l}{R}\right)}{\left\{1 + \left(\frac{l}{R}\right)^2\right\}^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi R} \hat{\mathbf{k}} \int_{-\infty}^{+\infty} \frac{\mathrm{d}x}{(1 + x^2)^{3/2}} \qquad (5.7)$$

The integral[3]

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}x}{(1 + x^2)^{3/2}} = J$$

is a number Even if we do not know its value, from Eq. (5.7) we see that the magnetic field at a point P due to an infinite (very long) straight wire carrying a current I is proportional to I, and is inversely proportional to the perpendicular distance R of the point from the wire It is in a direction perpendicular to the wire and the radius vector **AP**. See Figs. (5.4, 5.5) The integral J has a value of 2, so that

$$\mathbf{B} = \frac{\mu_0}{2\pi} \frac{I}{R} \hat{\mathbf{k}} \qquad (5\ 8)$$

We notice the following features of the magnetic field **B**.

(a) It is proportional to the current I

(b) It is inversely proportional to the distance R

(c) Its size is rather small, e.g. for a sizeable current such as one ampere, at a small distance of 1mm from the wire carrying current, the field has a value $(2 \times 10^{-7}/10^{-3})$T $= 2 \times 10^{-4}$T, comparable to the earth's magnetic field which is of order 10^{-4}T.

(d) The magnetic field is in a direction perpendicular to both the straight wire and the vector **AP** (Figs. 5 4 and 5 5)

To get a picture of the field direction, imagine a plane which is perpendicular to the straight wire and passes through the point P where the field is to be measured (see Fig 5 6). Consider a circle of radius R $(= |\mathbf{AP}|)$ drawn in this plane, with the wire as centre Clearly, the magnitude of the field is the same at all points on the perimeter of this wire, since they are all at the same distance R from the straight wire. Further, the field direction at any point on the circle is tangential to it **Thus the lines of constant magnetic field B form concentric circles.** This is illustrated in Fig 5.6, and is also clear from symmetry. The spacing between circles becomes larger, the smaller the magnetic field. This indicates the decrease in the size of **B** as distance increases. The direction of **B** is easily remembered with the help of the right hand rule which goes as follows. Imagine grasping the conductor in your right hand

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[3] The integral

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}x}{(1 + x^2)^{3/2}} = J$$

can be calculated as follows. Make the substitution $x = \tan\theta$. Then $\mathrm{d}x = \sec^2\theta \mathrm{d}\theta$, and $(1 + x^2) = \sec^2\theta$ Further, the limits $-\infty$ and $+\infty$ for x become $-(\pi/2)$ and $+(\pi/2)$ for θ. Thus the integral becomes

$$J = \int_{-\pi/2}^{+\pi/2} \frac{\sec^2\theta \mathrm{d}\theta}{\sec^3\theta} = \int_{-\pi/2}^{+\pi/2} \cos\theta \mathrm{d}\theta$$

$$= \sin\theta \Big|_{-\pi/2}^{+\pi/2} = 1 - (-1) = 2.$$

with the thumb along the current in the conductor. Then the other fingers curve in the direction of **B** with the finger tips pointing along **B** (Fig. 5.7). The above result (Eq. (5.8)) is for an 'infinitely long' wire. Such a thing is an idealization which does not exist. For a long straight wire of length L, at a distance R small compared to L, Eq. (5.8) for **B** is accurate, corrections being of relative order $(R/L)^2$.

Finally, there is a great difference between the field lines of **E** due to electric charges (Fig. 1.9) and the field lines of **B** due to a straight current e.g. Fig. 5.6. **The former either 'originate' in or 'end' at charges. But B lines start and end at the same point, always forming closed loops** (circular loops in the present case). This is just another expression of the fact that there are no sources of magnetic fields analogous to electric charges as sources of electric fields.

5.2.3 Magnetic field due to a circular current loop

The second case for which we will calculate the magnetic field is a circular loop carrying a current I (Fig. 5.8).[4]

We calculate the magnetic field **B** along the axis of the circular current loop of radius a, i.e. along a line perpendicular to the plane of the loop, and passing through its centre. In Fig. 5.8, the point P is one such, and is on the axis OP which passes through the centre O of the circular loop. The distance OP is R. To begin with we consider the field d**B** due to a small element dl of the circle, centred at A. The field has, from Eq. (5.2), the magnitude

$$|d\mathbf{B}| = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl}{|\mathbf{AP}|^2} = \frac{\mu_0}{4\pi}\frac{Idl}{(R^2+a^2)} \qquad (5.9)$$

since the angle between the circular element dl and the distance vector **AP** is 90° so that $\sin\theta = \sin 90° = 1$. (The line element is perpendicular to the plane of the paper and is

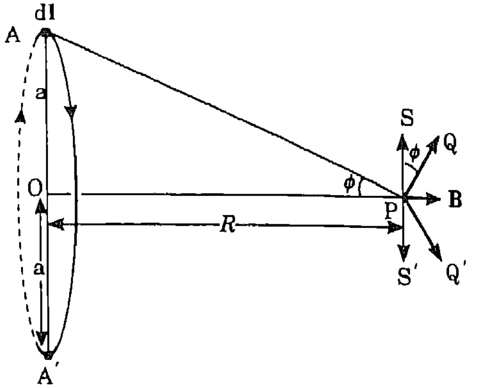

Figure 5.8: A circular current-carrying loop of radius a. We are interested in the magnetic field at the point P a distance R away from the centre O of the loop, on its axis. A small element at A produces a field along PQ and of equal length at A′ produces a field along PQ′. The net field due to this pair and similar pairs is along **OP**

coming out of it, and **AP** is in the plane of the paper, so these two are perpendicular.) The field is in a direction **PQ** perpendicular to the vector **AP** as well as to **dl**, this being shown in the figure (Fig. 5.8). This field can be resolved into two components, one along the axis OP, and the other (PS) perpendicular to it. The latter component is *exactly* cancelled by the perpendicular component (PS′) of the field due to a current element of the same size, radially opposite and centred at A′ (see Fig. 5.8). Thus we concentrate on the surviving component along OP. This has a magnitude

$$\begin{aligned} dB_{(\text{along OP})} &= \frac{\mu_0}{4\pi}\frac{Idl}{R^2+a^2}\{\sin\phi\} \\ &= \frac{\mu_0}{4\pi}\frac{Idl}{R^2+a^2}\frac{a}{(R^2+a^2)^{1/2}} \\ &= \frac{\mu_0}{4\pi}\frac{Ia}{(R^2+a^2)^{3/2}}dl. \qquad (5.10) \end{aligned}$$

We notice that the field is just proportional to the length of the circular current element and does not depend on where this element is. Thus the field along OP due to the entire circular wire of total perimeter $2\pi a$ is obtaine by replacing dl of Eq (5.8) by $2\pi a$, namely

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[4] In fact, there are wires leading the current in and out of the loop. If these leads are close together, the net field due to them is zero, since they carry equal currents in exactly opposite directions

$$\mathrm{B} = \frac{\mu_0}{4\pi}\frac{Ia}{(R^2+a^2)^{3/2}}2\pi a\hat{\mathbf{i}}$$

$$\mathrm{B} = \frac{\mu_0}{2}\frac{Ia^2}{(R^2+a^2)^{3/2}}\hat{\mathbf{i}} \qquad (5.11)$$

In Eq. (5.11), $\hat{\mathbf{i}}$ is a unit vector along OP, the x axis. Since, as discussed above, this is the only surviving component of the field, Eq. (5.11) describes completely the magnetic field due to the circular current loop of radius a at a point which is a distance R away, and is on its axis (i e. on a line perpendicular to the plane of the circle and passing through its centre). The direction of the field is as shown. On the other side of the loop at any point on the axis, the direction of **B** is the same. At distances R large compared to a we can approximate $(R^2+a^2)^{3/2}$ by R^3, and the field **B** then has the magnitude

$$\mathrm{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{(2I)(\pi a^2)}{R^3} \qquad (5.12a)$$

$$\mathrm{B} = \frac{\mu_0}{4\pi}\frac{2IA}{R^3} \qquad (5.12b)$$

where A is the area of the circular current loop. The direction of the field is as indicated in (Fig 5 8) We shall see later that this form suggests an interesting analogy with the electric field due to an electric dipole At the other extreme of distance, namely $R = 0$, i e. at the centre of the loop the magnitude of the field is

$$\mathrm{B} = \frac{\mu_0 I}{2a} \qquad (5.13)$$

which depends on loop radius a in the same way as the magnetic field of a straight long wire depends on the distance R from it

5.3 Forces on currents and the Lorentz force

5.3.1 Force laws

Current carrying conductors exert forces on each other The dependence of this force on currents, on the shape and size of the conductors as well as on their distance was first determined for special cases by Ampere in the period 1820–25. This force can be described in terms of a magnetic field, just as the force on a charge due to another can be described in terms of the electric field at the former due to the latter We describe this force law below, then illustrate it by a simple example, and finally show that it implies a fundamental law for force on a moving charge This last realization is due to the Dutch physicist H A Lorentz

Suppose the magnetic field at a particular point is **B**. The force on a small current-carrying wire $d\mathbf{l}$ situated at that point is found to be

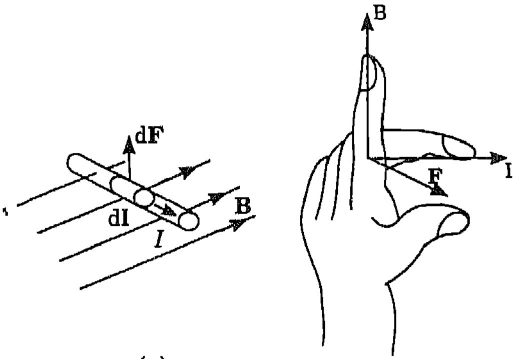
$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \qquad (5.14)$$

where I is the electric current passing through the small conducting segment This is the magnetic analogue of the electric force law $\mathbf{F} = q\mathbf{E}$ We notice that the force is proportional to I, $d\mathbf{l}$ and **B** and is perpendicular to both $d\mathbf{l}$ and **B** The force law (Eq. (5 14)) is illustrated in Fig. 5 9a A well known way of remembering the directions of the vectors involved is Fleming's left hand rule Stretch out the central finger, *f*orefinger and thumb of the left hand to be mutually perpendicular Then the first is along the current, the second along the *f*ield, and the third along the *mo*tion (or the force). The rule is illustrated in Fig. 5 9b

Clearly, if the field **B** is due to another current element, e.g Eq. (5.1), the force can be expressed entirely in terms of current elements and distances between them The general force law looks complicated and will not be mentioned here. We consider a special case, namely two *parallel* short wires of lengths dl_1 and dl_2 ($d\mathbf{l}_1$ is parallel to $d\mathbf{l}_2$) If they are a distance $\mathbf{r}_{21}$ apart (i.e., the vector $\mathbf{r}_{21} = (\mathbf{r}_2 - \mathbf{r}_1)$), and carry currents I_1 and I_2, it follows from Eq (5.14) that

$$d\mathbf{F}_2 = -\frac{\mu_0}{4\pi}I_1 I_2\, dl_1 dl_2 \frac{\mathbf{r}_{21}}{|\mathbf{r}_{21}|^3} = -d\mathbf{F}_1 \qquad (5.15)$$

The forces on the two current elements are equal and opposite, and for I_1 and I_2 of same



(a) (b)

Figure 5.9: (a) Force dF on a small current-carrying element dl, carrying an electric current I The current-carrying element is in the magnetic field B shown (b) Fleming's left hand rule

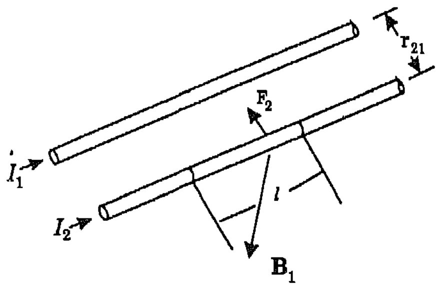


Figure 5.10: Two long parallel wires r_{21} apart, carrying currents I_1 and I_2 in the same direction. The magnetic field $\mathbf{B}_1$ at wire 2 due to wire 1, and the consequent force $\mathbf{F}_2$ on the former, are shown.

sign, are *attractive*. They are repulsive if the currents are in opposite directions For two *very long*, straight, parallel conductors carrying currents I_1 and I_2 (Fig. 5.10), and again a distance $\mathbf{r}_{21}$ apart, the force *per unit length* on wire 2 due to wire 1 is

$$\mathbf{f}_2 = -\frac{\mu_0}{2\pi}\frac{I_1 I_2}{|\mathbf{r}_{21}|}\hat{\mathbf{r}}_{21} = -\mathbf{f}_1. \tag{5.16}$$

The force on a length l is given by

$$\mathbf{F}_2 = -\frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{|\mathbf{r}_{21}|}\hat{\mathbf{r}}_{21}. \tag{5.17}$$

Actually, Eqs (5.15) and (5.16) are the fundamental relations which directly relate the mutual forces on current-carrying conductors to electrical currents and distances. They are similar to the Coulomb law of forces between electrical charges. The fundamental SI electrical unit, the ampere, is defined by Eq. (5.16). Namely, "the *ampere* is that steady current which, flowing in two infinitely long straight parallel conductors of negligible cross-sectional area placed 1 m apart in a vacuum, causes each wire to exert a force of 2×10^{-7} N on each metre of the other wire". This definition *assigns* a definite value $4\pi \times 10^{-7}$ N/A^2 to μ_0, the constant of proportionality between the currents and the force. Suppose we write

$$F_2 = k\frac{I_1 I_2}{R}l \tag{5.18}$$

where R is the distance between two parallel straight conducting wires carrying currents I_1 and I_2, and l is the length of the wire 2 on which the force is being measured For simplicity, assume $I_1 = I_2 = I$. If $R = l = 1$ m, and $F_2 = k = \mu_0/2\pi = 2 \times 10^{-7}$ N, the current $I = 1$ ampere. The ampere also *defines* the Coulomb as ampere second Further, the coefficient of proportionality ϵ_0 appearing in Eqs (1 3) and (1 7) has the exact value (μ_0/c^2) where c is the velocity of light (see Appendix D)

Example 5.1: What is the force on a wire of length 4 0 cm placed inside a solenoid near its centre, making an angle of 60° with its axis? The wire carries a current of 12 A and the magnetic field due to the solenoid has a magnitude of 0 25 T.

Answer: The force on a conductor of length l placed in a magnetic field of magnitude B, and carrying current I, is given by

$$F = IlB \sin\alpha$$

where α is the angle the conductor makes with the direction of B. Since the field due to a solenoid near its centre is along its axis, here $\alpha = 60°$. Therefore,

$$F = 12 \times 0.04 \times 0\ 25 \sin 60° \cong 0.10\text{N}.$$

The direction of force on the wire can be ascertained using Fleming's left hand rule. (*Note*: This only represents the magnetic force on the wire. The wire, of course, is subject to the force of gravity.)

Example 5.2: Two straight wires A and B of lengths 10 m and 12 m carrying currents of 4.0 A and 6.0 A, respectively, in opposite directions lie parallel to each other at a distance of 3.0 cm. Estimate the force on a 15 cm section of the wire B near its centre.

Answer: Since the ratio of length of the wires to the separation between them is large (more than 300), one can estimate approximately the force on a section of either of the two wires (near their centres) by using the exact result for force per unit length for two infinitely long wires carrying currents I_1 and I_2

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{R}.$$

where F/l is the force per unit length on either wire. Now, $\mu_0 = 4\pi \times 10^{-7}\text{NA}^{-2}$, $I_1 = 4.0$ A, $I_2 = 6.0$ A, $R = 0.03$ m. Therefore

$$\frac{F}{l} = \frac{2 \times 10^{-7} \times 4.0 \times 6.0}{0.03}\text{Nm}^{-1}$$
$$= 1.6 \times 10^{-4}\text{Nm}^{-1}.$$

Force on a 15 cm section of wire B (near its centre) is.

$$F = 1.6 \times 10^{-4} \times 0.15\text{N} = 2.4 \times 10^{-5}\text{N}$$

Since the currents are in opposite directions, the force is repulsive; its direction is normal to the wire away from A.

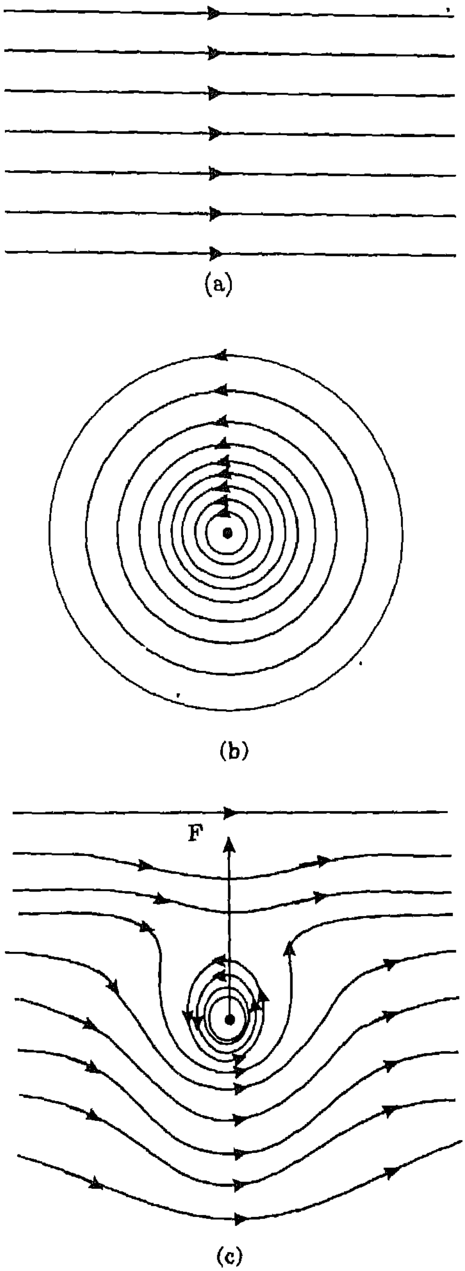
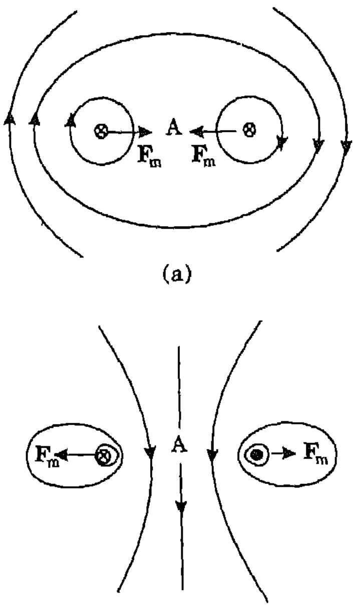


Figure 5.11: Force on a straight current-carrying conductor in a uniform magnetic field (a) Uniform field. (b) Field lines for straight conductor coming out of the paper. (c) Resultant field when the straight conductor is placed in the uniform field, i e , vector sum of (a) and (b)

★ There is a simple way of understanding how a current-carrying conductor placed in a uniform magnetic field (Fig 5.11a) experiences a force A long straight wire carrying a current produces a magnetic field symmetrically around itself as has been shown in Fig 5 11b The wire does not experience any force due to its own magnetic field However, when the current-carrying conductor is placed in the magnetic field (Fig 5 11a), this symmetry is disturbed as shown in Fig 5 11c This figure shows the resulting magnetic field lines Adding the uniform field (Fig 5 11a) and the field due to a straight wire (Fig 5 11b) we see, as shown in Fig 5 11c that these two are in the same direction in the lower half of the figure, and in the opposite direction in the upper half of the figure At the point P shown, the two exactly cancel To find out the direction in which the straight wire moves, we use a result, not proved by us, that in a magnetic field **B**, the magnetic energy density (i e , magnetic energy per unit volume) is $(\mu_0 B^2/2)$. A similar result for the electric field **E** has been derived in chapter 2 (Eq (2.85)) This result is true even if **E** or **B** are not uniform but vary in space Now consider Fig 5 11c For any given radial distance from the wire, the size of magnetic field is smaller on the upper half of the figure than on the lower half (the two fields tend to add in the lower half, they tend to substract in the upper half) Thus the magnetic energy density is larger in the lower half than in the upper half, for any given radial distance Thus the force, directed from higher to lower energy density direction is as shown A similar argument can be used to show that two parallel current carrying conductors attract if the currents flow in the same direction, and repel if currents flow in the opposite direction The total magnetic field (sum of that due to each conductor) is shown in Fig 5 12a for the parallel current case, and in Fig 5 12b for the antiparallel current case In the former case, the magnetic field vanishes at the point A which lies midway between the wires, the magnetic field energy density is the least there, and therefore mutual force on either wire tends to push them towards A When the currents are antiparallel, the magnetic field energy density is largest at A, so that both the conductors tend to move away from this region. They repel each other Indeed the actual force can be calculated as the (negative of) the gradient of the magnetic field energy ★



(a)

(b)

Figure 5.12: Force between two long straight wires each carrying a current I. Resultant magnetic field lines when the currents are in the same direction (a), and when they are in the opposite directions (b)

5.3.2 Lorentz Force

An electrical current is just the drift of electrical charges with a certain (drift) velocity. So if there are forces on currents placed in a magnetic field, this clearly means that there are forces on *electrical* charges *moving* in a *magnetic* field (There is obviously no force on an electrical charge *at rest* in a magnetic field.) This unusual basic fact, which underlies the observed forces on currents, was first clearly recognized by the Dutch physicist Hendrik Antoon Lorentz nearly a century ago. We deduce the force law now from the known force on a current element (Eq. (5.14)) and the relation derived earlier in Chapter 3 (Eq (3.11)) connecting an electrical current to the drift velocity of charges carrying it.

If charges (each of charge q) drift in the conductor with a speed v, the current I is

$$I = nqAv \tag{5.19}$$

from Eq. (3.11). Here A is the cross-sectional area of the conductor and there are n charges per unit volume. If the current is carried by electrons, as happens most often, the charge q is just the electric charge e on an electron, and the actual direction of drift of electrons is opposite to that of the current, i.e. v is negative if I is positive. Suppose the current flows in a straight conductor $\mathbf{l}$ placed in a uniform magnetic field $\mathbf{B}$. From Eq. (5.19), we have

$$I|\mathbf{l}| = Il = [nAl]\,qv \qquad (5.20)$$

The quantity in square brackets above is just the number N of charges in the length l, since its volume is $|Al|$ and the number density of charges is n. We thus have

$$I\mathbf{l} = Nq\mathbf{v} \qquad (5.21)$$

where Nq is the total charge in the line element and this charge has drift velocity $\mathbf{v}$.

The force equation Eq. (5.14) can be then rewritten as

$$\mathbf{F}_N = I(\mathbf{l}\times\mathbf{B}) = Nq\mathbf{v}\times\mathbf{B} \qquad (5.22)$$

where Nq is the total electrical charge in the current-carrying conductor, drifting with velocity $\mathbf{v}$. This fundamental force on an electric charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is called the *magnetic Lorentz force* after its discoverer. The total force (often called Lorentz force) on a charge q moving with velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is

$$\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}). \qquad (5.23)$$

The magnetic Lorentz force is perpendicular to both the velocity $\mathbf{v}$ (or the *direction* of motion) and the magnetic field $\mathbf{B}$. Because of the former fact, it does no work, at least for a steady magnetic field. Some effects of the Lorentz force are discussed in Section 5.6.

5.3.3 Relative sizes of magnetic and electrical forces

It is interesting to compare electric and magnetic forces under similar conditions. The magnitude of the force between charges q_1 and q_2, a distance $\mathbf{r}_{12}$ apart is given by

$$F_e = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}_{12}|^2}. \qquad (5.24)$$

The magnetic force between two parallel current elements $d\mathbf{l}_1$ and $d\mathbf{l}_2$ carrying currents I_1 and I_2, the same distance $\mathbf{r}_{12}$ apart has a magnitude

$$F_m = \frac{\mu_0 I_1 I_2}{4\pi}\frac{|d\mathbf{l}_1|\,|d\mathbf{l}_2|}{|\mathbf{r}_{12}|^2} \qquad (5.25)$$

In order to compare this with the electrical or Coulomb force (Eq. (5.24)), we should clearly require the currents I_1 and I_2 to be due to the *same* amounts of electric charges as cause the Coulomb force. We have just seen (Eq. (5.21)) that if the charges q_1 and q_2 drift with speeds v_1 and v_2, $I_1|d\mathbf{l}_1| = q_1 v_1$ and $I_2|d\mathbf{l}_2| = q_2 v_2$. Thus the magnetic force can be rewritten as

$$F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{|\mathbf{r}_{12}|^2}v_1 v_2 \qquad (5.26)$$

so that the *ratio* of magnetic and electrical forces is

$$\frac{F_m}{F_e} = (\mu_0\epsilon_0)\,(v_1 v_2) \qquad (5.27)$$

Since this ratio should be dimensionless, we expect $(\mu_0\epsilon_0)$ to have dimensions of v^{-2} where v is a velocity. We shall argue later, in the chapter on electromagnetic waves (Chapter 9) that $(\mu_0\epsilon_0)^{1/2}$ is the speed c of electromagnetic waves in vacuum. Light is an electromagnetic wave, and so we are saying that c is the speed of light, which is approximately 3×10^8 m/s. (In SI units c is *defined* to be 2.9979245×10^8 m/s, the unit of length being defined in terms of this.) We thus have

$$\frac{F_m}{F_e} = \frac{v_1 v_2}{c^2} \qquad (5.28)$$

Now typically, electron drift velocities v_d in metals are of order 10^{-4} m/s (e.g. in a copper wire of 2×10^{-3} m radius carrying a large current of 1 A, $v_d \simeq 10^{-5}$ m/s) so that under such fairly typical conditions magnetic forces are *much* weaker, by many orders of magnitude.

One might then wonder as to why magnetic forces are ever observed at all, and are not totally overwhelmed by electrical forces. There are two reasons. Matter is electrically neutral to a very high degree of accuracy. Especially in metallic conductors, no region can have a net positive or negative charge since free electrons (which carry the electrical current) nearly instantaneously move into or out of such a region to fully neutralize or screen the net charge in that region. Thus Coulomb forces are generally hardly in evidence, especially between conductors. (As discussed in Chapter 1, conductors can have only surface charges. They can have bulk currents.) Electrical currents do not disturb the neutrality of matter and are easily produced Secondly, in an electrical current, a large number of electrons are drifting together, so that the very small individual forces add up

Finally, Eq. (5 28) suggests that for charges moving with speeds comparable to that of light, electrical and magnetic forces are comparable.

5.4 Ampere's circuital law; The solenoid

The basic Biot–Savart relation (Eq (5.1)) between electric current and magnetic field was rephrased by Ampere This new relation, called Ampere's circuital law, will be derived below from a special case, namely the field B due to a long straight conductor carrying current

5.4.1 Ampere's circuital law

Consider a very long straight conductor carrying an electric current I As shown in the Section 5 1, the field lines form concentric circles with the wire as centre (Fig 5 6). The magnitude B of the magnetic field **B** anywhere on (the perimeter of) such a circle of radius R is $(\mu_0 I/2\pi R)$, as derived in Eq. (5.8). Now, consider the quantity $B(2\pi R)$ This is B times the perimeter of the circle R. It has, for the present case, the numerical value $(\mu_0 I/2\pi R)\times(2\pi R) = \mu_0 I$ This quantity is μ_0 times the current flowing through the plane of the circular disc of radius R. The Ampere circuital theorem is a general statement of this relation **For the case where B is (a) always directed along the tangent to the perimeter of a closed curve, and (b) constant in magnitude all along the perimeter with a value B, the Ampere circuital law states**

$$B \times (\textbf{perimeter of closed circuit}) = \mu_0 \times \left(\begin{array}{l}\textbf{total current passing}\\ \textbf{through any surface}\\ \textbf{of which the closed}\\ \textbf{circuit is the}\\ \textbf{perimeter}\end{array}\right) \tag{5.29}$$

We shall see that this theorem is very useful in calculating B There is a more general form of Ampere's circuital theorem, valid when neither the condition (a) nor (b) above is satisfied for B We shall not discuss this general form here, but in a starred paragraph, show how Eq (5 29) can be extended in a particular case. We apply the law to one very important practical example, namely the magnetic field due to a solenoid (toroidal or straight)

One question which naturally occurs is, Is the Eq (5 29), which connects magnetic field and current, independent of the Biot–Savart law which also connects these two quantities? Or do both Ampere's circuital theorem and the Biot–Savart law express the *same* physical consequences of electrical currents? The answer is no to the former question and yes to the latter We can guess these answers from the fact that the Biot–Savart law based Eq. (5.8) for the magnetic field was used above to argue for Ampere's circuital theorem (Eq. (5 29)) So, we have two different ways of describing the magnetic effects of electrical currents, just as in Chapters 1 and 2, we found that there are two ways of describing the electrical effects of electric charges, namely the Coulomb's law and the Gauss's theorem

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★ A simple generalization of Eq (5 29) for an arbitrary planar circuit around a straight wire, not just a circular one as discussed above is as follows

Consider *any* planar closed circuit enclosing a straight current-carrying wire Divide the cir-

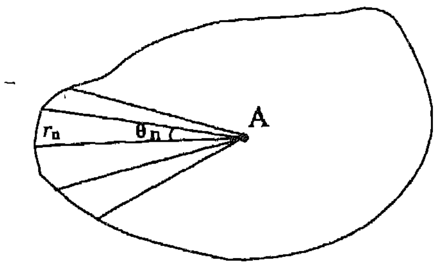

Figure 5.13: An arbitrary plane circuit, whose plane is perpendicular to a straight current carrying wire A, and which surrounds this wire. The circuit is divided into a large number of circular segments centred at A. A few of these are shown. One has radius r_n and subtends on angle θ_n

cuit into a large number of circular segments with centre at A, and with different radii (Fig. 5.13). Suppose the angle subtended at A by a particular segment is θ_n. The contribution of this segment (of a circle of radius r_n) to the circuit integral is

$$B_n(r_n\theta_n) = \frac{\mu_0 I}{2\pi r_n}(r_n\theta_n) = \mu_0 I\left(\frac{\theta_n}{2\pi}\right). \tag{5.30}$$

Thus the total circuit integral is $\mu_0 I$ since the sum of angles θ_n around the closed planar circuit is 2π. Finally, we note that the loop need not be planar, the same kind of argument will go through even then. ★

5.4.2 Toroidal and straight solenoids

Toroidal solenoid (or toroid): We now apply Ampere's circuital theorem to calculate the magnetic fields for two commonly used systems, namely toroidal and straight solenoids.

A toroidal solenoid is a hollow circular ring with a large number of turns of a wire carrying electric current wound around it. (see Fig. 5.14) We are interested in the magnetic field B inside the toroidal solenoid. Consider any point P on the central circle of the toroid. It is clear from symmetry that the direction of the magnetic field is as shown, tangential to the circle. (The field lines form concentric circles, as shown!) Suppose we form the circuit

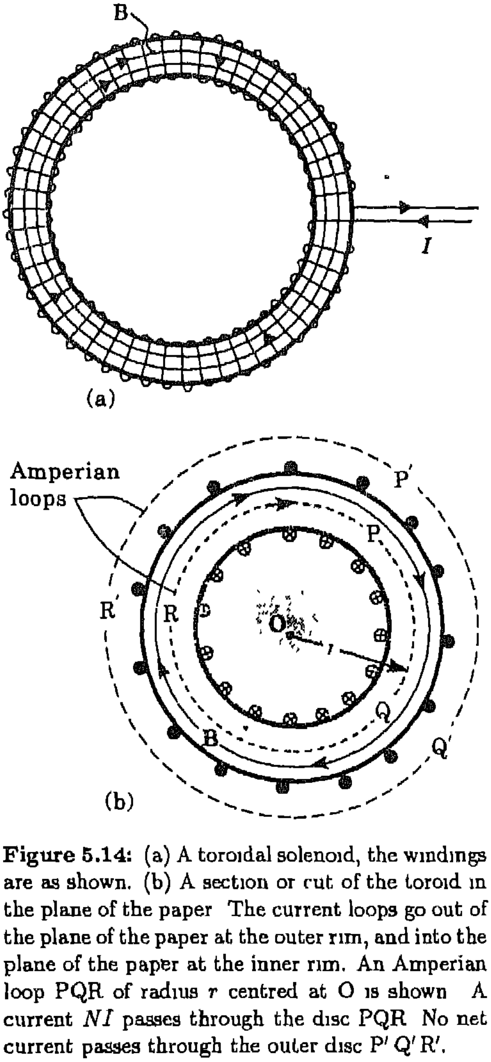

Figure 5.14: (a) A toroidal solenoid, the windings are as shown. (b) A section or cut of the toroid in the plane of the paper. The current loops go out of the plane of the paper at the outer rim, and into the plane of the paper at the inner rim. An Amperian loop PQR of radius r centred at O is shown. A current NI passes through the disc PQR. No net current passes through the outer disc P′ Q′ R′.

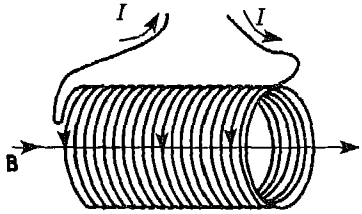

Figure 5.15: A solenoid. It consists of a current-carrying wire wound in parallel circles on a hollow straight cylinder (not shown)

PQR (called an Amperian loop). Since B has (again from symmetry) the same magnitude at all points on this circle, and is tangential to it, the circuit integral of B is $B(2\pi r)$ This must equal μ_0 times the current going through the disc enclosed by the circle PQR. (Shown by a dotted area in Fig. 5 14b). Suppose the total number of turns of wire winding round the toroid is N. Then the current enclosed is NI Since the mid-circumference of the toroid is $2\pi r$, and the number of turns N is expected to be proportional to $2\pi r$, we write $N = 2\pi r n$ where n is the number of turns per unit length So, Ampere's theorem tells us that

$$B(2\pi r) = \mu_0 I N = \mu_0 I (2\pi r n)$$

or

$$B = \mu_0 I n \tag{5.31}$$

The direction of **B** is as indicated. We see from Eq. (5 31) that B does *not* depend on the radius of the toroid, but only on the current I and the number n of turns per unit length By increasing these, a high magnetic field can be obtained.

Interestingly, there is no magnetic field outside the toroid. To see this, imagine a circle P′ Q′ R′ outside the toroid but with the same centre O. The net current passing through this circular disc is *zero*, since a current NI passes in, and the same current passes out Thus, using Ampere's circuit law, the field B is zero.

Straight solenoid (or solenoid) A straight solenoid (Fig 5 15) is also very common. In order to find the magnetic field B inside it, we can imagine it to be a piece of a toroidal solenoid of infinite radius. Since the magnetic field B of the latter does not depend on the radius, we have, from Eq. (5 31) that

$$B = \mu_0 I n \tag{5.32}$$

where again I is the current flowing through the conducting wire with n turns per unit length. The direction of the field is along the axis of the solenoid, as shown in Figs. 5.15 and 5 16 A straight solenoid is finite, it has ends Thus the expression Eq (5.32) for the field can be expected to be correct far from the two ends, near the middle of the solenoid, and at points close to the axis As one moves away from the middle, the magnetic field decreases. Fig. 5.16 shows the magnetic field lines for a solenoid We notice the relative uniformity of the field inside, and its smallness outside. (The field line spacing is inversely proportional to the size of the field **B**, namely if field lines are closely spaced the field is large and if they are far apart, the field is small) The region over which the field does not vary much is sizeable (centred around the middle of the solenoid) for a long and thin solenoid We do not estimate the field at a general point, but argue that exactly *at* either end it is

$$B_{\text{end}} = \frac{\mu_0 n I}{2} \tag{5.33}$$

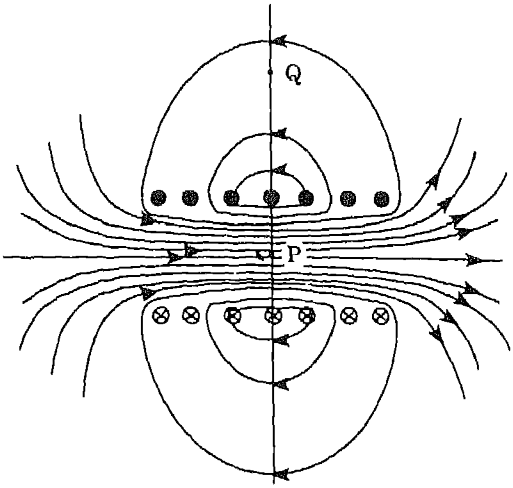

Figure 5.16· Magnetic field lines for a solenoid. The field is strong and uniform at internal points such as P, but is relatively weak outside, e g at Q

Imagine dividing a long solenoid into two equal halves. Consider the field at the centre of the plane common to the two halves, at the axial point The total field is the sum of those due to the two halves separately (i.e , calculated as if the other half were not there). Now by symmetry, the two halves produce identical fields, same in magnitude and along the solenoid axis Thus the field due to each part is half of $\mu_0 nI$, hence the result Eq (5 33).

Solenoids and toroids are used in transformers, windings of electromagnets, etc , as we shall see later. Clearly, the current carrying wire in a solenoid is to be surrounded by an insulating sheath, so that each turn carries the same, externally given current and the different windings do not electrically touch each other. In many solenoids, to increase the magnetic field B, a magnetic material such as iron is introduced inside the cylindrical or toroidal region enclosed by the conductor windings. We shall see in Chapter 6 how this increases B by a large factor.

Example 5.3: A solenoid 50 cm long has 4 layers of windings of 350 turns each. The radius of the lowest layer is 1 4 cm If the current carried is 6 0 A, estimate the magnitude of B (a) near the centre of the solenoid on its axis, and off its axis, (b) near its ends on its axis, and (c) for outside the solenoid near the middle, or point Q (Fig. 5 16)

Answer: The ratio of length to radius of the solenoid is quite large (about 35). Therefore to estimate B approximately, we can use the exact results for a closely wound infinitely long solenoid

(a) At the centre or near it, from Eq. (5 32)

$$B = \mu_0 nI$$

where n is the number of turns per unit length. Note, the radius of the wire does not enter this equation. Therefore, to get n, simply multiply number of turns per layer by the number of layers and divide the product by the length of the solenoid

$$n = \frac{350 \times 4}{0.50} = 2800 m^{-1}.$$

Now $I = 6.0$ A, and $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}, which gives B = 2.1×10^{-2} T.

Note· This estimate of B is for *both* on and off the axis, since for an infinitely long solenoid, the internal field near the centre is uniform over the entire cross section

(b) At the ends of the solenoid, from Eq. (5.33)

$$B = \frac{\mu_0 nI}{2} = 1\ 05 \times 10^{-2} \mathrm{T}$$

(c) The outside field near the middle of a long solenoid is negligible compared to the internal field

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5.5 Current loop, magnetic dipole moment, torque and galvanometer

We now show that a closed loop of wire carrying current has a magnetic dipole moment The magnetic field due to such a loop, and the torque acting on it when placed in a uniform magnetic field, are respectively identical in form with the electric field due to an electric dipole and the torque on it in a uniform electric field (see Chapter 1 for the latter, for example Eq. (1 34)) The torque effect is used, among other things, in galvanometers, which are devices for measuring electric currents (and voltages), and in motors

5.5.1 Current loop and magnetic dipole

We notice first that the magnetic field lines due to a solenoid (Fig 5 16) are very similar to those of a bar magnet (see for example Fig. 6.6) Now as you know, the bar magnet can be thought of as a 'dipole', i e., the field lines resemble very closely those of an electric dipole (Fig 1.17) We try to make this connection more precise by comparing the magnetic field due to a circular current loop at a distance R from it, and on its axis, with the electric field due to an electric dipole, at the same distance R along the dipole axis The former, from Eq. (5.12b), is

$$B = \frac{\mu_0}{4\pi}\frac{2IA}{R^3}. \qquad (5.34)$$

The electric dipole field, from Eq (1.30), is

$$E = \frac{1}{4\pi\epsilon_0}\frac{2p_e}{d^3} \qquad (5.35)$$

Comparing the two, we see that we can associate a **magnetic dipole moment m with the circular current loop carrying a current I and of area A. The magnitude of m is**

$$|\mathbf{m}| = IA. \qquad (5.36)$$

Further, the direction of the magnetic dipole moment is perpendicular to the plane of the the loop. The magnetic dipole moment equation (5.36a) is often written in the vector form

$$\mathbf{m} = I\mathbf{A} \qquad (5.37)$$

where **A** is a vector perpendicular to the plane of the loop, and has a magnitude A, i.e., equal to the area of the loop. If the current is directed along the four curved fingers of the *right* hand, the magnetic moment is directed along the thumb sticking out We have proved the result (Eq. (5.36)) only for a circle but it is actually true for *any planar current loop.* (We present an argument below.)

5.5.2 Torque and magnetic dipole

We how show that a closed current loop placed in a uniform magnetic field experiences a torque Comparing this with the torque experienced by an electric dipole in a uniform *electric* field, we again show that a current loop is a magnetic dipole, of size given by Eq. (5 36). This torque effect is very important and is the basis of action of electric motors, as well as of galvanometers which are used to measure electric currents and voltages.

Consider a rectangular loop of sides a and b, carrying a current I, placed in a uniform magnetic field B. The angle between the direction of B and the vector perpendicular to the plane of the loop is θ (see Figs 5.17a and b). Clearly, the various parts of the rectangular loop experience forces in the magnetic field, the size and direction being given by Eq (5.14). The elements PQ and RS experience forces in opposite directions. They are collinear and equal in magnitude, so the net force and torque due to these are zero. The force on SP is in the direction shown (Fig. 5.17b), it is equal in magnitude and opposite in sign to the force on QR. So the net force on the rectangular loop is zero. Since the forces are *not* along the same *line*, there is a net torque, whose magnitude and direction are easily calculated. The force on PS has, from Eq. (5.14), the magnitude

$$F = IaB$$

The 'lever arm' of the torque is $(b \sin\theta)$ (see Fig. 5.17b), so that the torque has a magnitude

$$\begin{aligned} \tau &= IaBb\sin\theta \\ &= IAB\sin\theta \\ &= |\mathbf{m}|B\sin\theta \qquad (5.38) \end{aligned}$$

where we have used the definition (Eq. (5.37)) for the magnitude of the magnetic dipole moment. The direction of the torque is such as to rotate the loop clockwise with the dotted

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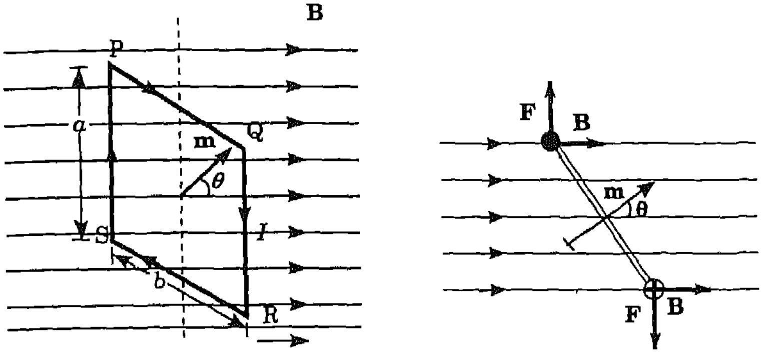
★ Ampere noticed, as early as 1820, that the magnetic field due to an electric current loop and that due to a bar magnet are similar. He then made the bold guess that *all* magnetism is due to current loops or circulating electrical currents, and that there need not be magnetic analogues of electrical charges, i e., magnetic north or south poles So in a magnet made of iron, it is assumed that the magnetic dipole arises as the sum of its magnetic dipole moments due to individual iron atoms In each iron atom, there is a circulating electrical current which causes this magnetic dipole moment. If such magnetic dipoles are all aligned parallel to each other, the piece of iron is fully magnetized This speculation is basically correct! However, it turns out that in addition to atomic magnetic moments caused by orbital electron circulation or orbital angular momentum, there is also magnetic moment connected with *intrinsic* angular momentum of the electron, called spin This magnetic moment is present even if the particle (say the electron or proton) has no orbital motion In the case of iron metal, for example, the observed magnetic moment is almost entirely due to the intrinsic angular momentum of the unfilled *d* shell electrons ★



(a) (b)

Figure 5.17: (a) A rectangular current-carrying loop PQRS of sides a and b, with the loop plane at an angle θ to the uniform magnetic field **B** in which it is placed
(b) Top down view of the rectangular loop shown in Fig 5 17a The current element SP (current up and out of the plane of paper) and QR (opposite) are shown by a(•) and a ⊗ respectively The forces on them are shown, as is the effective magnetic moment **m** of the loop

line of Fig. 5.17a as axis The torque τ is fully defined by the equation

$$\tau = \mathbf{m} \times \mathbf{B} \tag{5 39}$$

where the magnetic moment is a vector perpendicular to the plane of the rectangular loop, and of magnitude IA. Now Eq. (5.39) is identical in form with the torque on an electric dipole **p** placed in a uniform electric field **E** (Eq. (1 33)), namely

$$\tau = \mathbf{p} \times \mathbf{E}. \tag{5.40}$$

If the rectangular loop has N turns each carrying current I, the torque increases N fold, and has a magnitude

$$\tau = NIBA \sin\theta. \tag{5.41}$$

It can be shown that the expression (Eq. (5.39)) for the torque is correct for any planar loop of area A (not necessarily a rectangle), in a uniform magnetic field with which its normal makes an angle θ.

★ We can easily generalize Eqs (5 37) and (5 38) to plane current loops of arbitrary shape. The argument is simple Consider a general loop (Fig 5 18) This can be divided into a large number of rectangular loops through each of which the same current I circulates, *without* introducing any new currents (Fig. 5 18) There is no net force or torque from 'internal' line currents such as CD since on each such line the current flows in one direction as part of one rectangle (eg , FECD) and in the opposite direction as part of the adjacent rectangle (eg , DCAB) So this construction does not change either the torque on the loop, or the current flow However, we can now calculate the torque as being the sum of those on the rectangular strips These all point in the same direction, their magnitudes are proportional to the

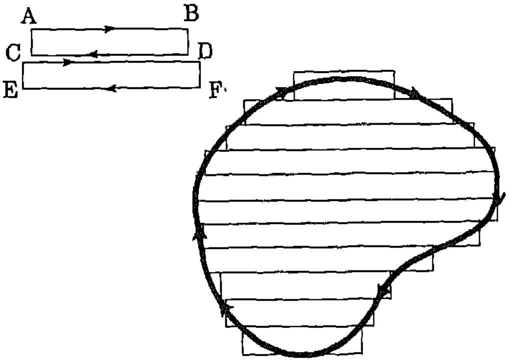
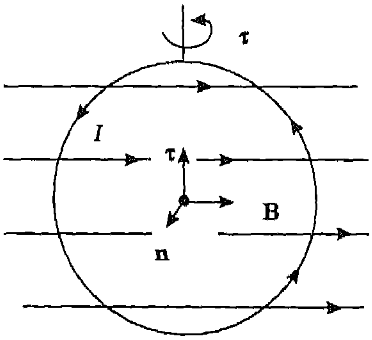


Figure 5.18: A general planar loop carrying a current I imagined to be split up into a large number of parallel rectangular loops, some of which are shown Two of them are shown expanded on the left, so that the oppositely directed current elements are clearly seen

areas of the rectangles. The magnitude of total torque is then proportional to the total area of the loop, and has the form Eq (5.39) with $\mathbf{m} = I\,\mathbf{A}$, even though the loop is not rectangular. ★

Example 5.4: A circular coil of 25 turns and radius 6.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 1.2 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on the coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning? (see figure)



Answer: The net force on the coil in a uniform magnetic field is zero. To see this, consider any element dl of the wire. The force on this element is $I\mathrm{d}\mathbf{l} \times \mathbf{B}$. For each element dl, there is another length element on the closed loop given by − dl. Since **B** is uniform, the forces cancel for each pair of such elements, and the net force on the coil is zero.

The torque $\boldsymbol{\tau}$ on a plane loop of any shape carrying a current I in a magnetic field **B** is given by Eqs (5.36) and (5.38), i.e.,

$$\boldsymbol{\tau} = IA\hat{\mathbf{n}} \times \mathbf{B}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the plane of the loop (direction of motion of a right-handed screw rotating in the sense of the current). For a circular coil of radius R and N turns, $A = N\pi R^2$. From the figure, $\hat{\mathbf{n}}$ is normal to the plane of the paper, coming out of it. The angle between $\hat{\mathbf{n}}$ and **B** is thus 90°. The magnitude of $\boldsymbol{\tau}$ is then given by

$$\begin{aligned}|\tau| &= IAB\sin\alpha \\ &= 10 \times 1.2 \sin 90^\circ \\ &\quad \times 25 \times \pi(0.06)^2\,\mathrm{N\,m} \\ &= 3.4\,\mathrm{N\,m}.\end{aligned}$$

The direction of $\boldsymbol{\tau}$ is vertically upwards, with its turning effect as shown by the thick curved line at the top of the figure. To prevent the coil from turning, a balancing torque $\boldsymbol{\tau}' = -\boldsymbol{\tau}$ must be applied.

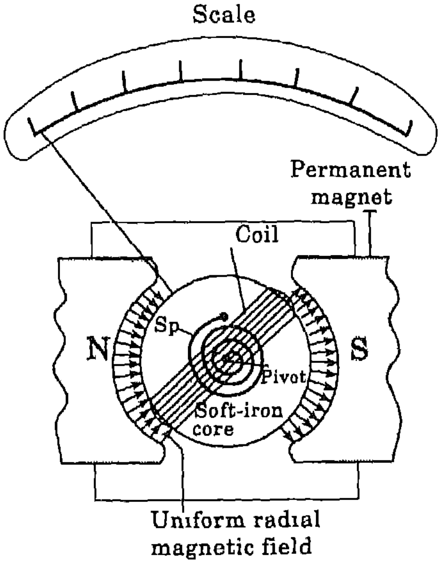


Figure 5.19: The rudiments of a galvanometer. Depending on the external circuit, this device can be wired up as either a voltmeter or an ammeter.

5.5.3 Galvanometer

The torque on a current loop in a uniform magnetic field is used to measure electrical currents. This current measuring device is called a (moving coil) galvanometer. The phrase 'moving coil' is used because the current loop or coil rotates (moves) in a magnetic field, as a result of the torque on it.

The galvanometer consists of a coil of wire, often rectangular, carrying the current to be measured. There are generally many turns in the coil to increase its sensitivity The coil is placed in a magnetic field such that the lines of B remain nearly parallel to the plane of the wire as it turns (Fig. 5.19). This is achieved by having a soft iron cylinder placed at the centre of the coil. Magnetic field lines tend to pass through the iron cylinder, producing the field configuration shown. The moving coil is hung from a spring, which winds up as the coil rotates; this winding up produces a restoring torque proportional to the winding up (or twisting) of the spring, i.e. to the angular deflection ϕ of the coil. The coil comes to equilibrium when this restoring torque k balances the torque due to the magnetic field (Eq. (5 41)). Since by design, field lines are radial, we have $\sin\theta \simeq 1$, so that for equilibrium

$$k\phi = INBA \qquad (5.42a)$$

$$\phi = \left(\frac{NBA}{k}\right) I \qquad (5.42b)$$

Thus the deflection ϕ of the galvanometer is proportional to the electric current I passing through it. By proper design, currents as low as 10^{-10} to 10^{-11} A can be measured! We have seen in Chapter 3 (Section 3) how a galvanometer can be made into an ammeter or a voltmeter.

Example 5.5: A moving coil meter has the following particulars. Number of turns, $N = 24$; area of the coil, $A = 2.0 \times 10^{-3}\text{m}^2$; magnetic field strength, $B = 0.20$ T; resistance of the coil, $R = 14\ \Omega$.

(a) Indicate a simple way to increase the current sensitivity of the meter by 25% (It is not easy to change A or B).

(b) If in so doing, the resistance of the coil increases to 21 Ω, is the voltage sensitivity of the modified meter greater or less than the original meter?

Answer: (a) Current sensitivity of a meter is the deflection of the meter per unit current, i.e., ϕ/I. We have

$$\frac{\phi}{I} = \frac{NAB}{k}$$

where k is the torsion constant of the springs connected to the coil which provide the balancing torque ($= k\phi$) to the torque ($NABI$) produced by the field on the current-carrying coil. Since it is easier to change N than k, A or B, the current sensitivity can be increased simply by increasing N To increase it by 25%, N should be increased from 24 to 30.

(b) Voltage sensitivity of a meter is analogously defined as the deflection of the meter per unit voltage, i.e , ϕ/V. We have

$$\frac{\phi}{V} = \frac{\phi}{RI} = \frac{NAB}{kR}$$

Since k, A, B are the same for the two meters, we need compare only the ratio N/R.

$N/R = 24/14 = 1.7$ (original meter)

$N/R = 30/21 = 1\ 4$ (modified meter)

Thus the new meter's voltage sensitivity is less than that of the original meter By increasing the number of turns, it has gained in current sensitivity but lost in voltage sensitivity.

Note: In the example, while N increases by 25%, R increases by 50% If we expect R to be proportional to N, then it is clear that while current sensitivity of the meter can be increased by increasing N, voltage sensitivity will remain practically constant, as long as other parameters (k, A and B) remain unchanged).

5.6 Moving charges in magnetic and electric fields

Forces acting on charges moving in magnetic and electric fields are used to deflect and focus beams of electrons in many practical devices. Perhaps the most common (but rather complicated) example of this is the TV tube. The image on the face of the tube is due to a beam of electrons hitting a coating of a material called phosphor which emits light on being struck. The beam of electrons is deflected in a very complex way by electric fields applied to parallel plates between which the beam passes In this section, we describe some very simple examples of the effects electric and magnetic fields have on moving charges The first two examples are just those of the motion of an electric charge in a uniform magnetic field, and the last describes a moving charge in magnetic *and* electric fields. You will also encounter the example of an electron moving in magnetic and electrical fields in Chapter 12 on electrons and photons.

5.6.1 Moving charge in a magnetic field

Consider a charge q moving in a uniform magnetic field **B**. We can assume, perfectly generally, that a coordinate axis, say the z axis, is along **B**. We are interested in the path or trajectory of the charged particle Suppose its initial velocity is v (Fig. 5 20a) Since the magnetic Lorentz force is $q\mathbf{v} \times \mathbf{B}$ and **B** is along z, there is *no* force in the z direction. The z component of momentum does not change, so we have

$$v_z(t) = v_z. \qquad (5.43)$$

The magnetic field affects the x and y components of the velocity. The force on the charged particle is *always perpendicular* to its velocity (the force is in the xy plane). This has two consequences. Firstly, the force does no work, so the charged particle energy (here kinetic energy) does not change. Since the kinetic energy is

$$\text{K E} = \frac{m}{2}v^2 = \frac{m}{2}\left(v_z{}^2 + v_x{}^2 + v_y{}^2\right) \quad (5.44a)$$

and v_z is unchanged (Eq. (5 44b)), we have

$$v_x{}^2 + v_y{}^2 = \mathbf{v}_\perp{}^2 = v_\perp^2 = \text{constant}. \quad (5.44b)$$

Now the magnitude of the magnetic Lorentz force is

$$F = q\, v_\perp B \sin\theta = q\, v_\perp B \qquad (5.45)$$

since $\mathbf{v}_\perp$ (a vector in the xy plane) and **B** are perpendicular. Since $|\mathbf{v}_\perp|$ is constant (Eq. (5 44b)), the Lorentz force has a constant magnitude and is *always* directed perpendicular to the velocity. As we know from our study of circular motion, such a force gives

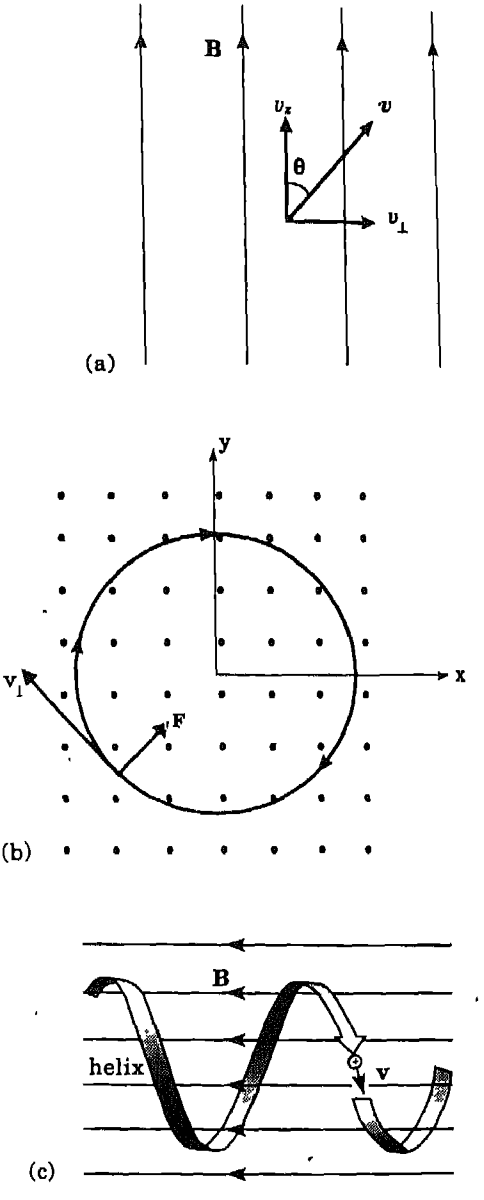


Figure 5.20: (a) A charged particle in a magnetic field, its initial velocity v making an angle θ with the field **B** directed along the z axis (b) The circular orbit in the x, y plane The magnetic field comes out of the plane of the paper. The orbit is clockwise as shown for a positively charged particle (c) A schematic picture of the helical orbit, for v_z opposed to **B**.

rise to circular orbits From Newton's second law, the centripetal acceleration $(v_\perp^2/r) = v_\perp\omega$ produced by this force is given by

$$v_\perp\omega = (qv_\perp B/m) \qquad (5\ 46a)$$

or

$$\omega = (qB/m) \qquad (5\ 46b)$$

The charged particle thus executes a circular orbit in the xy plane (plane perpendicular to B) with constant angular frequency ω given by Eq (5.46b) (Fig. 5 20b) The frequency (qB/m) (called cyclotron frequency and given a special symbol ω_c) is independent of the particle's velocity, depending only on its charge q, mass m, and on the magnetic field B. The circular orbit is described in a clockwise direction in the xy plane if the particle has a positive charge, and the magnetic field is along the positive z direction

The radius r of this cyclotron orbit is easily calculated in terms of $v_\perp$, since

$$r = (v_\perp/\omega_c) = (mv_\perp/qB). \qquad (5.47)$$

The particle moves in a circular orbit with angular frequency (qB/m) and radius $(mv_\perp/qB)$ in the xy plane while at the same time moving in the z direction with velocity v_z. Its orbit in space is thus a helix, of radius $r = (v_\perp/w_c)$ and pitch $p = (v_z/w_c)$ (Fig. 5.20c) The cyclotron frequency of an electron in small magnetic fields is in the ultrahigh radio frequency or microwave range. For example, for $B = 0.01\text{T}$, $\nu = \omega/2\pi \simeq 2\ 8 \times 10^8$ Hz.

Example 5.6: In a chamber a uniform magnetic field of 8 0 G (1G = 10^{-4}T) is maintained An electron with a speed of 4.0 $\times 10^6$ ms^{-1} enters the chamber in a direction normal to the field

(a) Describe the path of the electron

(b) What is the frequency of revolution of the electron?

(c) What happens to the path of the electron if it progressively loses its energy due to collisions with the atoms or molecules of the environment?

Answer: (a) The path of the electron is a circle of radius r given by

$$r = \frac{mv}{eB}$$

For the electron $m = 9.1 \times 10^{-31}$kg, $e = 1.6 \times 10^{-19}$C.
Therefore,

$$r = \frac{9\ 1 \times 10^{-31} \times 4\ 0 \times 10^6}{1\ 6 \times 10^{-19} \times 8.0 \times 10^{-4}} = 2.8\,\text{cm}$$

The sense of rotation of the electron in its orbit can be ascertained from the direction of the centripetal force, $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$ Thus if $\mathbf{B}$ is along the z axis, the electron revolves counter clockwise

(b) The frequency of revolution of the electron in its circular orbit is

$$\nu = \frac{v}{2\pi r} = \frac{eB}{2\pi m}$$
$$= \frac{1.6 \times 10^{-19} \times 8.0 \times 10^{-4}}{2\pi \times 9.1 \times 10^{-31}} = 22\text{MHz}.$$

(c) Due to collisions with the atomic constituents of the environment, the electron loses its speed progressively If the velocity vector of the electron after collisions remains in the plane of the initial circular orbit, the radius of the circular orbit will decrease in proportion to the decreasing speed. However, in general, the velocity of the electron will not remain in the plane of the initial orbit after collision. In that case, the component of velocity normal to B will determine the radius of the orbit while the component of velocity parallel to B remains constant. Thus the path of the electron between two collisions is, in general, helical. **Notice, however, the important fact: the frequency of orbital revolution remains the same, whatever be the speed of the electron.**

5.6.2 The cyclotron

The cyclotron is a device used largely in the past sixty or so years to accelerate charged particles to high energies, using the fact that

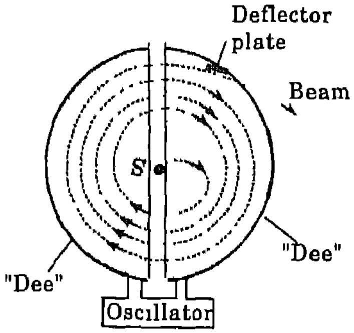


Figure 5.21: The elements of a cyclotron, showing the ion source S and the dees. A uniform magnetic field emerges from the plane of the figure The circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between them

the frequency of cyclotron motion is independent of the particle velocity (or more precisely, momentum).

The cyclotron consists of two evacuated, semicircular, hollow, flat metal D shaped objects (naturally called dees). They are open on their straight edges The dees form part of an electrical oscillator which establishes an alternating potential difference across the gap between the dees (see Fig. 5.21). The dees are in a uniform magnetic field perpendicular to their flat planes (i.e. perpendicular to the plane of the paper in Fig 5 21). Charged particles (e.g. protons) are injected into the dees near the centre, and start off on a circular orbit in one of the dees. When they reach the space between the dees after completing a semicircle they are accelerated by the electric field there (if the voltage has the right sign). They complete a semicircle in the other dee in a time $t = (\pi/\omega_c)$, and if they reach the space between the dees just as the voltage between them changes sign relative to its value during their previous arrival there, the particles will be accelerated again. Thus, if the ac voltage has the cyclotron frequency (resonance condition) the charged particles will arrive between the dees exactly in time to be repeatedly accelerated, and gain energy. This energy gain does not affect the resonance condition, merely the orbit size (it increases with energy, see Eq. 5.47). The accelerated particles reach the periphery of the dees, where the beam can be deflected out by a suitable magnet. The accelerated or high energy charged particle beam can be used for many experiments, e g. nuclear collision reactions, ion implantation. For example, if the voltage between the dees is 1000 V and a proton goes through the dees 10^3 times, the energy gained is 2×10^6 eV = 2 MeV! These devices as such are not in wide use now, but others based on the cyclotron resonance principle are.

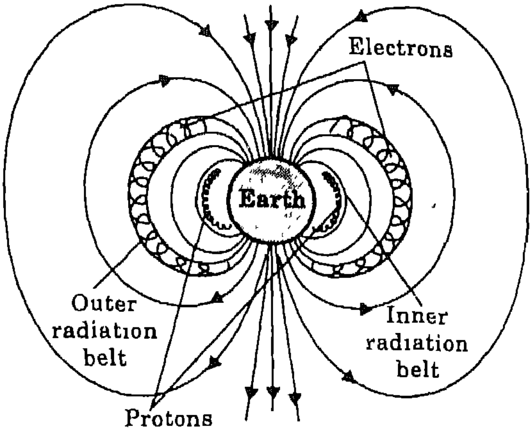


Figure 5.22: The earth's magnetic field showing spiralling, trapped protons and electrons The magnetic north and south poles of the earth are shown The figure is one of revolution about the N-S axis (magnetic north is close to geographic south)

Cyclotron motion is not uncommon in nature Charges in the upper atmosphere, produced by electrical discharges near the poles, or ejected by the sun, move in the way shown in the earth's magnetic field (Fig. 5.22). We notice the circular motion perpendicular to the magnetic field lines, and the change in the pitch as the field changes.

5.6.3 A solid in perpendicular electric and magnetic fields; the Hall effect

When charges in a conductor move under mutually perpendicular electric and magnetic fields, a voltage is developed across leads which lie in a direction perpendicular to both. The situation is illustrated in Fig. 5.23. A current I flows through a conducting strip $KLMN$ in the direction shown, i.e., along KM, because of an electric field or voltage applied in that diection. A uniform magnetic field **B** is applied perpendicular to the plane of the strip (as shown by the dots). Clearly this causes a magnetic Lorentz force on the moving charge carriers which are responsible for the electric current I. The direction of the Lorentz force $\mathbf{F}_B$, is perpendicular to both **B** and the electric current (e.g., Eq. 5.22) and is shown in Fig. 5.23. Suppose electrons are the charge carriers. Under this force, they will move to the end N, so that this end is negatively charged relative to L (Fig. 5.23a). Thus an electric field is set up along LN. You can see that it is such as to oppose further charge motion. An equilibrium is reached when the force $\mathbf{F}_E$ on electrons due to this electric field balances the magnetic Lorentz force $\mathbf{F}_B$. The final result is an electric field in the direction LN, there is a voltage difference between the ends L and N. This voltage V_H, observed first by the American Scientist H E Hall in 1879, is called the Hall voltage, the phenomenon being known as the Hall effect. The direction of $\mathbf{F}_E$ implies that the electric field is along LN so that $V_L > V_N$, (Fig. 5.23a). This is often found to be the case. However, in many systems, eg., the elements beryllium, cadmium and tungsten, it is found that the Hall voltage has the opposite sign, i.e., $V_L < V_N$, (see Fig. 5.23b). This unexpected anomaly in the *sign* of the Hall voltage is understood and involves the motion of *holes*, which have a charge opposite to that of electrons. They are the current carriers in many metals and semiconductors.

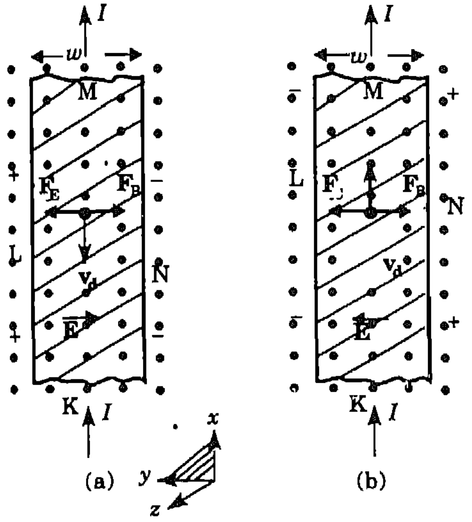


Figure 5.23: A conducting strip $KLMN$ of width w carrying a current I, and placed in a uniform magnetic field **B** perpendicular to it (shown by dots). In equilibrium, the magnetic Lorentz force $\mathbf{F}_B$ is balanced by $\mathbf{F}_E$ due to the electric field set up by the charge separation shown. (a) If electrons carry the current the side L is positive relative to N and $V_L > V_N$, (b) If carriers have positive charge, $V_L < V_N$, (c) Useful coordinate axes: the strip is in the xy plane and **B** is along the z axis.

The *size* of the Hall voltage depends on the drift velocity of the carriers in the direction KM, and on the magnetic field. For a given current I, the drift velocity depends *inversely* on the density n of carriers (number of carriers per unit volume). This follows from the discussion of the Ohm's law in chapter 3, in particular Eq. (3.12). Thus we have the important result that the Hall voltage is inversely proportional to carrier density. This result is used to measure the density (number density) of charge carriers. The Hall effect is thus very useful in experimentally determining the basic electrical characteristics of metals and semiconductors, namely current carrier type and carrier density.

★ We analyze in detail the experimental arrangement discussed above, and show exactly how the Hall voltage can be related to the current I, the carrier density n, and the charge q of a carrier We first find the megnetic force $\mathbf{F}_B$ on each drifting charge carrier, equate it to $\mathbf{F}_E$ and thus find the electric field along LN, or the voltage difference $(V_L - V_N)$

We have seen in section 3 3 4 that the current density j or current per unit area is given by the relation

$$j = (I/A) = nqv_d \tag{5 48}$$

where n is the density of carriers of charge q drifting with speed v_d A is the cross sectional area wd of the strip with width w and thickness d We thus have

$$v_d = (I/nqA) \tag{5 49}$$

The direction of v_d is shown in Fig 5 23a for electrons carrying the current Since the magnetic field $\mathbf{B}$ is perpendicular to it, the magnetic Lorentz force is along LN, as shown (for either sign of the charge) and has a magnitude

$$qv_dB = \frac{qIB}{nqA} = \frac{IB}{nA} \tag{5.50}$$

This force causes the charges in the conductor to move along LN, and accumulate at the ends if they are not connected from the outside. This accumulation clearly produces an electric field E_H opposing further charge motion. Equilibrium is attained when the two forces balance There is no further charge motion and E_H does not increase This happens when

$$F_E = qE_H = F_B = \left(\frac{IB}{nA}\right) \tag{5 51}$$

$$\text{or } E_H = \left(\frac{IB}{nqA}\right) \tag{5 52}$$

Since this field is across a distance w, the voltage difference between L and N is

$$\begin{aligned} V_H &= (V_L - V_N) = -E_H w \\ &= -\left(\frac{Iw}{A}\right)\left(\frac{B}{nq}\right) \end{aligned} \tag{5 53}$$

The ratio (V_H/I) of the Hall voltage V_H to the electric current I is naturally called the Hall resistance. The ratio of the Hall electric field E_H to the current density $j = (I/A)$ is called the resistivity ρ_H The Hall resistance is

$$\left(\frac{V_H}{I}\right) = -\left(\frac{w}{A}\right)\left(\frac{B}{nq}\right) \tag{5 54}$$

The Hall resistivity ρ_H is

$$\rho_H = \frac{E_H}{j} = \frac{-V_H/w}{(I/A)} = \frac{B}{nq} \tag{5 55}$$

The Hall resistivity per unit magnetic field, (ρ_H/B) is called the Hall coefficient, and is given the symbol R_H It has the value

$$R_H = \left(\frac{1}{nq}\right). \tag{5 56}$$

The Hall coefficient depends on the carrier density n, and on the charge q of the carrier Fig 5 23a shows the forces, voltages and currents for the case that electrons are the carriers, i e, $q = e$ If the current is carried by positively charged objects, the corresponding physical quantities are as in Fig 5 23b

The Hall effect is due to the magnetic Lorentz force on the charges drifting in a perpendicular electric field, and can be used to measure the number density and sign of the charge of current carriers in solids. For example, in a metal like copper, the current is most likely carried by unfilled outer shell electrons, of which there is one per Cu atom in copper metal Thus given the density of copper, (nq) can be calculated and compared with that obtained from the Hall resistance of a copper strip The agreement is fairly good as it is for some other metals In some substances, e g beryllium, cadmium and tungsten, the *sign* of the Hall effect suggests positive charge carriers But we know that say beryllium, has two *electrons* outside the closed shell Then why is the Hall voltage sign anomalous? The answer to this question involves the idea of 'holes , mentioned also in Chapter 14 of this book When the number density of carriers is small, the Hall voltage or resistance is large This is what happens for semiconductors such as Si or Ge In these substances (when pure), n depends exponentially on temperature, i e ,

$$n \propto \exp(-E_g/k_BT) \tag{5 57}$$

where E_g is known as the energy gap of the semiconductor. By measuring n as a function of temperature T in these systems, E_g can be determined The Hall effect is thus very useful in understanding some properties of metals and semiconductors ★

Summary

1 **Oersted's observation** 'A magnetic needle brought close to a straight current-carrying wire aligns itself perpendicular to the wire, reversing the current reverses the direction of the needle

2 **Biot-Savart Law** The magnetic field **dB** at a distance **r** from a current element of length **dl** carrying a current I is given by

$$\mathbf{dB} = \frac{\mu_o}{4\pi}\frac{I\mathbf{dl}\times\hat{\mathbf{r}}}{r^2}, \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$$

where $\mu_0/4\pi$ is exactly 10^{-7} Tm A^{-1}, T denotes the SI unit of **B** (Tesla) The magnetic field **B** due to a general current distribution is obtained by superposing fields due to its elementary current elements

(i) **B due to a straight (infinitely) long wire carrying a current I** It has magnitude

$$B = \frac{\mu_0 I}{2\pi R}$$

where R is the perpendicular distance of the point from the wire Thus points of constant B form concentric circles with the wire as centre The direction of **B** at any point on the circle is tangential to it, determined by the right-hand rule grasp the conductor in your right hand with the thumb up and along the current Then other fingers curve in the direction of **B**.

The field lines of **E** due to electric charges begin or end at charges In contrast, field lines of **B** always form closed loops Thus there are no sources of magnetic fields which are analogous to electric charges, i e. there are no magnetic monopoles

(ii) **B due to a circular current loop of radius a** At a point on the axis at a distance R from the centre of loop, it has a magnitude

$$B = \frac{\mu_0 I a^2}{2(R^2 + a^2)^{3/2}}$$

$$\text{For } R \gg a,\ B = \frac{\mu_0 I a^2}{2R^3} = \frac{\mu_0 I A}{2\pi R^3}$$

where $A = \pi a^2$ is the area of the loop

At the centre of the loop,

$$B = \frac{\mu_o I}{2a}$$

The direction of **B** at the centre (or a point on the axis) is the direction of motion of a right-handed screw rotated along the sense of the current in the loop

3. **Forces on Currents** The force on a small current carrying element dl carrying a current I in a magnetic field **B** is

$$\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$$

To remember the direction of **F**, use Fleming's left-hand rule Central finger along current, forefinger along field and thumb of the left hand along motion (or force)

For two *parallel* short wires of lengths dl_1 and dl_2 a distance $\mathbf{r}_{21}$ apart, carrying currents I_1 and I_2 respectively, the force on wire 2 is given by

$$d\mathbf{F}_2 = -\frac{\mu_0}{4\pi}\frac{I_1 I_2 dl_1\, dl_2}{r_{21}}\hat{\mathbf{r}}_{21} = d\mathbf{F}_1$$

where $\hat{\mathbf{r}}_{21}$ is a unit vector in the direction from 1 to 2 If I_1 and I_2 are in the same direction, the mutual force is attractive, if they are in opposite directions, it is repulsive For two very long straight parallel conductors a distance r_{12} apart, carrying currents I_1 and I_2, the force *pre unit length* on wire 2 due to wire 1 is

$$\mathbf{f}_2 = -\frac{\mu_0}{2\pi}\frac{I_1 I_2}{|\mathbf{r}_{21}|}\hat{\mathbf{r}}_{21} = -\mathbf{f}_1$$

The last relation is used in the SI definition of the unit of current, ampere

The total force (**Lorentz force**) on a charge q with velocity $\mathbf{v}$ in electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Since the magnetic force $q\mathbf{v} \times \mathbf{B}$ is normal to $\mathbf{v}$, it does no work

If we compare the electric force $\mathbf{F}_e$ between charges q_1 and q_2 with the magnetic force $\mathbf{F}_m$ between the same amounts of charges with drift speeds v_1 and v_2, we find

$$\frac{|\mathbf{F}_m|}{|\mathbf{F}_e|} = \frac{v_1 v_2}{c^2}.$$

Thus under typical conditions of $v \sim 10^{-5}$ m/s, magnetic forces are much weaker. We still observe magnetic forces between current carrying conductors, since electric forces are absent between them, matter being electrically neutral

4. **Ampere's Circuital Law:** If **B** is always directed along the tangent to the perimeter of a closed curve and is constant in magnitude all along the perimeter,

   B × perimeter of the closed circuit

   $= \mu_0 \times$ total current passing through any surface with the closed circuit as its boundary.

   This is the simple form of a more general result called Ampere's circuital Law, which is derived from the Biot-Savart Law.

   The field inside a toroidal solenoid can be obtained by using this law and symmetry·

   $$B = \mu_0 I n$$

   where I is the current in the windings and n is the number of turns per unit length The field lines form concentric circles. The magnetic field outside the space enclosed by the windings is zero

   For a straight solenoid, the field near the centre is given by the same formula as above $B = \mu_0 I n$

At either end $B_{end} = \mu_0 nI/2$. Magnetic field lines due to a solenoid are similar to those of a bar magnet

5 Any planar loop of area A carrying current I is associated with a magnetic dipole moment $\mathbf{m}$ whose magnitude is IA $\mathbf{m}$ is along the thumb sticking out

A closed current loop placed in a uniform magnetic field $\mathbf{B}$ experiences a torque $\boldsymbol{\tau}$ given by

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

(which is analogous to the formula for torque on an electric dipole in a uniform electric field $\mathbf{E}$ $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$.) If a rectangular loop carries current I and has area A and N turns,

$$|\tau| = NIAB \sin\theta$$

where θ is the angle between $\mathbf{B}$ and the normal to the plane of the loop.

A moving coil galvanometer consists of a coil of wire of area A and n turns carrying the current I to be measured The coil is placed in a magnetic field so designed that $\mathbf{B}$ remains parallel to the plane of the coil as it turns $(\sin\theta = 1)$. The coil is hung from a spring. In equilibrium, the restoring couple balances the torque due to $\mathbf{B}$·

$$k\phi = NIAB$$

where ϕ is the deflection of the coil and k is the torsion constant of the spring.

6. **A charge q of mass m moving in uniform B** Along $\mathbf{B}$, there is no force, so the velocity along $\mathbf{B}$ is unchanged In the plane normal to $\mathbf{B}$, the particle executes a circular orbit of angular frequency $\omega = qB/m$ The orbit of the particle in space is a helix Note that ω is independent of the particle's momentum. This fact is used in the cyclotron, a device to accelerate charged particles to high energies.

7 **Hall effect**· If a strip carrying current I is placed in a magnetic field $\mathbf{B}$ normal to the direction of current, a voltage V_H develops in the transverse direction (i.e., normal to $\mathbf{B}$ and current) The effect arises due to the magnetic force experienced by the moving charge carriers in the strip In equilibrium, the magnetic force is balanced by the transverse electric field produced

The sign of the Hall voltage V_H depends the sign of the charge of the charge carriers, and its size inversely on the carrier density It can therefore be used to determine n and the sign of charge carriers in solids

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★ The Hall resistance is equal to

$$\frac{V_H}{I} = \frac{Bw}{A}\frac{1}{nq}$$

where w is the width of the strip, A is the cross-sectional area, n is the number density of carriers each of charge q and B is the magnetic field

The Hall effect can be used to determine n and the sign of charge carriers in solids in terms of the measured quantities V_H, I, B, w and A ★

Exercises

5.1 State the Biot–Savart law for the magnetic field due to a current-carrying element. Use this law to obtain a formula for magnetic field at the centre of a circular loop of radius a carrying a steady current I.

5.2 A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40A. What is the magnitude of the magnetic field **B** at the centre of the coil?

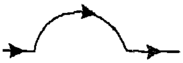
5.3 Give the formula for the magnetic field produced by a straight infinitely long current-carrying wire. Describe the lines of field **B** in this case.

5.4 A long straight wire carries a current of 35A. What is the magnitude of the field **B** at a point 20 cm from the wire?

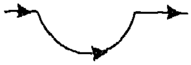
5.5 A long straight wire in the horizontal plane carries a current of 50A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

5.6 A horizontal overhead power line carries a current of 90A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

5.7 A straight wire carrying a current of 12 A is bent into a semicircular arc of radius 2.0 cm as shown in Fig. a. What is the direction and magnitude of **B** at the centre of the arc? Would your answer change if the wire were bent into a semicircular arc of the same radius but in opposite way as shown in Fig. b?



(a)



(b)

5.8 What is the magnitude of magnetic force per unit length on a wire carrying a current of 8A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?

5.9 A 3.0 cm wire carrying a current of 10A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

5.10 Two long and parallel straight wires A and B carrying currents of 8.0A and 5.0A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

5.11 A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A, estimate the magnitude of **B** inside the solenoid near its centre.

5.12 A square coil of side 10 cm consists of 20 turns and carries a current of 12A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

5.13 Two moving coil meters M_1 and M_2 have the following particulars.
$R_1 = 10\Omega$, $N_1 = 30$,
$A_1 = 3.6 \times 10^{-3}\text{m}^2$, $B_1 = 0\ 25$ T,
$R_2 = 14\Omega$, $N_2 = 42$,
$A_2 = 1.8 \times 10^{-3}\text{m}^2$, $B_2 = 0\ 50$T,
(The spring constants are identical for the two meters). Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

5.14 In a chamber a uniform magnetic field of 6.5G ($1\text{G} = 10^{-4}\text{T}$) is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6\text{ms}^{-1}$ normal to the field. Explain why the path of the electron is a circle Determine the radius of the circular orbit ($e = 1\ 6 \times 10^{-19}\text{C}$, $m_e = 9\ 1 \times 10^{-31}\text{kg}$).

5.15 In 5 14 obtain the frequency of revolution of the electron in its circular orbit Does the answer depend on the speed of the electron? Explain.

5.16 (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6 0A is suspended vertically in a uniform horizontal magnetic field of magnitude 1 0 T. The field lines make an angle of 60° with the normal of the coil Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered)

Additional Exercises

5.17 Two concentric circular coils X and Y of radii 16cm and 10cm respectively lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A, coil Y has 25 turns and carries a current of 18 A The sense of the current in X is anticlockwise, and in Y clockwise, for an observer looking at the coils facing west Give the magnitude and direction of the net magnetic field due to the coils at their centre.

5.18 A magnetic field of 100 G ($1\text{ G} = 10^{-4}\text{T}$) is required which is uniform in a region of linear dimension about 10cm and area of cross section about 10^{-3}m^2. The maximum current-carrying capacity of a given coil of wire is 15A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is *not* ferromagnetic.

5.19 For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{3/2}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small compared to R, and is given by

$$B = 0\ 72\frac{\mu_0 N I}{R}\text{approximately}$$

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils.]

5.20 A toroid has a core (non-ferromagnetic) of inner radius 25cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (i) outside the toroid, (ii) inside the core of the toroid, and (iii) in the empty space surrounded by the toroid.

5.21 Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

5.22 Answer the following questions

(a) A cloud chamber photograph shows a pair of circular tracks emerging from a common point. The tracks have similar density at droplets but curve in opposite directions in a plane normal to the magnetic field maintained in the chamber. If one of the ionizing particles is established to be an electron, guess the high energy event that took place at the common point of the tracks.

(b) A similar event as in (a) photographed in a liquid-hydrogen bubble chamber shows spiral tracks instead of circular tracks. Explain why?

(c) A flat copper strip carries current in the direction shown in the Fig. 5.23. If a uniform magnetic field is set up normal to the plane of the strip in the direction shown, specify the polarity of the Hall voltage set up across the strip.

(d) If the polarity of Hall voltage in a material is opposite to that obtained for a copper strip, what conclusion can you draw?

5.23 An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30° with the initial velocity.

5.24 A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator? ($e = 1.60 \times 10^{-19}$C, $m_p = 1.67 \times 10^{-27}$ kg). Express your answer in units of MeV (1 MeV = 1.602×10^{-13}J).

5.25 A magnetic field set up using

Helmholtz coils is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single-species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9.0 \times 10^5 \text{V m}^{-1}$, make a simple guess as to what the beam contains. Why is the answer not unique?

5.26 A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before. (Ignore the mass of the wires.) $g = 9.8 \text{ ms}^{-2}$

5.27 The wires which connect the battery of an automobile to its starting motor carry a current of 300A (for a short time). What is the force per unit length between the wires if they are 70cm long and 1.5cm apart? Is the force attractive or repulsive?

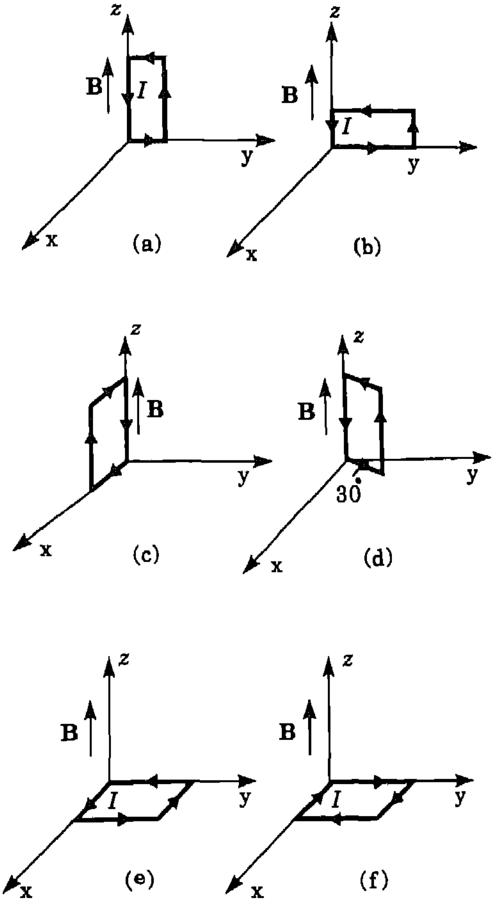
5.28 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

5.29 A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5cm carries a current of 12 A. What is the torque on the loop in the different cases shown in the figure? What is the force on each case? Which case corresponds to stable equilibrium?



5.30 A rectangular coil of sides 8cm and 6cm having 2000 turns and carrying a current of 200 mA is placed in a uniform magnetic field of 0.2 T directed along the +ve x-axis.

(a) What is the maximum torque the coil can experience? In which orientation does it experience the maximum torque?

(b) For which orientations of the coil is the torque zero? When is this equilibrium stable and when is it unstable?

5.31 A rectangular coil of area $5.0 \times 10^{-4} m^2$ and 60 turns is pivoted about one of its vertical sides The coil is in a radial horizontal field of 90 G ('radial' means the field lines are in the plane of the coil for any orientation) What is the torsional constant of the hairsprings connected to the coil if a current of 0 20 mA produces an angular deflection of 18°.

5.32 Given a uniform magnetic field of 100 G in an east to west direction and a 44cm long wire with a current-carrying capacity of at most 10 A, what is the shape and orientation of the loop made of this wire which yields maximum turning effect on the loop? What is the magnitude of the maximum torque?

5.33 A short conductor of length 5 0cm is placed parallel to a long conductor of length 1 5 m near its centre The conductors carry currents 4 0A and 3.0A respectively in the same direction What is the total force experienced by the long conductor when they are 3.0cm apart?

5.34 On a smooth plane inclined at 30° with the horizontal, a thin current-carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0.15 T in the vertical direction For what value of current can the rod remain stationary? The mass per unit length of the rod is 0.30 kg m^{-1}

5.35 A circular coil of 20 turns and radius 10cm is placed in a uniform magnetic field of 0 10 T normal to the plane of the coil If the current in the coil is 5.0 A, what is the

(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field? (The coil is made of copper wire of cross-sectional area $10^{-5} m^2$, and the free electron density in copper is given to be about $10^{29} m^{-3}$)

5.36 A flat silver strip of width 1.5cm and thickness 1.5 mm carries a current of 150 A A magnetic field of 2.0 T is applied perpendicular to the flat face of the strip. The voltage developed across the width of the strip is measured to be 17.9 μV (Hall effect) Estimate the number density of free electrons in the metal.

5.37 A solenoid 60cm long and of radius 4 0cm has 3 layers of windings of 300 turns each. A 2.0cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? $g = 9.8\text{ms}^{-2}$

5.38 Describe qualitatively the path of a charged particle moving in

(a) a uniform electrostatic field, with initial velocity (i) parallel to the field, (ii) perpendicular to the field, (iii) at an arbitrary angle with the field direction;

(b) a uniform magnetic field, with initial velocity (i) parallel to the field, (ii) perpendicular to the field, (iii) at an arbitrary angle with the field direction;

(c) a region with uniform electrostatic and magnetic fields parallel to each other, with initial velocity (i) parallel, (ii) perpendicular, (iii) at an arbitrary angle with the common direction of the fields;

5.39 (a) State Ampere's law connecting the line integral of **B** over a closed path to the net current crossing the area bounded by the path

(b) Use Ampere's law to derive the formula for magnetic field due to an infinitely long straight current-carrying wire

(c) Explain carefully why the same derivation as in (b) is not valid for magnetic field in a plane normal to a current-carrying straight wire of finite length, and passing through the mid-point of the wire.

5.40 (a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e turns about the vertical axis)?

(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

(d) Is displacement current, like conduction current, a source of magnetic field?

5.41 A rectangular loop of sides 25cm and 10cm carrying current of 15A is placed with its longer side parallel to a long straight conductor 2.0cm apart carrying a current of 25A What is the net force on the loop?

André Marie Ampère (1775-1836)

French physicist who founded and named the science of electrodynamics or electromagnetism, and for whom the unit of electric current is named. Within a week of hearing about Oersted's discovery he formulated the law of magnetic force between electric currents, based on his own experiments He was the first to develop techniques for measuring the flow of electricity. He speculated that all magnetism is due to electrical currents

CHAPTER 6

Magnetism

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6.1 Introduction

In the previous Chapter we learnt about the magnetic effects of electric currents. This and other connections between electricity and magnetism, as we saw, were discovered and understood only from the beginning of the nineteenth century. Long before this, however, several electric and magnetic phenomena were known but were thought to be unrelated.

In this Chapter we study this early knowledge of magnetism. Apart from electric currents, the simplest sources of magnetic fields are permanent (bar) magnets. To some extent we shall deal with magnets and magnetic fields directly, without continually referring to the fact that they are ultimately consequences of electric charges in motion. At suitable points, however, we will refer to the material of the previous Chapter. Thus there will be some overlap between these two Chapters.

We shall study the basic features of the magnetic field of a bar magnet, emphasizing its dipole form. Then we look at the definition of magnetic field, lines of force, magnetic flux and their units We compare the roles of point electric charges and magnetic dipoles as alternative test objects to explore a magnetic field. The absence of isolated magnetic charges leads us to an analogue of Gauss's Theorem in electrostatics.

The most important features of the earth's magnetic field, and then the magnetic properties of different materials, are taken up next. We conclude with a brief description of the tangent galvanometer, which is a modification of the moving coil galvanometer described in the previous Chapter.

6.2 Field of a bar magnet - magnetic field lines - magnetic flux

The simplest examples of magnetic effects are: (a) the property of a freely suspended or piv-

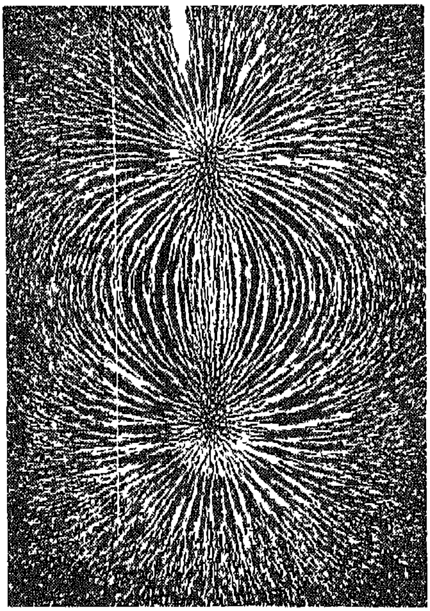


Figure 6.1: Pattern of iron filings near a bar magnet

oted magnetic compass needle, able to swing in a horizontal plane, to point approximately along the geographical North-South direction; (b) the characteristic pattern produced by iron filings sprinkled on a sheet of paper or on a glass pane held over a bar magnet, as shown in Fig 6 1. As described in the previous Chapter, the similarity of this pattern to the electric field lines outside an electric dipole, indicated in Fig. 1.17, is striking. We have also learnt in Section 5.5 that a (small) planar current loop behaves just like a (small) magnetic dipole or bar magnet. Its magnetic moment, according to Eq. (5.37), is given by multiplying the loop area by the amount of current in the loop We note also the close similarity between the magnetic field lines due to a solenoid (Section 5.4.2 and Fig. 5 16), and the pattern of iron filings shown in Fig 6.1. Since the iron filings point and concentrate in the direction of the magnetic field, this means that the magnetic field due to a bar magnet and a solenoid have a similar pattern in space.

★ We mentioned in Chapter 1 that knowledge about magnetic substances and effects goes back to ancient times Thales of Miletus, who knew about static electricity, was also aware of the ability of pieces of magnetite (an iron ore) to attract small pieces of iron The words "magnetite", "magnetic" and "magnetism" all come from the name of the island of magnesia in Greece where this ore was found We also recalled that the Chinese were using magnetic compass needles for navigation almost a thousand years ago In fact the alternative name "lodestone" for magnetite means "leading stone"

William Gilbert, in his book "De Magnete" in 1600, suggested that the earth itself is a gaint magnet Its field is roughly like that of a bar magnet Just as an electric field exerts a torque on an electric dipole, the earth's magnetic field exerts a torque on a freely hanging compass needle This makes the needle point roughly in the North - South direction ★

Permanent magnets: Permanent magnets are made of hard steel or special alloys One can make a magnet out of a steel rod by holding it in the North - South direction and repeatedly hammering it. Here we take advantage of the earth's field. Once it becomes a magnet in this way, it retains this property unless heated to a high temperature. Another is to stroke the steel rod with one end of a bar magnet a large number of times, always in the same sense

A more efficient way is to place the steel rod inside a solenoid and run a current. Then the magnetic field of the solenoid magnetizes the rod as well.

The properties of soft iron, in contrast, are quite different Filings (long thin pieces) of soft iron easily become magnets when placed in a magnetic field, for example near a permanent magnet (Conversely it requires a strong field to magnetize a steel rod). But when removed from the field, they lose their magnetization just as easily. It is a case of "easy come easy go"

★ The microscopic picture of the situation is this As we learnt in the previous Chapter, the ultimate sources of magnetism are the intrinsic magnetic moments of electrons due to their spin, and the moments caused by their motion within the atoms An unmagnetized steel rod contains many small "atomic" magnets, like so many compass needles Groups of these are aligned parallel to each other; each such group is called a domain (see section 6.4) Different domains are oriented in different direction Magnetising such a system means orienting the domains in more or less the same direction, making use of the torque on the atomic magnets exerted by the outside agency, e g , a solenoid or a permanent magnet The magnetization achieved in this way is then retained This change from the unmagnetised to the magnetised condition is depicted in Fig. 6.2 ★

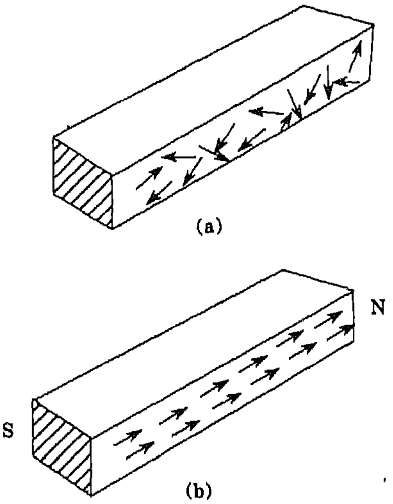


Figure 6.2: Picture of a steel rod (a) before, (b) after, magnetization

Magnetic field lines and flux: Suppose we have a magnetic field created by some electric currents and/or permanent magnets. What can we use as a test object to measure the field at different points in space?

We might place a magnetic compass needle at various positions, and see how it orients itself at each point. This tells us the direction of the field at different points in space. (This is like sprinkling iron filings on a sheet of paper over a bar magnet, as shown in Fig. 6.1). We can now construct magnetic field lines, analogous to electric lines of force. A magnetic field line is a (imaginary) line in space, such that at each point on it the tangent is parallel to the magnetic field **B** there. Such lines were frequently mentioned in Chapter 5. As in the electric case, magnetic field lines crowd together where the field is strong, and thin out where it is weak. Fig. 5.12 shows some field lines due to a solenoid

However this procedure gives us only the *direction*, and not the *magnitude*, of **B** at each point To get the complete vector **B**, we can proceed in this way· We hold the compass needle in various directions at each point, and for each direction measure the torque exerted on it by the field. If the magnetic dipole moment of the needle is **m**, the torque is given by Eq. (5 36), namely,

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \qquad (6.1)$$

In this way we can in principle *infer* the direction and magnitude of **B**. (When we allow the needle to freely align itself along **B**, the torque vanishes). The similarity in the behaviour of electric and magnetic dipoles in electric and magnetic fields respectively can be made use of to obtain an expression for the potential energy of a magnetic dipole in a given magnetic field. We have seen in chapter 2 that the potential energy of an electric dipole of moment **p** at the point **r** in an electric field **E** is $-\mathbf{p}\cdot\mathbf{E}(\mathbf{r})$, and that this is consistent with (cf Eq. 2.57) the fact that there is a torque $\mathbf{p}\times\mathbf{E}(\mathbf{r})$ on the dipole, as shown in Eq. (1.33) It then follows, by a strictly similar argument, that a (small) magnetic dipole of moment **m** situated at the point **r** in the field **B** has the potential energy $-\mathbf{m}\cdot\mathbf{B}$:

Potential energy of a small magnetic

dipole in a field **B**

$$= -\mathbf{m}\cdot\mathbf{B} \qquad (6.2)$$

Just like the torque $\mathbf{m} \times \mathbf{B}$, this too has the correct dimensions, namely the Joule.

Example 6.1: A magnetic needle pivoted through its centre of mass and free to rotate in a plane containing a uniform magnetic field of 100G is displaced slightly from its stable equilibrium. The frequency of its angular oscillations of small amplitudes is measured to be 1.5 s^{-1}. If the moment of inertia of the needle about its axis of rotation is 0.75 × 10^{-5} kgm^2, determine the magnetic moment of the needle.

Answer: The frequency of small angular oscillations of a magnetic needle about its stable equilibrium is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{mB}{I}}$$

(I here is moment of inertia)

from which we get

$$\begin{aligned} m &= \frac{4\pi^2\nu^2 I}{B} \\ &= \frac{4\pi^2(1.5)^2(0.75\times10^{-5})}{10^{-2}} \\ &= 0.067\,\text{Am}^2 = 0.067\,\text{JT}^{-1} \end{aligned}$$

Example 6.2: A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T experiences a torque of magnitude 0.032 J. (a) Estimate the magnetic moment of the magnet. (b) If the bar were free to rotate, which orientations would correspond to its (i) stable, and (ii) unstable equilibrium? What is its potential energy in the field for cases (i) and (ii)?

Answer: (a) The torque on a magnetic dipole of magnetic moment **m** in an external field **B** is

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

so that the magnitude of the torque is given by

$$\tau = mB\sin\theta$$

where θ is the angle between **m** and **B**. Here $\tau = 0.032$J, $B = 0.16$T, $\theta = 30°$. Therefore, $m = 0.032/(0.16 \times 1/2) = 0.40\,\text{JT}^{-1}$. The direction of **m** is from the south pole of the magnetic to its north pole.

(b) Potential energy of a magnetic dipole in an external field **B** is given by

$$U = -\mathbf{m}\cdot\mathbf{B}$$

(i) The bar is in stable equilibrium when its magnetic moment **m** is parallel to **B** ($\theta = 0$); its potential energy then is minimum:

$$U = -mB = -0.40 \times 0.16 = -0.064\,\text{J}$$

(ii) The bar is in unstable equilibrium when **m** is antiparallel to **B** ($\theta = 180°$); its potential energy then is maximum:

$$U = +mB = +0.064\,\text{J}$$

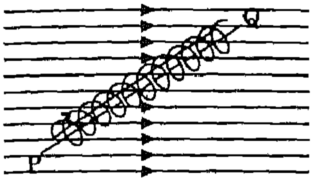
Example 6.3: A closely wound solenoid of 1000 turns and area of cross-section 2.0 × 10^{-4}m^2 carries a current of 2.0 A. It is placed with its horizontal axis at 30° with the direction of a uniform horizontal magnetic field of 0.16 T, as shown.
(a) What is the torque experienced by the solenoid due to the field?
(b) If the solenoid is free to turn about the vertical direction, specify its orientations of stable and unstable equilibrium. What is the amount of work needed to displace the solenoid from its stable orientation to its unstable orientation?

Answer: The magnitude of magnetic moment of the solenoid is given by

$$m = NIA.$$

With the given values of N, I and A,

$$\begin{aligned} m &= 1000 \times 2.0 \times 2.0 \times 10^{-4} = 0.40\,\text{Am}^2 \\ &= 0.40\,\text{JT}^{-1}. \end{aligned}$$



The direction of **m** is related to the sense of flow of current via the right-handed screw rule. In the figure, **m** is along the axis of the solenoid in the direction PQ. (a) The solenoid behaves like a bar magnet of magnetic moment 0.40 JT^{-1}, with its end P as south pole and end Q as north pole. (Thus it is equivalent to the bar magnet of example 6.2 which was found to have $m = 0.40\ JT^{-1}$). The magnitude of torque experienced is

$$\begin{aligned}\tau &= mB \sin\theta \\ &= 0.40 \times 0.16 \times 1/2 = 0.032\,\mathrm{J}\end{aligned}$$

(same as in example 6.2). The direction of τ is vertically downwards; its turning effect will be to bring the axis along the N-S direction of the external field.

(b) Like for a bar magnet, the stable orientation of the solenoid corresponds to **m** parallel to **B**, the unstable orientation corresponds to **m** antiparallel to **B**. In the former case PQ is aligned with N-S direction with P facing N and Q facing S. In the latter (unstable) case, QP is aligned with N-S direction; now Q faces N and P faces S. The potential energy for the stable orientation is $-mB = -0.40\times 0.16 = -0.064$J; while for the unstable orientation it is $+mB = +0.064$J The amount of work needed to displace the solenoid from its stable to unstable orientation $= +0.064 - (-0.064) = +0.128$J.

Another way is suggested by the Lorentz force laws, Eqs (5.21, 5.22). A moving point charge q feels a force due to any electric field **E** present, as well as a force due to any magnetic field **B**. The former acts whether the charge is at rest or is moving; the latter acts *only* when it is moving Thus we can imagine "firing" the point charge q with various velocities **v** through a given point in space, and see whether it feels any "side ways" force having the form $q\mathbf{v} \times \mathbf{B}$. If it does, then we can find **B** at that point

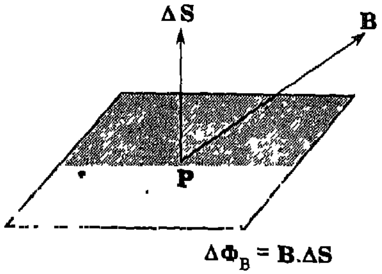


Figure 6.3: Magnetic flux through small area element

Thus the situation with magnetism is somewhat more complicated than that in the electric case For electric fields the simplest sources, as well as test objects, are point charges. In magnetic fields the simplest sources, as well as test objects, are magnetic dipoles or electric charges in motion (equivalently an electric current element). As we learnt in Chapter 5, Section 5, a (small) closed planar current loop is a (small) magnetic dipole.

Magnetic flux is defined in a similar way to electric flux in Chapter 2. A simple and qualitative definition is in terms of field lines. the magnetic flux through any (open or closed) surface S is just the number of magnetic field lines cutting S

To improve the definition, consider first a small area element ΔS around a point P where the field is **B**. The magnetic flux $\Delta\Phi_B$ through ΔS is defined as

$$\begin{aligned}\Delta\Phi_B &= \left\{\begin{array}{l}\text{magnetic flux through} \\ \text{small area element } \Delta S\end{array}\right. \\ &= \mathbf{B}\cdot\Delta\mathbf{S}. \qquad (6.3)\end{aligned}$$

This is shown in Fig. 6.3. The similarity to the electric case, Eq. (2.1) and Fig. 2.2, is clear. For a finite (open or closed) surface S, we have to divide it into many small elements ΔS, calculate the individual fluxes, and add them up

$$\begin{aligned}\Phi_B &= \left\{\begin{array}{l}\text{total magnetic flux}\\ \text{through surface S}\end{array}\right.\\ &= \sum_{\substack{\text{all area}\\ \text{elements } \Delta S}} \Delta\Phi_B\\ &= \sum_{\substack{\text{all area}\\ \text{elements } \Delta S}} \mathbf{B}\cdot\Delta\mathbf{S} \qquad (6.4)\end{aligned}$$

Dimensions and SI units: The SI unit for magnetic field was introduced in Section 5.2 it is the tesla, written as T Magnetic flux has dimensions of field times area Its SI unit is the weber, written Wb:

$$\begin{aligned}[\Phi_B] &= [\text{magnetic field}] \times (\text{metre})^2\\ &= \text{tesla} \times (\text{metre})^2.\\ &= \text{weber};\end{aligned}$$

$$1\ \text{Wb} = 1\ \text{T m}^2,\ 1\ T = 1\ \text{Wb/m}^2 \qquad (6.5)$$

So the magnetic field can also be expressed in webers per square metre

---

★ An older unit for magnetic field is the *gauss* It is related to the tesla by

$$1\ \text{T} = 10^4\ \text{gauss}. \qquad (6.6)$$

It is a convenient unit to describe the earth's magnetic field, which is typically a fraction of a gauss

In different text books, various names are used for the magnetic field **B** 'magnetic induction', 'magnetic induction field', 'magnetic flux density' For simplicity we have used the name 'magnetic field', and realise that it means the same as magnetic flux density

In addition to Eqs (6.5) above, there are simple connections among the ampere, the tesla, the weber, and mechanical units (This has been mentioned at Eq. (5.15)) Leaving aside the coulomb (which is ampere second), we have the relations,

$$\begin{aligned}1\ \text{tesla} &= 1\ \text{newton/ampere metre}\\ &= 1\ \text{joule/ampere (metre)}^2\\ &= \text{SI unit of magnetic field } \mathbf{B};\\ 1\ \text{weber} &= 1\ \text{joule/ampere}\\ &= \text{SI unit of magnetic flux } \Phi_B,\\ 1\ \text{ampere} &= 1\ \text{joule/weber}\\ &= 1\ \text{joule/tesla (metre)}^2\\ &= \text{SI unit of current} \qquad (6.7)\end{aligned}$$

★

Absence of individual magnetic charges: The two ends of a compass needle (or a permanent magnet) are called the north (N) and south (S) poles. When the needle is freely suspended about its centre, the former points towards the earth's geographic north pole, the latter towards the geographic south pole Thus magnetic north (and south) poles are geographically north seeking (and south seeking) poles

From early studies using thin long magnets, so that one could see how individual poles affect one another, it was known that two like poles (two N or two S poles) repel each other, while two unlike poles (an N and an S pole) attract each other This is similar to the electric coulomb forces between electric charges (For historical reasons we speak of "North" and "South" magnetic poles instead of positive and negative magnetic charges). Using Gilbert's picture of the earth as a giant magnet, this means that the earth's magnetic south pole is near its geographic north pole, and vice versa

Some idea of the dependence of magnetic forces on the distance between the poles was also known Working again with long thin magnets, it was found (by Coulomb) that these forces obey an inverse square law. (These studies were carried out well before the discoveries of Oersted, Biot and Savart, and Ampere, linking magnetism to electric currents) This would suggest that, as with electric charges, there are two types of magnetic charges, say N and S types And the force law would be similar to Coulomb's electrostatic law.

However (as stressed in Section 5.2) all efforts to find single magnetic charges, or to isolate the poles of a magnet from one another, have failed. In fact, breaking a long bar magnet into two pieces just produces two new magnets: additional N and S poles appear, so each piece has one N and one S pole This is shown in Fig 6.4.

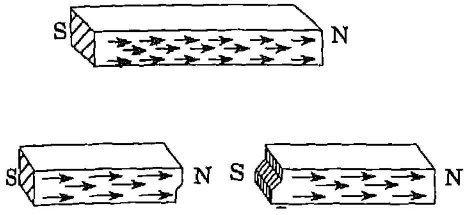


Figure 6.4: Breaking a bar magnet produces two new magnets

★ We can appreciate the absence of separate N and S poles in this way We picture a small atomic magnet as a tiny current loop, a macroscopic magnet is a collection of many of them more or less uniformly aligned For an atomic magnet, we define that face of the current loop whose outward normal points towards the *head* of the magnetic moment vector **m** as the N pole (or N face) The other face of the loop, towards the *tail* of **m**, is the S pole (or S face) Now given a small planar current loop, it clearly is impossible to physically separate its two faces! This is the ultimate reason why magnets are *dipoles* which cannot be separated into single magnetic poles

Still, *purely for convenience* we may imagine that in every magnet there are two artificial "poles" of *equal and opposite strengths*, then the dipole moment is the product of pole strength and distance of separation This is literally true, of course, for electric dipoles ★

The inverse square law dependence of magnetic forces among fictitious magnetic charges was mentioned above In electrostatics, Coulomb's inverse square law of force led to Gauss's Theorem. Therefore in magnetism too we have a corresponding Gauss's Theorem The difference is that there are no single magnetic charges This leads to the result:

Gauss's Theorem in magnetism: **The total magnetic flux Φ_B through any closed surface S is always exactly zero.**

In terms of magnetic field lines, there are as many lines entering S as leaving it, as illustrated in Fig 6 5.

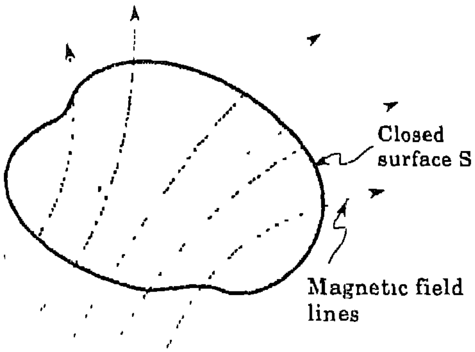


Figure 6.5: Illustrating Gauss's Theorem in magnetism

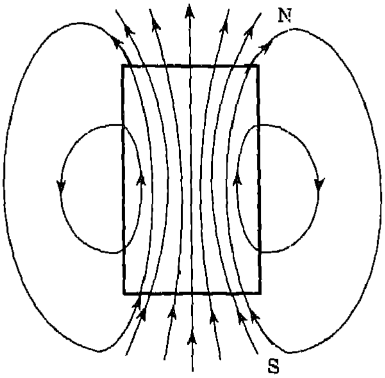


Figure 6.6: Field lines for a bar magnet

We learnt in Section 1.7 that electric lines of force begin and end on electric charges (or at infinity), and are otherwise continuous The absence of individual magnetic charges, expressed above in Gauss's Theorem, means that magnetic field lines never end *they are always closed continuous loops*. For the case of a solenoid this is seen in Fig 5.12. We show in Fig. 6 6 the field lines for a bar magnet· here too they are closed loops passing right through the magnet!

This discussion reinforces the observation that the simplest magnetic object is a magnetic *dipole*, in contrast to individual point electric charges. It is directly realised as a small planar current loop characterised by a *moment* whose dimensions (according to Eq. (5.34)) are:

$$\begin{aligned}[m] &= \text{dimensions of magnetic moment}\\ &= \text{ampere (metre)}^2\\ &= \text{joule/tesla.} \qquad (6.8)\end{aligned}$$

6.3 Magnetic field of the earth

The earth's magnetic field is approximately like that of a giant magnetic dipole. Its axis, however, does not coincide with the earth's axis of rotation, which determines the geographic North and South poles, N_g and S_g, and the geographic equator. The angle between the two axes is about 20°. An approximate picture of the magnetic field lines around the earth, showing the difference between geographic and magnetic axes and some other features described later, is given in Fig 6 7. The strength of this field is typically a fraction of 10^{-4}T, that is, a fraction of a gauss For example, at Delhi it is 0.35 gauss

The points where the magnetic axis cuts the earth's surface are called *magnetic poles*. The great circle on the earth's surface, perpendicular to this axis, is the *magnetic equator* It passes near Trivandrum in South India The magnetic equator divides the earth's surface into two (magnetic) hemispheres. As Fig. 6.7 shows, the field lines come out of the magnetic hemisphere containing S_g, and go into the one containing N_g

The North (seeking) pole of a freely suspended magnetic compass needle points approximately towards N_g. Therefore it is appropriate to call the magnetic pole near N_g the *South magnetic pole*, S_m. Then the one near S_g is the *North magnetic pole*, N_m. These have been marked in Fig. 6.7. At present S_m is located at a point in Northern Canada, longitude 96°W and latitude 70 5°N Therefore N_m is at the diametrically opposite point, 84°E 70 5°S (In these and several later statements we ignore departures from a perfect dipole field). The longitudes of S_m and N_m imply that the geographic and magnetic equators intersect at two points with longitudes 6°W, 174°E respectively.

★ Here are some interesting additional remarks. The earth's field is more or less what would have been produced by a giant fictitious bar magnet located deep inside the earth In actual fact, however, it is believed that the source of the field is ***not*** any large solid mass of magnetic material. Rather, it seems to be an electric current of molten charged metallic fluid inside the earth's core This core has a radius of about 3500 km, compared to the earth's radius of 6400 km

One of the most spectacular effects caused by the earth's field can be seen in the arctic regions, near the magnetic poles S_m and N_m. It is unfortunately not visible in India It is called the "aurora borealis" or "northern lights" in the north, and "aurora australis" in the south The aurora consists of enormous patterns of coloured lights, in the shape of giant curtains or flowing sarees, high up in the atmosphere Streams of charged particles coming from the sun – electrons and protons making up the so called ***solar wind*** – get trapped above the atmosphere near the two magnetic poles. On their way down they strike the molecules in the atmosphere, which in turn give out the light seen in the aurora

The approximate dipole form of the earth's field lasts to a distance of almost five times the earth's radius, that is, about 32,000 km Beyond this, the solar wind severely disturbs the dipole pattern. One can say that by then we are not alone! ★

Local features of earth's field - magnetic meridian, declination, dip angle. At any point P on the earth's surface we can geographically determine the following: the vertical axis or up-down direction; the horizontal plane, perpendicular to the vertical axis, and the geographic meridian plane, which is a plane containing the vertical axis and the meridian of longitude (north-south direction) through P. At the poles N_g and S_g, the meridian plane is undefined.

The magnetic field **B** at P similarly determines certain directions and planes there. The *magnetic meridian* is the vertical plane containing **B** and the vertical axis at P. The *magnetic declination* is the angle between the magnetic and geographic meridian planes

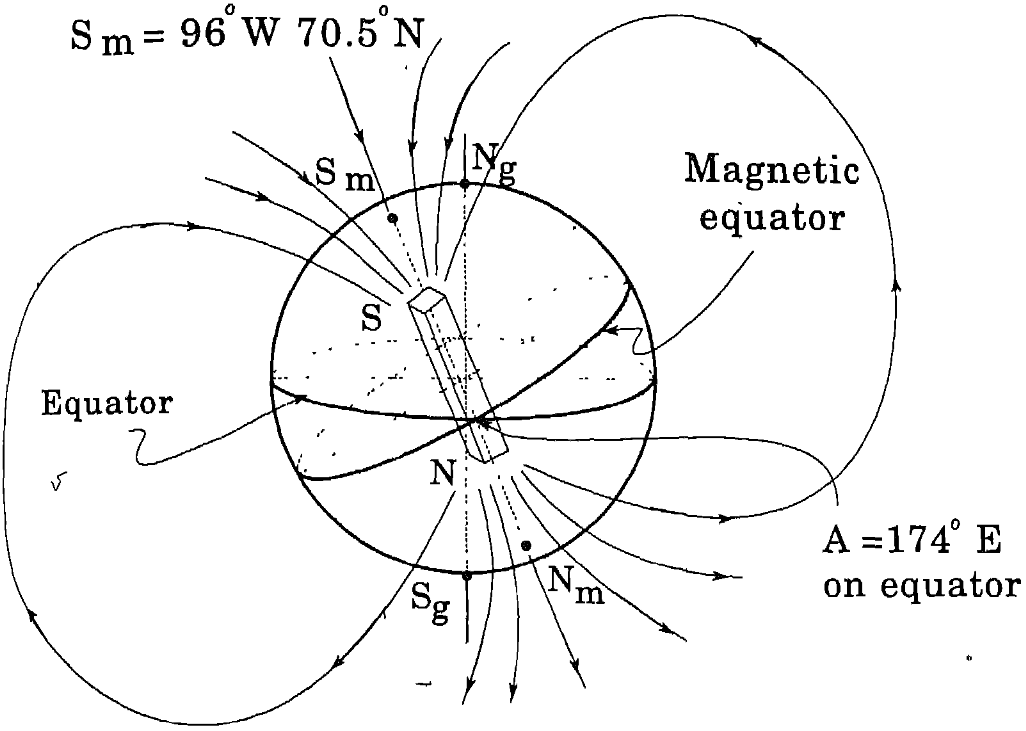
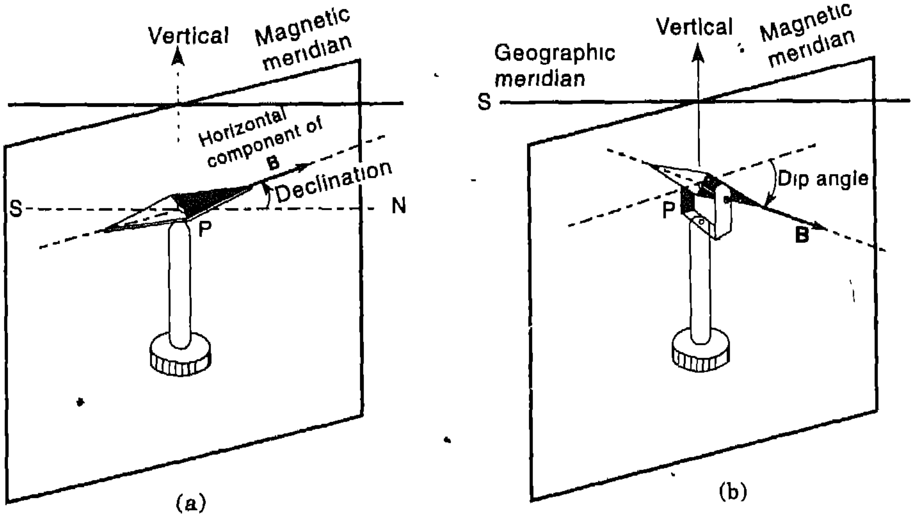


Figure 6.7: The earth's magnetic field



(a)

(b)

Figure 6.8: Magnetic meridian, declination and dip angle at a general point P on the earth's surface

at P We can define declination also in another way Express **B** as the resultant (vector sum) of a horizontal component (lying in the horizontal plane) and a vertical component (along the vertical axis) Then the declination is the angle between the horizontal component of **B** and the north-south direction.

Suppose we set up a compass needle at P free to rotate in the horizontal plane, about the vertical axis. Then it will come to rest pointing along the horizontal component of **B**. The angle it then makes with the north-south direction is the declination

Both magnetic meridian and declination are defined at all points except the magnetic poles S_m and N_m. At S_m, **B** points vertically downwards, while at N_m it points vertically upwards.

Now the vector **B** at P will generally not lie in the horizontal plane. The *dip angle* at P is the angle between **B** and its horizontal component, that is, between **B** and the horizontal plane It tells us by how much **B** goes down below (or comes up out of) the horizontal plane. To measure it we use a *dip circle* This is a compass needle pivoted about a horizontal axis perpendicular to the magnetic meridian, so the needle can move freely in that plane. (This means we must first determine the declination, then set up the dip circle). Ultimately it will come to rest pointing along **B**. The angle it then makes with the horizontal plane is the dip angle

The earth's field is quite irregular, and not as smooth as in the idealized Fig. 6 7 So the meridian, declination and dip vary irregularly from place to place, and also change with time. On the magnetic equator, the field **B** is horizontal, so the dip angle is zero At S_m and N_m it is 90° as **B** is vertical downwards and upwards respectively At Delhi, the declination is about 0°15′E and the dip angle is about 42° (below the horizontal).

The magnetic meridian, declination and dip angle are illustrated in Fig 6.8.

Time variations of the earth's field · In addition to changing irregularly from place to place, the earth's field also changes with time. This happens both in the short and in the long term. Thus in the short term the poles S_m and N_m keep shifting their positions. Detailed charts of the direction and strength of the field over the earth's surface have been made, but they have to be revised every thirty years or so.

Apart from this, geological studies of rocks have shown that major changes take place over hundreds of thousands of years. These are geological time scales Thus at irregular intervals the direction of the earth's field has even *reversed* itself! This happens once in 10^5 to 10^6 years These reversals mean that once in a long while the electric currents deep in the earth's core slow down, come to a halt, then pick up in the opposite direction!

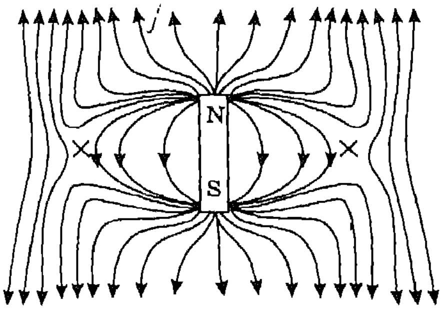
Example 6.4: A magnetic needle free to rotate about the vertical direction (compass) point 3 5° west of the geographic north Another magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 18° with the horizontal. The magnitude of the horizontal component of the earth's magnetic field at the place is known to be 0 40 G What is the direction and magnitude of the earth's magnetic field at the place?

Answer: The first observation (magnetic declination) specifies the vertical plane containing the magnetic field direction (magnetic meridian). The magnetic meridian at the place makes an angle of 3.5° with the geographic meridian towards the west In the magnetic meridian, if δ is the angle **B** makes with the horizontal component B', (this angle is called the angle of dip or inclination)

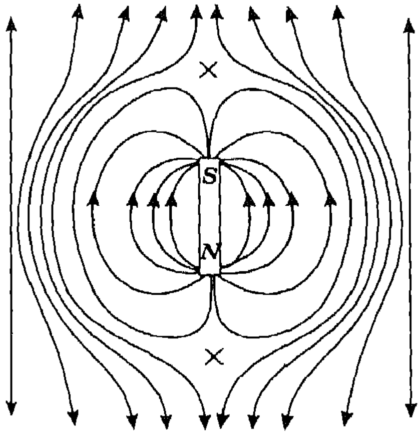
$$\cos\delta = \frac{B'}{B} \quad \text{or} \quad B = B'\sec\delta$$
$$= 0.40\sec 18^\circ$$
$$= 0.42\,\text{G}.$$

Thus **B** has a magnitude of 0 42G, and makes an angle of 18° with the horizontal direction (in the magnetic meridian) towards the ground.

Example 6.5: A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equatorial line (i.e., its normal bisector) at 12.5 cm from the centre of the magnet. The earth's magnetic field at the place is 0.38 G, and the angle of dip is zero. (a) What is the total magnetic field at points on the axis of the magnet located at the same distance (12.5cm) as the null-points from the centre? (b) Locate the null-points when the bar is turned around by 180°. Assume that the length of the magnet is negligible compared to the distance of the null-points from the centre of the magnet.



(a)



(b)

Answer: (a) From figure (a) it is clear that null-points are obtained on the normal bisector when the magnet's north and south poles face magnetic north and south respectively. Now, magnetic field on the normal bisector at a distance r from the centre is given by

$$\mathbf{B} = -\frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3} \tag{1}$$

provided r is much greater than the length of the magnet. (Formula strictly true only for a point dipole.)

At a null-point this field is balanced by the earth's field. That is

$$\frac{\mu_0}{4\pi}\frac{m}{r^3} = 0.38 \tag{2}$$

(Since the dip angle is zero, the horizontal component of the earth's field equals the field itself.)

Next, magnetic field due to a magnet on its axis at points distant r from the centre is given by

$$\mathbf{B} = \frac{\mu_0}{2\pi}\frac{\mathbf{m}}{r^3} \tag{3}$$

provided r is much greater than the length of the magnet. (Formula strictly true only for a point dipole.) From figure (a) it is clear that on the axis this field adds up to the earth's field.

Thus the total field at a point on the axis has a magnitude equal to

$$\left(\frac{\mu_0}{2\pi}\frac{m}{r^3} + 0.38\right)\,\text{G} \tag{4}$$

and direction along $\mathbf{m}$ (which is parallel to the earth's field in case (a)).

Thus for the same distance on the axis as the distance of the null-point, the total field, using Eqs (2) and (4) is

$$(0.76 + 0.38)G = 1.14\,\text{G directed along } \mathbf{m}$$

(Note, we did not require the given value of 12.5 cm for the null-point distance, except in so far that this was assumed to be much greater than the length of the magnet.)

(b) When the bar is turned around by 180°, the magnet's north and south poles face magnetic south and north respectively, i.e., in this case **m** is antiparallel to the earth's field From figure (b), it is clear that the null-points now lie on the axis of the magnet at a distance r' given by

$$\frac{\mu_0}{2\pi}\frac{m}{r'^3} = 0.38 \qquad (5)$$

where we have made use of formula (3). Comparison of Eqs (2) and (5) shows that

$$r'^3 = 2r^3$$

$$\text{or} \quad r' = \sqrt[3]{2}\,r$$

For $r = 12.5$cm, $r' = 15.7$cm.

6.4 Magnetic materials, the H-field

With respect to behaviour in a magnetic field, materials fall into three categories. diamagnetic, paramagnetic and ferromagnetic. We describe them briefly

Diamagnetic substances are those in which the individual atoms (or ions or molecules) do not possess any net magnetic moment on their own. When a sample of such material is placed in an external (uniform) field **B**, the effect is to create in each atom (or ion or molecule) a small magnetic moment proportional to **B** but *pointing in the opposite direction* This happens as a result of the Lorentz force, due to **B**, acting on each circulating electron in the atom. These induced magnetic moments then create their own field which partially cancels the external field. Thus the field within the sample is decreased from **B** to a smaller value The reduction is generally very slight, from the external value to one less than it by about one part in 10^5. We say that a diamagnetic substance *expels* field lines, and this is shown in Fig 6 9a. If such a substance is placed in a nonuniform external field, the tendency is to move from the high to the low field region

Paramagnetic substances are those in which each atom (or ion or molecule) has a net nonzero magnetic moment on its own. When such a material is placed in an external **B**-field, the latter tends to align the individual atomic moments in its own direction However, thermal agitation tends to knock the dipoles into random directions. For strong enough **B** and/or low enough temperature, there is a net average magnetic dipole moment density *in the same direction* as **B**. So the field within the sample gets enhanced, and we have a pulling in of field lines as shown in Fig 6.9b. If a paramagnetic sample is placed in a nonuniform field, it tends to move from the low to the high field region In both respects, then, the behaviour is opposite to the diamagnetic case.

By far the most interesting class of materials is the *ferromagnetic* type In such materials – mainly iron, nickel and cobalt and some special alloys – each individual atom has a nonzero magnetic moment, as in a paramagnetic substance. However, by means of a delicate quantum mechanical effect, they interact with one another in such a way as to spontaneously align themselves in a common direction over macroscopic volumes In the first instance this effect leads to small *domains* of uniform magnetization A typical domain may contain some 10^{11} atomic moments, corresponding to a volume of about 10^{-12}cm^3 or a size of about 10^{-4}cm Domain sizes range from about 10^{-5} cm to 10^{-2}cm, depending on the magnetic material All the atomic moments within a domain are lined up, leading to a net moment, but this varies randomly from domain to domain, as shown in Fig 6 10 So there is no bulk magnetic moment

When we magnetize a ferromagnetic material, which has domains pointing in random directions so that there is no net magnetization, what happens is roughly the following. The domains oriented in the direction of the magnetizing field grow in size at the response of others; and even the others change their orientation so as to point along the magnetizing field. This arrangement persists even after the magnetizing field is removed, in permanent or "hard" magnetic materials In others, the domains reorient randomly on removing the magnetizing agent. These are "soft" materials used for magnets, etc. This leads to spontaneous bulk magnetization. This is the origin of macroscopic magnets, both the natu-

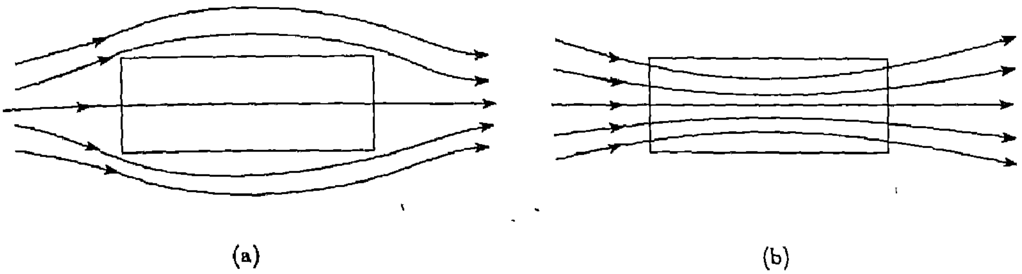

(a) (b)

Figure 6.9: Behaviour of (a) diamagnetic (b) paramagnetic substance in a magnetic field

Table 6.1: Properties of magnetic materials and comparison with dielectrics

	Examples	Individual atom/ molecule	Behaviour in external field	Behaviour in nonuniform field	Comparison to dielectrics
Diamagnetic substances	Bismuth, copper, lead, water, sodium chloride	No magnetic moment	Field reduced, field lines repelled	Movement from high to low field region	Nonpolar type
Paramagnetic substances	Aluminium, sodium, copper chloride, liquid oxygen	Nonzero magnetic moment	Field enhanced, field lines concentrated	Movement from low to high field region	Polar type
Ferromagnetic substances	Iron, nickel, cobalt	Nonzero moments organized into domains	Field strongly enhanced, field lines highly concentrated	Movement from low to high field region	Ferroelectric type

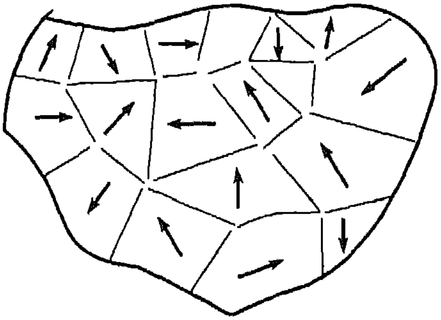


Figure 6.10: Randomly oriented domains in an unmagnetized ferromagnetic material

rally occurring lodestone and artificially made permanent magnets. The bulk magnetization disappears beyond a characteristic temperature called the Curie temperature; then the substance becomes simply paramagnetic. For iron, this temperature is 1043 K.

In Table 6.1 we list the main properties of these three kinds of magnetic materials, giving also some examples of each. In the last column a comparison with properties of dielectric substances is indicated.

Diamagnetism is always present and always weak. It is hard to detect if the substance is para or ferromagnetic. In the comparison with dielectrics given in the table, we must note an important point: when placed in an external electric field, both nonpolar and polar dielectrics *decrease* the field inside the sample, unlike the opposite behaviours of dia- and paramagnets in a magnetic field. This is why, as seen in Chapter 2, the capacitance of a capacitor is always increased by inserting a dielectric in it, whether polar or nonpolar. Ferroelectric substances are the electric analogues of ferromagnetic ones.

Ferromagnets are made of special alloys chosen for their specific properties. The material is placed in the field of a solenoid, or any other magnetic field, long enough to get magnetized, and then slowly removed from the field. An alloy (Alnico) of iron, aluminium, nickel, cobalt and copper is used for making permanent magnets. Such magnets are used in various instruments, such as galvanometers, and also in industry in magnetic separators. For certain other applications one needs "soft" ferromagnets (rather than "hard" or permanent ones which can retain their magnetization). These can be quickly, easily and highly magnetized, but they can change, lose, or reverse their magnetization equally quickly when the external field changes. Soft magnets are often made using an alloy of iron and nickel, and are needed in transformers, motors, recording heads in tape recorders, etc.

★ The proper understanding of magnetism and its microscopic origin depends on quantum mechanics, but we will be content with a classical description. As mentioned in Chapter 5 and earlier in this Chapter, one source of magnetic moment in an atom is the circulating electronic motion. This is like a small loop of electric current and so can be pictured as a tiny magnetic dipole. The other comes from the intrinsic spin, which is like a spinning top-like motion of an electron around its own axis. This is in fact the more important source. Thus each electron has both an orbital and a spin magnetic moment.

In an atom or ion with a closed shell, and in many molecules, the individual magnetic moments cancel out, i.e., vectorially add to zero. These are diamagnetic. If the outermost shell of electrons in the atom or ion is only partially filled or as in some molecules (like oxygen) there is a net magnetic moment, or in metallic solids (which have free electrons with spin magnetic moment) the substance containing such atoms, ions, molecules or free electrons is paramagnetic.

When any substance is placed in an external magnetic field **B**, there is *always* a Lorentz force that acts on the electrons circulating around the nuclei. One can show that the effect of this Lorentz force is to create a small magnetic moment tending to cancel **B**. This is why diamagnetism, though weak, is present in all materials.

Turning to ferromagnetic materials, the direct interaction between atomic magnetic moments is too weak to lead to an alignment and formation of magnetic domains. What comes into play is a stronger indirect interaction which can only be explained using quantum mechanics. In contrast, in ferroelectric substances it is the direct interaction between electric dipoles, plus some other effects, that leads to a co-operative large scale alignment of all the dipoles ★

The **H**-*field*: Suppose we have a magnetized body, which may be dia, para or ferromagnetic. At each point of the body there is some average macroscopic magnetic moment density, **M** say. (According to Eq. (6.6), **M** has dimensions, ampere/metre). It is now possible to replace this **M** by an *equivalent electric current density* $\mathbf{J}_M$, either in the interior of the body or on its surface or both, such that the magnetic field produced at any point by the magnetized body is the same as that produced by $\mathbf{J}_M$ via Ampere's Law. $\mathbf{J}_M$ is determined by **M**. The *total* magnetic field **B** at any point is due to *all* currents taken together. "true" or "free" currents flowing in coils or conductors or free space, and the above equivalent current density $\mathbf{J}_M$ representing magnetized bodies.

It now turns out that there is a combination of **B** and **M** which is directly related to the "free" currents alone. It is given by

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad (6.9)$$

From a practical point of view, this **H** field is very important, especially in dealing with ferromagnetic materials. It is what we can directly control by adjusting the free current, say in coils or solenoids. Except in magnetized bodies, **H** and **B** are simply proportional

The field **H** is called the *magnetic intensity*. It has the same dimensions, ampere/metre, as magnetic moment density **M**. Thus the fields **B** and **H** have different dimensions, since μ_0 carries dimensions tesla × metre/ampere.

We mentioned in Eq (6.4) that an older unit for magnetic field **B** is the gauss, which is 10^{-4} T The corresponding older unit for magnetic intensity **H** is the *oersted* one oersted is one gauss divided by μ_0 So the relation of the oersted to SI units is

$$\begin{aligned} 1 \text{ oersted} &= 1 \text{ gauss}/\mu_0 \\ &= 10^{-4}\ \text{T}/\mu_0 \\ &= \frac{10^{-4}}{4\pi \times 10^{-7}} \text{ amp/metre} \\ &\simeq 80 \text{ amp/metre.} \end{aligned} \qquad (6.10)$$

★

In nonferromagnetic substances, if the fields are not too strong, the three vectors **B**, **M** and **H** are multiples of one another at each point. It is conventional to express **M** in terms of **H** rather than of **B**. So for a linear nonferromagnetic material we have

$$\begin{aligned} \mathbf{M} &= \chi_m \mathbf{H}, \\ \mathbf{B} &= \mu_0(1+\chi_m)\mathbf{H} \\ &= \mu_0 \kappa_m \mathbf{H} \\ &= \mu \mathbf{H}, \\ \kappa_m &= 1+\chi_m,\ \mu = \mu_0 \kappa_m \end{aligned} \qquad (6.11)$$

Three convenient constants-χ_m, κ_m and μ-have been introduced here, but only one is independent. The dimensionless constant χ_m is the *magnetic susceptibility* of the material. It is positive for para and negative for dia magnetic materials. Correspondingly, the dimensionless *relative magnetic permeability* κ_m is slightly more or slightly less than unity for para and dia magnetic substances respectively. Finally, μ is the *magnetic permeability* of the material. So one calls μ_0 *the magnetic permeability* of vacuum.

Example 6.6: The core of a toroid having 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field in the core for a current of 0.70A is 2.5T. What is the relative permeability of the core?

Answer: The magnetic field in the empty space enclosed by the windings of the toroid

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★ Just as different books use different names for **B**, they do so for **H** too. Some use the names magnetic induction and magnetic field strength for **B** and **H** respectively Others prefer magnetic field for **B** and magnetic intensity for **H**, as we have done is given by

$$B = \mu_0 n I$$

where n is the number of turns per unit length and I is the current. If the space is filled by a core of permeability μ the formula modifies to

$$B = \mu n I$$

Here $B = 2.5\,\mathrm{T}$, $I = 0.70\,\mathrm{A}$

$$n = \frac{3000}{2\pi \times 11.5 \times 10^{-2}}\mathrm{m}^{-1}$$

where we have ignored the variation of B across the cross-section of the toroid and taken the radius of the toroid to be the mean of inner and outer radii. We then have

$$\mu = \frac{B}{nI} = 8.61 \times 10^{-4}\,\mathrm{TmA}^{-1}$$

The relative permeability of the core is defined to be μ/μ_0. With $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{TmA}^{-1}$, we get

$$\frac{\mu}{\mu_0} = 685.$$

★ *Comparison to description of dielectrics*: It is helpful to briefly compare the use of the three vectors **B**, **M** and **H** here with the use of **E**, **P** and **D** in describing the behaviour of dielectric materials. As we learnt in Section 2.8, when a dielectric substance is placed in an electric field, a polarization or electric dipole moment density **P** is created within the material To calculate the electric field created by **P**, we can replace it by an equivalent or effective surface charge density at the surface of the dielectric (If **P** is not uniform, there is also an equivalent volume charge density inside the dielectric) The electric displacement vector **D** defined in Eq (2 94) is a combination of the true total electric field **E** and the polarization **P**, which depends only on the free charges on conductors or in free space.

We can now see that the creation of a magnetic moment density **M** in a magnetic material when placed in a magnetic field is similar to the creation of polarization **P** in a dielectric in an electric field. The difference is that in the former case **M** can sometimes be opposite to the external field This is what a diamagnetic substance does, and for this there is no dielectric analogue.

From a basic point of view the two fields **E** and **B** are analogous. In material media we find it useful to introduce fields **D** and **H**, in a way to remove the contributions of polarization **P** and magnetization **M** to **E** and **B** respectively So **D** and **H** are analogous They are the fields due respectively to external or free charges (**D**) and external currents (**H**). In free space we have $\mathbf{D} = \epsilon_0\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu_0$.

Hysteresis: In ferromagnetic materials, for instance soft iron, the relation between **B** and **H** is quite complicated, and not linear. It can even depend on the history of the sample! The relative magnetic permeability κ_m is of the order of 10^4, far greater than values just above unity for paramagnetic substances

Suppose we take a core of unmagnetized soft iron, and pass a current through a coil wound around it. To begin with the fields **H** and $\mathbf{B}/\mu_0$ in the iron both increase steadily, but the latter is much larger than the former This is because even with a moderate current in the coil, leading to a moderate **H** field, all the tiny magnetic moments in different domains line up more or less along **H** This leads to a large magnetization **M** in the iron parallel to **H**. In turn this is equivalent to a large current density $\mathbf{J}_M$, and since this too contributes to the **B** field, this field is much larger than $\mu_0\mathbf{H}$ The initial build-up of **B**, as **H** is increased, is shown by the solid curve in Fig. 6.11, from the origin O outwards.

After a while, however, the curve of **B** against **H** "saturates", that is, **M** reaches a maximum. It does not increase any more with increasing **H**. (So actually it is **M** that saturates, but since $\mathbf{B}/\mu_0$ is so much larger than **H**, we may effectively say that **B** saturates. We see for example from Fig 6.11 that $\mathbf{B} \simeq 1.5\mathrm{T}$ or $(\mathbf{B}/\mu_0) \simeq 1.25 \times 10^6\,\mathrm{A/m}$, **H** is only about 200A/m. This corresponds to the flattening out of the upper part of the solid curve in Fig 6 11, near point a. It evidently happens when all the tiny magnetic moments in individual domains are essentially perfectly lined up along **H**

If we now slowly decrease the (free) current in the coil and reduce **H** to zero, some magnetization **M** remains in the iron, so **B** does not fall to zero! We are at the point b in Fig. 6 11 To make **B** vanish, we have to ***reverse*** the current in the coil and build up **H** in the opposite direction to a certain strength — this corresponds to point c

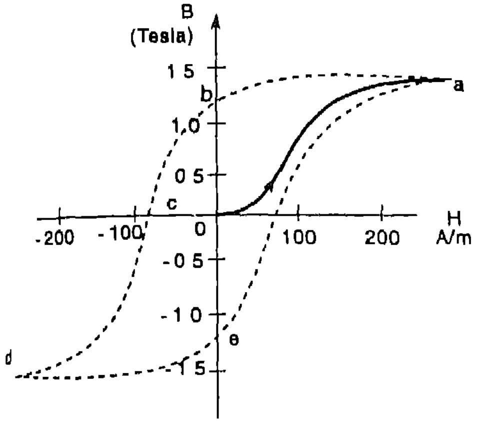


Figure 6.11: The B - H cycle in ferromagnetic materials - the hysteresis loop

In Fig. 6.11 Saturation in the opposite direction is reached with further increase in the magnitude of **H**, by then, namely point d in Fig. 6.11, the magnetization **M** would be opposite to what it was at point a. Now a gradual decrease of **H** to zero and then a build up in the original direction is described by the lower dotted curve in Fig. 6.11, linking point d via point e to point a.

So one complete cycle of change in **H** - the thing we can control by changing the strength and direction of the current in the coil — makes **B** follow a closed loop abcdea, not up and down a curve in the **B** — **H** diagram. For each value of **H**, the value of **B** is not unique but depends on the previous history of the sample. This phenomenon is called *hysteresis*. ★

6.5 The tangent galvanometer

In Section 5.5 we described how electric currents can be measured using a moving coil galvanometer. Taking advantage of the earth's magnetic field, another instrument called the *tangent galvanometer* can be used for the same purpose. It consists of a circular coil of n turns and radius a, say, through which we pass the current to be measured. The coil is mounted so as to lie in the magnetic meridian. At its centre is a magnetic compass needle pivoted about a vertical axis, and able to swing freely in the horizontal plane. The arrangement – as viewed from "above" – is shown in Fig. 6.12.

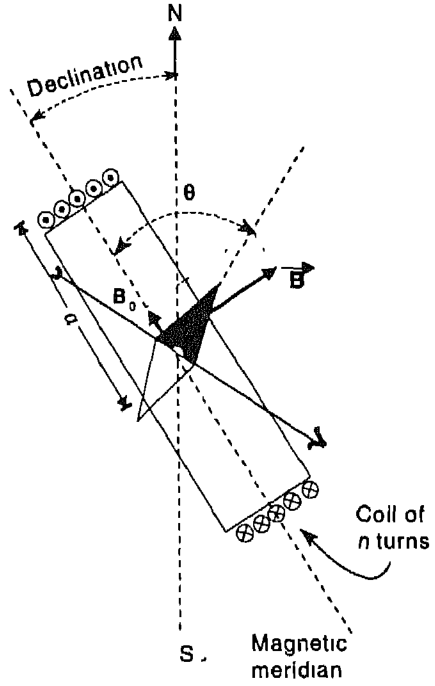


Figure 6.12: The tangent galvanometer arrangement

When there is no current, the needle rests in the plane of the coil, namely the magnetic meridian. It lies parallel to the horizontal component, $\mathbf{B}_0$ say, of the earth's field. If now we pass a current I we know from Section 5.2 that a horizontal magnetic field **B**, perpendicular to the plane of the coil, is produced at the centre. Its magnitude is given by Eq. (5.12) (with the replacements $r \to 0$, $I \to nI$):

$$|\mathbf{B}| = \mu_0 nI/2a. \tag{6.12}$$

Clearly $\mathbf{B}_0$ and **B** are mutually perpendicular. The needle will now feel torques due to both fields. It will thus come to rest pointing along the resultant of $\mathbf{B}_0$ and **B**. Thus it will swing away from the magnetic meridian by an angle θ given by

$$\tan\theta = |\mathbf{B}|/|\mathbf{B}_0| = \mu_0 nI/2a|\mathbf{B}_0|. \qquad (6.13)$$

This can be solved for the current I:

$$I = |\mathbf{B}_0|\,\frac{2a}{n\mu_0}\,\tan\theta. \qquad (6.14)$$

The name of this instrument is explained by this expression. While $|\mathbf{B}_0|$ depends on the location of the instrument, $2a/n\mu_0$ is a constant characteristic of the instrument itself.

To use the tangent galvanometer, one has to determine the magnetic meridian and the horizontal part of the earth's field quite accurately, and these may vary in time. In addition it is easily affected by magnetic materials or current carrying conductors placed nearby. Therefore it is not a very useful instrument

Summary

1. Simple features of magnetism have been known for a long time such as· magnets attract small pieces of iron, a freely suspended magnet points in the N-S direction; like poles of two magnets repel while unlike poles attract each other; breaking a long bar magnet into two pieces produces two new magnets i.e. North and South poles of a magnet cannot be isolated.

2. After Oersted's discovery, it was realized that magnetism was an effect of electric charges in motion Ampere suggested that the elementary 'atomic' magnets in magnetic materials were tiny circulating currents. In a bar magnet, they are aligned *in the same direction*. This picture (with some modifications) is quite correct (see 7).

3. In a magnetic field $\mathbf{B}$, the torque on a magnet of magnetic moment $\mathbf{m}$ is given by $\tau = \mathbf{m} \times \mathbf{B}$. Potential energy is given by $U = -\mathbf{m} \cdot \mathbf{B}$.

4 Magnetic dipoles are the most elementary sources of magnetic field i e. there are no magnetic monopoles This fact implies that the flux of magnet i.e , field through any *closed* surface is always zero. The unit of flux is Weber.

   $1\ \text{Wb} = 1\ \text{Tm}^2$

5 The strength of the earth's magnetic field is about 10^{-4}Wb/m^2. Two angles are needed to specify its direction. The angle between the geographical north and the direction of the compass's north pole is called the *magnetic declination* at that point This determines the vertical plane of $\mathbf{B}$. In this plane, the angle which $\mathbf{B}$ makes with the horizontal direction is called the dip angle.

6. A tangent galvanometer consists of a circular coil of radius a and n turns carrying current I to be measured. It is placed in the vertical plane of the earth's magnetic field $\mathbf{B}_0$. The angle θ which a compass needle at the centre of the coil makes with the plane of the coil is related to I by

$$I = B_0 \frac{2a}{n\mu_o} \tan\theta$$

7 A circulating electron in an atom has a magnetic moment both due to its orbital motion and intrinsic spin

Diamagnetism: In atoms of some materials (e.g. Bi, Cu, Pb), magnetic moments due to different electrons cancel out. When such a substance is placed in $\mathbf{B}$, an orbital magnetic moment is induced, proportional to $\mathbf{B}$ but *in opposite direction*. Diamagnetism is a weak but universal effect.

Paramagnetism: Atoms or molecules of some materials (e.g. Al, Na, $CuCl_2$) have a non-zero magnetic moment. When such a substance is placed in $\mathbf{B}$, the latter tries to align individual magnetic dipoles in its direction, while thermal motion randomizes their directions. There is a net magnetization in *the same direction* as $\mathbf{B}$.

Ferromagnetism: In some materials (e.g. Fe, Ni, Co) the non-zero atomic magnetic dipoles interact with one another and spontaneously produce magnetization over small domains of sizes of a few microns. The different domains may have random orientations resulting in zero bulk magnetic moment. Macroscopic magnetization arises when the domains are made to align themselves in a common direction. In ferromagnetic materials, for each value of magnetization, there is no unique value of the field **B**, it depends on the history by which the magnetization is reached This is the phenomenon of ***hysteresis***

Exercises

6.1 A magnetized needle of magnetic moment 4.8×10^{-2} J T^{-1} is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. What is the torque acting on the needle?

6.2 In 6.1, suppose the needle is pivoted through its centre of mass and is free to rotate in the plane of magnetic field. If the needle is displaced slightly from its stable equilibrium and released, predict the angular frequency of its oscillations (of small amplitude). The moment of inertia of the needle about its axis of rotation is given to be 2.25×10^{-5} kg m^2.

6.3 A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

6.4 A short bar magnet of magnetic moment $m = 0.32$ J T^{-1} is placed in a uniform external magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientations would correspond to its (i) stable and (ii) unstable equilibrium ? What is the potential energy of the magnet in each case?

6.5 A planar loop of irregular shape encloses an area of 7.5×10^{-4} m^2, and carries a current of 12 A. The sense of flow of current appears to be clockwise to an observer. What is the magnitude and direction of the magnetic moment vector associated with the current loop ?

6.6 A circular coil of 300 turns and diameter 14 cm carries a current of 15 A. What is the magnitude of magnetic moment associated with the coil?

6.7 A closely wound solenoid of 800 turns and area of cross-section 2.5×10^{-4} m^2 carries a current of 3.0 A. Explain in what sense does the solenoid act like a bar magnet. What is its associated magnetic moment?

6.8 If the solenoid in 6.7 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of the torque on the solenoid when its axis makes an angle of 30° with the direction of the applied field?

6.9 A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required to turn the magnet so as to align its magnetic moment (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

6.10 A closely wound solenoid of 2000 turns and area of cross-section 1.6×10^{-4} m^2, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

6.11 A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the strength of the earth's magnetic field at the place.

6.12 At a certain location in Africa, a compass points 12° West of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

6.13 A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (i) the axis (ii) the equatorial line (normal bisector) of the magnet.

6.14 A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0 36 G and the angle of dip is zero What is the total magnetic field on the normal bisector of the magnet at the same distance as the null-points (i.e 14 cm) from the centre of the magnet?

6.15 If the bar magnet in 6.14 is turned around by 180°; where will the new null-points be located?

6.16 A short bar magnet of magnetic moment 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet on (a) its normal bisector, (b) its axis is the resultant field inclined at 45° with the earth's field. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

6.17 A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

Additional Exercises

6.18 A long straight horizontal cable carries a current of 2.5 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (Ignore the thickness of the cable)

6.19 A telephone cable at a place has four long straight horizontal wires carrying a current of 1 0 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G, and the angle of dip is 35°. The magnetic declination is nearly zero. What are the resultant magnetic fields at points

4.0 cm below, and above the cable?

6.20 A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45° with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.

(a) Determine the horizontal component of the earth's magnetic field at the location.

(b) The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the place to be zero

6.21 A current-carrying planar loop suspended in a vertical plane normal to the magnetic meridian is in stable equilibrium. The horizontal component of the earth's field is 0.32 G. Another horizontal magnetic field of magnitude 0.48 G parallel to the plane of the loop is set up along magnetic west to east. Specify the new orientation of the loop when it comes to stable equilibrium. What happens if the current in the loop is reversed?

6.22 A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60°, and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

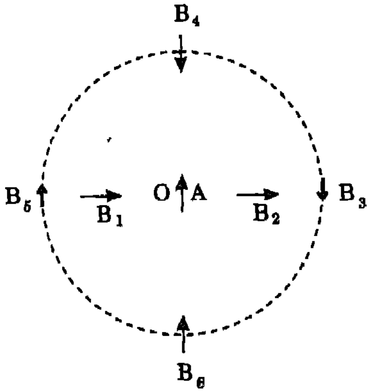
6.23 A monoenergetic (18 keV) electron beam initially in the horizontal direction is subject to a horizontal magnetic field of 0.40 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C). [**Note:** Data in this exercise are so chosen that the answer will give you an idea of the effect of earth's magnetic field on the motion of the electron beam from the electron gun to the screen in a TV set].

6.24 The figure shows a small magnetized needle A placed at a point O. The arrow shows the direction of its magnetic moment. The other arrows show different positions (and orientations of the magnetic moment) of another identical magnetized needle B.

(a) In which configurations is the system not in equilibrium?

(b) In which configurations is the system in (i) stable, and (ii) unstable equilibrium?

(c) Which configuration corresponds to the lowest potential energy among all the configurations shown?



6.25 A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5 $\times 10^{-23}$JT^{-1} The sample is placed under a homogeneous magnetic field of 0.84 T, and cooled to a temperature of 4 2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K ? (assume Curie's law)

6.26 A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field (**B**) in the core for a magnetizing current of 1 2 A?

6.27 The magnetic moment vectors $\boldsymbol{\mu}_s$ and $\boldsymbol{\mu}_l$ associated with the intrinsic spin angular momentum **S** and orbital angular momentum **l** respectively of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by

$$\boldsymbol{\mu}_s = -(e/m)\,\mathbf{S},\ \boldsymbol{\mu}_l = -(e/2m)\,\mathbf{l}$$

Which of these relations is in accordance with the result expected 'classically'? Outline the derivation of the classical result.

6.28 Answer the following questions:

(a) What happens if a bar magnet is cut into two pieces

(i) transverse to its length (ii) along its length?

(b) What happens if an iron bar magnet is melted. Does it retain its magnetism?

(c) A magnetized needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?

(d) Must every magnetic field configuration have a north pole and a south pole? What about the field due to a toroid?

(e) Can you think of a magnetic field configuration with three poles?

(f) Two identical-locking iron bars A and B are given, one of which is definitely known to be magnetized (We do not know which one). How would one ascertain whether or not both are magnetized? If only one is magnetized, how does one ascertain which one? [Use nothing else but the two bars A and B].

6.29 Answer the following questions regarding earth's magnetism.

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18°. Would you expect a greater or lesser dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) Which direction would a compass point to if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8 $\times 10^{22}$ J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

6.30 Answer the following questions:

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e the source of energy) to sustain these currents?

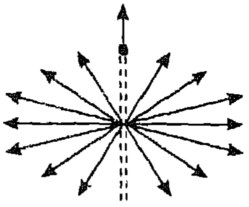
(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

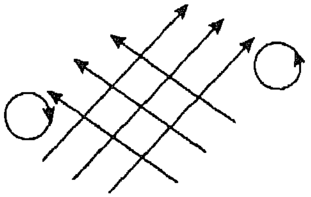
(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T Can such a weak field be of any significant consequence? Explain.

[**Note:** Exercise 6.30 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]

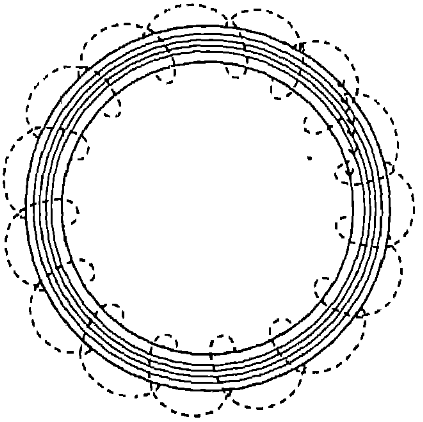
6.31 Many of the figures below show magnetic field lines *wrongly* (thick lines in the figures). Point out what is wrong with them. Some of them may describe electrostatic field lines correctly. Point out which ones Remember we are talking of only *static* electric or magnetic fields.



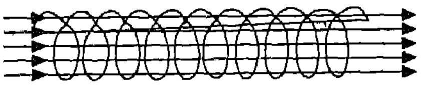
(a)



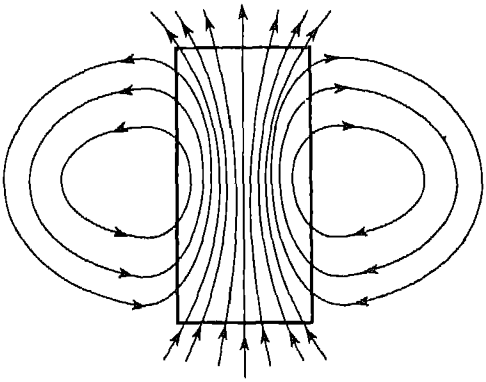
(b)



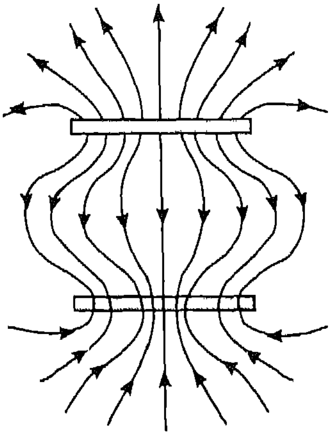
(c)



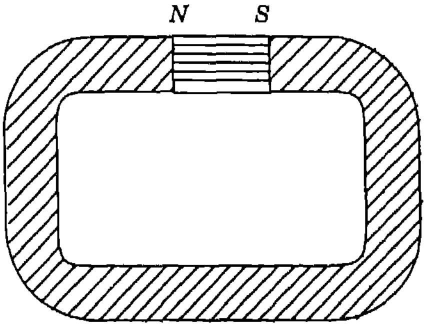
(d)



(e)



(f)



(g)

6.32 Answer the following questions carefully:

(a) Magnetic field lines show the direction (at every point) which a small magnetized needle takes up (at that point). Do the magnetic field lines also represent the '*lines of force*' on a moving charged particle at every point?

(b) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

(c) If magnetic monopoles existed, how would Gauss's law of magnetism be modified?

(d) Does a bar magnet exert a torqe on itself due to its own field? Does one element of a current-carrying wire exert a force on another element of the *same wire*?

(e) Magnetic field arises due to charges in motion. Can a system have magnetic moments even though its net charge is zero?

(f) Magnetic force is always normal to the velocity of a charge and therefore does no work. An iron nail held near a magnet, when released, increases its kinetic energy as it moves to cling to the magnet What agency is responsible for this increase in kinetic energy if not the magnetic field?

6.33 Answer the following questions:

(a) Why does a paramagnetic sample display greater magnetisation (for the same magnetizing field) when cooled?
(b) Why is diamagnetism, in contrast, almost independent of temperature?

(c) If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

(d) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point). Why?

(f) Would the maximum possible magnetization of a paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?

6.34 Answer the following questions·

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetization curve of a ferromagnet.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetization, which piece will dissipate a greater heat energy?

(c) 'A system displaying a hysteresis loop such as a ferromagnet is a device for storing memory?' Explain the meaning of this statement.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?

(e) A certain region of space is to be shielded from magnetic fields. Suggest a method

Appendix A

THE INTERNATIONAL SYSTEM OF UNITS (SI)

THE SI BASE UNITS

QUANTITY	NAME	SYMBOL	DEFINITION
Length	metre	m	" the length of the path traveled by light in vacuum in 1/299, 792,458 of a second " (1983)
mass	kilogram	kg	" this prototype [a platinum-iridium cylinder] shall henceforth be considered to be the unit of mass " (1989)
time	second	s	" the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom " (1967)
electric current	ampere	A	".. that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length " (1946)
thermodynamic temperature	kelvin	K	" the fraction 1/273 16 of the thermodynamic temperature of the triple point of water." (1967)
amount of substance	mole	mol	" the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12 " (1971)
luminous intensity	candela	cd	" the luminous intensity, in the perpendicular direction, of a surface of 1/600,000 square metre of a blackbody at the temperature of freezing platinum under a pressure of 101,325 newtons per square metre " (1967)

SOME SI DERIVED UNITS WITH SPECIAL NAMES

Quantity	SI Unit			
	Name	Symbol	Expression in terms of other units	Expression in terms of SI base units
frequency	hertz	Hz	—	s^{-1}
force	newton	N	—	$kg.m/s^2$
pressure	pascal	Pa	N/m^2	$kg/m.s^2$
energy, work, quantity of heat	joule	J	N m	$kg.m^2/s^2$
power, radiant flux	watt	W	J/s	$kg.m^2/s^3$
quantity of electricity, electric charge	coulomb	C	—	A.s
electric potential, potential difference, electromotive force	volt	V	W/A	$kg.m^2/A.s^3$
capacitance	farad	F	C/V	$A^2.s^4/kg.m^2$
electric resistance	ohm	Ω	V/A	$kg.m^2/A^2.s^3$
conductance	siemens	S	A/V	$A^2\ s^3/kg.m^2$
magnetic flux	weber	Wb	V s	$kg\ m^2/A\ s^3$
magnetic field	tesla	T	Wb/m^2	$kg/A.s^2$
inductance	henry	H	Wb/A	$kg.m^2/A^2.s^2$

Appendix B

CONVERSION FACTORS

Conversion factors are written as equations for simplicity; relations marked with an asterisk are exact.

Length

1 km = 0.6215 mi
1 mi = 1.609 km
1 m = 1.0936 yd = 3.281 ft = 39.37 in
*1 in = 2.54 cm
*1 ft = 12 in = 30.48 cm
*1 yd = 3 ft = 91.44 cm
1 lightyear = 1 c·y = 9.461×10^{15} m
*1 Å = 0.1 nm

Area

*1 $m^2 = 10^4\ cm^2$
1 $km^2 = 0.3861\ mi^2$ = 247.1 acres
*1 $in^2 = 6.4516\ cm^2$
1 $ft^2 = 9.29 \times 10^{-2}\ m^2$
1 $m^2 = 10.76\ ft^2$
*1 acre = 43,560 ft^2
1 mi^2 = 460 acres = 2.590 km^2

Volume

*1 $m^3 = 10^6\ cm^3$
*1 L = 1000 $cm^3 = 10^{-3}\ m^3$
1 gal = 3.786 L
1 gal = 4 qt = 8 pt = 128 oz = 231 in^3
1 $in^3 = 16.39\ cm^3$
1 $ft^3 = 1728\ in^3$ = 28.32 L = $2.832 \times 10^4\ cm^3$

Time

*1 h = 60 min = 3.6 ks
*1 d = 24 h = 1440 min = 86.4 ks
1 y = 365.24 d = 31.56 Ms

Speed

1 km/h = 0.2778 m/s = 0.6215 mi/h
1 mi/h = 0.4470 m/s = 1.609 km/h
1 mi/h = 1.467 ft/s

Angle and Angular Speed

*π rad = 180°
1 rad = 57.30°
1° = 1.745×10^{-2} rad
1 rev/min = 0.1047 rad/s
1 rad/s = 9.549 rev/min

Mass

*1 kg = 1000 g
*1 tonne = 1000 kg = 1 Mg
1 u = 1.6606×10^{-27} kg
1 kg = 6.022×10^{23} u
1 slug = 14.59 kg
1 kg = 6.852×10^{-2} slug
1 u = 931.50 MeV/c^2

Density

*1 g/cm^3 = 1000 kg/m^3 = 1 kg/L
(1 g/cm^3)g = 62.4 lb/ft^3

Force

1 N = 0.2248 lb = 10^5 dyn
1 lb = 4.4482 N
(1 kg)g = 2.2046 lb

Pressure

*1 Pa = 1 N/m^2
*1 atm = 101.325 kPa = 1.01325 bars

$$\begin{aligned} 1\ \text{atm} &= 14.7\ \text{lb/in}^2 = 760\ \text{mmHg} \\ &= 29.9\ \text{inHg} = 33.8\ \text{ftH}_2\text{O} \end{aligned}$$

1 lb/in^2 = 6.895 kPa
1 torr = 1 mmHg = 133.32 Pa
1 bar = 100 kPa

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$$

BINOMIAL THEOREM

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad (x^2 < 1)$$

$$(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} + \cdots \qquad (x^2 < 1)$$

EXPONENTIAL EXPANSION

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

LOGARITHMIC EXPANSION

$$\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots \qquad (|x| < 1)$$

TRIGONOMETRIC EXPANSIONS

(θ in radians)

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} - \cdots$$

PRODUCTS OF VECTORS

Let $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ be unit vectors in the x, y, and z directions. Then

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1, \quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0,$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0,$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}, \ \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}, \ \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}.$$

Any vector **a** with components a_x, a_y, and a_z along the x, y, and z axes can be written,

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}.$$

Let **a**, **b** and **c** be arbitary vectors with magnitudes a, b, and c. Then

$$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (\mathbf{a}\times\mathbf{b}) + (\mathbf{a}\times\mathbf{c})$$

$$(s\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(s\mathbf{b}) = s(\mathbf{a}\times\mathbf{b}) \qquad (s = \text{a scalar}).$$

Let θ be the smaller of the two angles betwen **a** and **b**. Then

$$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a} = a_xb_x + a_yb_y + a_zb_z = ab\cos\theta$$

$$\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_yb_z - b_ya_z)\hat{\mathbf{i}} + (a_zb_x - b_za_x)\hat{\mathbf{j}} + (a_xb_y - b_xa_y)\hat{\mathbf{k}}$$

$$|\mathbf{a}\times\mathbf{b}| = ab\sin\theta$$

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

ANSWERS TO EXERCISES AND ADDITIONAL EXERCISES

Chapter 1

1.1 9×10^9N

1.2 6×10^{-3}N (repulsive)

1.3 (a) 12 cm

(b) 0.2 N (attractive)

1.4 2.4×10^{39}. This is the ratio of electric force to the gravitational force (at the same distance) between an electron and a proton.

1.6 Charge is not created or destroyed. It is merely transferred from one body to another

1.8 Zero N

1.9 Varies as $\frac{1}{r^2}$

1.11 (a) 5.4×10^6 NC^{-1} along OB

(b) 8.1×10^{-3} N along OA

1.13 Total charge is zero. Dipole moment $= 7.5 \times 10^{-8}$ Cm along -z-axis

1.14 10^{-4} Nm

1.15 (a) 2×10^{12}, from wool to polythene

(b) Yes, but of a negligible amount ($= 2 \times 10^{-18}$ kg in the example).

1.16 (a) 1.5×10^{-2} N

(b) 0.24 N

1.17 5.7×10^{-3} N

1.18 Charges 1 and 2 are negative, charge 3 is positive. Particle 3 has the highest charge to mass ratio.

Answers to Additional Exercises

1.19 (a) Little change in the distribution of charges

(b) redistribution of charge on each sphere; positive and negative charges will 'face' each other

(c) charge on each sphere will be uniformly distributed.

1.20 9.81×10^{-4} mm

1.21 only (c) is right; the rest cannot represent electrostatic field lines. (a) is wrong because field lines must be normal to a conductor, (b) is wrong because lines of force cannot start from a negative charge, (d) is wrong because lines of force cannot intersect each other, (e) is wrong because electrostatic field lines cannot form closed loops.

1.22 The force is 10^{-2} N in the negative z-direction, that is in the direction of decreasing electric field. You can check that this is also the direction of decreasing potential energy of the dipole; torque is zero.

Chapter 2

2.1 25.98 Nm^2/C

2.2 Zero. The number of lines entering the cube is the same as the number of lines leaving the cube

2.3 (a) 0.07 μC

(b) No, only that the net charge inside is zero

2.4 2.2×10^5 N m^2/C

2.5 1.9×10^5 N m^2/C

2.6 (a) $-10^3 Nm^2/C$, because the charge enclosed is the same in the two cases.

(b) −8.8nC

2.7 −6.67nC

2.8 (a) $1.45 \times 10^{-3}C$

(b) 1.6×10^8 N m^2/C

2.9 $10\mu C/m$

2.10 (a) Zero, (b) Zero, (c) 1.9 N/C

2.11 9 cm from the first charge

2.12 2.7×10^6 V

2.13 (a) 4×10^4 V

(b) $8 \times 10^{-5}J$; No

2.14 (a) −0.7J

(b) 0.7J

2.15 (a) The plane normal to AB and passing through its mid-point has zero potential everywhere

(b) Normal to the plane in the direction AB

2.16 (a) Zero

(b) 10^5 NC^{-1}

(c) 4.4×10^4 NC^{-1}

2.17 96 pF

2.18 (a) 3 pF

(b) 40V

2.19 (a) 9 pF

(b) $2 \times 10^{-10}C$, $3 \times 10^{-10}C$, $4 \times 10^{-10}C$

2.20 18 pF, $1.8 \times 10^{-9}C$

2.21 (a) $V = 100V$, $C = 108pF$, $Q = 1.08 \times 10^{-8}C$

(b) $Q = 1.8 \times 10^{-9}C$, $C = 108 pF$, $V = 16.6V$

2.22 $1.5 \times 10^{-8}J$

2.23 $6 \times 10^{-6}J$.

Answers to Additional Exercises

2.24 1.2J; the point R is irrelevant to the answer.

2.25 Potential $= 4q/\sqrt{3}\,\pi\epsilon_0 b$, field is zero, as expected by symmetry.

2.26 (a) $2.4 \times 10^5 V$, $4.0 \times 10^5 Vm^{-1}$ from charge $2.5\mu C$ to $1.5\mu C$

(b) $2.0 \times 10^5 V$; $6.6 \times 10^5 Vm^{-1}$ in the direction that makes an angle of about 69° to the line joining charge $2.5\mu C$ to $1.5\mu C$

2.27 (a) Hint: choose a gaussian surface lying wholly within the conductor and enclosing the cavity.

(b) Gauss's law on the same surface as in (a) shows that q must induce $-q$ on the inner surface of the conductor.

(c) Enclose the instrument fully by a metallic surface.

2.28 (a) $-q/(4\pi r_1^2), (Q+q)/(4\pi r_2^2)$

(b) By Gauss's law, the net charge on the inner surface enclosing the cavity (not having any charge) must be zero. For a cavity of arbitrary shape, this is not enough to claim that electric field inside must be zero The cavity may have positive and negative charges with total charge zero. To dispose of this possibility, take a closed loop, part of which is inside the cavity along a field line and the rest inside the conductor. Since field inside the conductor is zero, this gives a net work done by the field in carrying a test charge over a closed loop We know this is impossible for an electrostatic field Hence there are no field lines inside the cavity (i.e., no field), and no charge on the inner surface of the conductor, whatever its shape.

2.30 Hint· consider the conductor with the hole filled up. Then the field just outside is $(\sigma/\epsilon_0)\ \hat{\mathbf{n}}$ and is zero inside View this field as a superposition of the field due to the filled-up hole plus the field due to the rest of the charged conductor Inside the conductor, these fields are equal and opposite. Outside they are equal both in magnitude and direction Hence the field due to the rest of the conductor is $(\sigma/2\epsilon_0)\ \hat{\mathbf{n}}$.

2.31 $\lambda/(2\pi\epsilon_0 r)$, where r is the distance of the point from the common axis of the cylinders The field is radial, perpendicular to the axis

2.33 p·uud, n.udd.

2.34 (a) −27.2eV,

(b) 13 6 eV,

(c) −13 6eV, 13 6 eV Note in the latter choice the total energy of the hydrogen atom is zero.

2.35 −19 2eV, the zero of potential energy is taken to be at infinity

2.36 (a) Hint Prove it by contradiction. Suppose the equilibrium is stable then the test charge displaced slightly in any direction will experience a restoring force towards the null-point That is, all field lines near the null-point should be directed inwards towards the null-point. That is, there is a net inward flux of electric field through a closed surface around the null-point. But by Gauss's law, the flux of electric field through a surface enclosing no charge must be zero Hence the equilibrium can not be stable.

(b) The mid-point of the line joining the two charges is a null-point. Displace a test-charge from the null-point slightly along the line. There is a restoring force But displace it say normal to the line. You will see that the net force takes it away from the null-point. Remember, stability of equilibrium needs restoring force in all directions.

2.37 The ratio of electric field of the first to the second is (b/a) A flat portion may be equated to a spherical surface of large radius, and a pointed portion to one of small radius.

2.38 (a) The atoms in the paper get polarized by the charged comb, resulting in a net force of attraction.

(b) To enable them to conduct a charge (produced by friction) to the ground.

(c) Reason similar to (b)

(d) current passes only where there is a *difference* in potential.

2.39 (a) On the axis of the dipole, potential is $(\pm 1/4\pi\epsilon_0)\ p/(x^2 - a^2)$ where $p = 2qa$ is the dipole moment, the $+$ sign when the point is closer to q and the $-$ sign when it is closer to $-q$. Normal to the axis, at points $(x, y, 0)$, potential is zero.

(b) The dependence on r is $1/r^2$ type

(c) Zero; No, because work done by electrostatic field between two points is independent of the path connecting the two points.

2.40 (a) $V_P - V_Q > 0$: $V_B - V_A > 0$

(b) $(\text{P.E.})_Q - (\text{P.E.})_P > 0$; $(\text{P.E.})_A - (\text{P.E.})_B > 0$

(c) Negative

(d) Positive

(e) Decreases

2.41 3J (Hint: the loss in potential energy of the system of dipoles appears as heat).

2.42 For large r, quadrupole potential goes like $1/r^3$, dipole potential goes like $1/r^2$, monopole potential goes like $1/r$

2.43 18 1μF capacitors arranged in 6 parallel rows, each row consisting of 3 capacitors in series.

2.44 1130 km^2

2.45 Equivalent capacitance = $\frac{200}{3}$pF;

$Q_1 = 10^{-8}$C, $V_1 = 100$ V ; $Q_2 = Q_3 = 10^{-8}$C;

$V_2 = V_3 = 50$V

$Q_4 = 2.55 \times 10^{-8}$C, $V_4 = 200$ V.

2.46 (a) 2.55×10^{-6}J

(b) $u = 0.113$Jm^{-3}, $u = (1/2\epsilon)\,E^2$

2.47 2.67×10^{-2}J

2.48 Hint: Suppose we increase the separation of the plates by Δx. Work done (by external agency) = $F\Delta x$. This goes to increase the potential energy of the capacitor by $ua\Delta x$ where u is energy density. Therefore $F = ua$ which is easily seen to be $(1/2)\,QE$, using $u = (1/2\epsilon)E^2$: The physical origin of the factor 1/2 in the force formula lies in the fact that just outside the conductor, field is E, and inside it is zero. So the average value $E/2$ contributes to the force.

2.49 (a) 5.5×10^{-9}F

(b) 4.5×10^2V

(c) 1.3×10^{-11}F.

2.50 (a) No, because charge distributions on the spheres will not be uniform.

(b) No.

(c) Not necessarily. (True only if the line of force is a straight line.) The line of force gives the direction of acceleration, not that of velocity, in general.

(d) Zero, no matter what the shape of the complete orbit

(e) No, potential is continuous

(f) A single conductor is a capacitor with one of the 'plates' at infinity.

(g) Water molectules have permanent dipole moments. However, detailed explanation of the value of dielectric constant requires microscopic theory and is beyond the scope of this book

2.51 1.2×10^{-10}F, 2.9×10^{4}V.

2.52 19 cm^2

2.53 (a) Planes parallel to x-y plane

(b) Same as in (a), except that planes differing by a fixed potential get closer as field increases.

(c) Concentric spheres centred at the origin

(d) A periodically varying shape near the grid which gradually reaches the shape of planes parallel to the grid at far distances

2.54 30 cm.

2.55 Hint: By Gauss's law, field between the sphere and the shell is determined by q_1 alone Hence potential difference between the sphere and the shell is independent of q_2 If q_1 is positive, this potential difference is always positive.

2.56 (a) Our body and the ground form an equipotential surface As we step out into the open, the original equipotential surfaces of open air change, keeping our head and the ground at the same potential.

(b) Yes The steady discharging current in the atmosphere charges up the aluminium sheet gradually and raises it's voltage to an extent depending on the capacitance of the capacitor (formed by the sheet, slab and the ground)

(c) The atmosphere is continually being charged by thunderstorms and lightning all over the globe and discharged through regions of ordinary weather. The two opposing currents are on an average, in equilibrium

(d) Light energy involved in lightning; heat and sound energy in the accompanying thunder

Chapter 3

3.1 30 A

3.2 17Ω, 8.5 V

3.3 (a) 6Ω

(b) 2V, 4V, 6V

3.4 (a) (20/19) Ω

(b) 10A, 5A, 4A; 19A.

3.5 1027°C

3.6 2.0×10^{-7}Ωm

3.7 0.0039°C^{-1}

3.8 867°C

3.9 Current in branch AB = (4/17)A.

in BC = (6/17)A, in CD = (−4/17)A, in AD = (6/17)A, in BD = (−2/17) A, total current = (10/17) A

3.10 (a) $X = 8.2\Omega$; to minimize resistance of the connections which are not accounted for in the bridge formula.

(b) 60.5 cm from A

(c) the galvanometer will show no current.

3.11 11.5V; the series resistor limits the current drawn from the external source. In its absence, the current will be dangerously high.

3.12 Resistance in series = 5988Ω

3.13 Shunt resistance = 10 mΩ

3.14 2.25V

3.15 (a) Electric field is established throughout the circuit almost instantly (with the speed of light) causing at every point a *local electron drift*. Establishment of a current does not have to wait for electrons from one end of the conductor travelling to the other end. However, it does take a little while for the current to reach its steady value.

(b) Each 'free' electron does accelerate increasing its drift speed until it collides with a positive ion of the metal. It loses its drift speed after collision but starts to accelerate and increase its drift again only to suffer a collision again and so on. On the average, therefore electrons acquire only a drift speed

(c) Simple Because the electron number density is enormous ~ 10^{29}m^{-3}.

(d) By no means The drift velocity is superposed over the large random velocities of electrons.

(e) In the absence of electric field, the paths are straight lines: in the presence of electric field, the paths are in general curved.

3.16 2.7×10^4s (7.5h)

Answers to Additional Exercises

3.17 (a) Electrons

(b) electrons and positive ions (protons) of hydrogen.

(c) H$^+$ and SO_4^{--}

(d) same as in (c); the direction of current opposite to that during discharging

(e) electrons and holes (holes are negative charge vacancies which effectively act as positive charge carriers.

(f) electrons

(g) electrons

3.18 Take the radius of the earth = 6.37×10^6 m and obtain total charge of the globe. Divide it by current to obtain time = 283 s. Still this method gives you only an estimate, it is not strictly correct. Why?

3.19 (a) 1.4 A, 11.9 V

(b) 0.005 A; impossible because a starter motor requires large current ($\sim$ 100 A) for a few seconds

3.20 (b) The term 'capacity' of a cell refers to the quantity of electrical energy it can supply. This depends on the size of the cell, among other things. A cell marked 3.5 Ah at 1h discharge rate willl supply 3.5 A for one hour. However depletion of energy is faster (than expected proportionately) for higher currents. Thus the cell will not supply 14 A for 15 min, but may supply 1.75 A for more than 2 h.

(c) (i) An accumulator (secondary cell) say the lead-acid type with very low internal resistance (ii) a 'dry battery' (primary cell) say the carbon-zinc cell.

3.21 $3.9 \times 10^2 \Omega$

3.22 The mass (or weight) ratio of copper to aluminium wire is $(1.72/2.63) \times (8.9/2.7) \simeq 2.2$. Since aluminium is lighter, it is preferred for long suspensions of cables.

3.23 Ohm's law is valid to a high accuracy; the resistivity of the alloy manganin is nearly independent of temperature.

3.24 Plot the straight line graph: $\mathcal{E} - V = Ir$ with $\mathcal{E}$ = 1.5V and $r = 0.5\,\Omega$ The intersection of this line with the (non-linear) V–I plot for the conductor gives the required voltage (0.9V) and current (1.2 A)

3.25 (a) Inside the cell there is no net field; the electrostatic field due to the plates is balanced by a field of non-electrostatic origin. (If this were not so, charge carriers inside the cell would constitute a current even when the circuit is open.) Outside the cell, the net field is just the electrostatic field of the plates

(b) When the circuit is closed from outside, current flows in the direction of electrostatic field outside, and *opposite* to the direction of electrostatic field inside the cell. The latter fact shows that there is a net field inside the cell *opposite* to the electrostatic field.

(c) They are prevented from being neutralized because the electrostatic field is opposed by a field in the source that has a complex non-electrostatic origin. The latter is related to the emf of the source.

(d) Here the situation is different from (b). Now the current inside the secondary cell during charging is in the direction of electrostatic field The electrostatic field is greater than the non-electrostatic field. The terminal voltage therefore is greater than emf of the secondary cell

(e) Emf does not have simple electrostatic origin. Therefore the concept of potential may not be strictly applicable. The emf is work done per unit charge.

3.26 (a) Only current (because it is given to be steady!). The rest depend on the area of cross section inversely.

(b) No, examples of non-ohmic elements: vacuum diode, semiconductor diode.

(c)Typical electrical insulators (e.g. glass) differ in their resistivity from metals enormously, by a factor of the order of 10^{22}. The corresponding factor for thermal insulators versus thermal conductors is only 10^{3}.

(d) The internal resistance of a car battery decreases with increase in temperature.

(e) Mainly in its internal resistance, but partly in its emf also

(f) Because the maximum current drawn from a source $= \mathcal{E}/r$

(g) Because, if the circuit is shorted (accidentally), the current drawn will exceed safety limits, if internal resistance is not large.

3.27 (a) greater (b) lower (c) reduces

(d) decreases (e) decreases

(f) nearly independent of

(g) 10^{22}

3.28 1.2 Ω

3.29 (a) 0.9 A

(b) No, there is no formula for emf and internal resistance of non-similar cells joined in parallel. For this situation, one must use Kirchhoff's rules.

3.30 (a) (i) in series, (ii) all in parallel; n^2

(b) (i) Join 1 Ω, 2 Ω in parallel and the combination in series with 3Ω

(ii) parallel combination of 2 Ω and 3 Ω in series with 1 Ω (iii) all in series, (iv) all in parallel.

(c) (i) (16/3) Ω, (ii) $5R$

3.31 Hint: Let X be the equivalent resistance of the infinite network. Clearly, $2 + X/(X + 1) = X$ which gives $X = (1+\sqrt{3})\Omega$, therefore the current is 3.7A

3.32 (a) Shunt resistance $= 4.00 \times 10^{-3}\ \Omega$ which is also nearly its net resistance; reads slightly less.

(b)Series resistance = 3988 Ω; net resistance = 4000 Ω; reads slightly less

3.33 For X, use (b), for Y use (a)

3.34 (a) The 20kΩ voltmeter gives the highest reading; the last case (both the voltmeters connected across AB) gives the lowest reading

(b) All cases will give the same reading if internal resistance is zero, otherwise answers similar to (a)

3.35 'Resistance per volt' is a different way of specifying the current at full scale deflection. Here it is (1/5000)A = 0.20mA. To convert it to a 20 V meter, put a resistance R in series with the meter so that $R \times 2 \times 10^{-4} = 15$ i.e., $R = 7.5 \times 10^{4} = 75,000\Omega$. The 'resistance per volt' of the new meter is the same as before. The higher the 'resistance per volt' of the meter, the lesser is the current it draws from the circuit and the better it is. So this meter is more accurate than the one graded as 2000 Ω/V.

3.36 (a) $\mathcal{E} = 1.25$ V

(b) To reduce current through the galvanometer when the movable contact is far from the balance point.

(c) No.

(d) No.

(e) No. If $\mathcal{E}$ is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire AB.

(f) The circuit as it is would be unsuitable, because the balance point (for $\mathcal{E}$ of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the emf to be measured. Then the balance point will be at larger length of the wire and the percentage error will be much smaller

3.37 $X = 11.7\Omega$. If there is no balance point, it means potential drops across R or X are greater than the potential drop across the potentiometer wire AB. The obvious thing to do is to reduce current in the outside circuit (and hence potential drops across R and X) suitably by putting a series resistor

3.38 1.7Ω

3.39 (a) Use a standard cell to first calibrate potential drop per unit cm of the potentiometer wire Then use an external circuit containing a cell and a variable resistor across which the voltmeter to be calibrated is connected. Tap off different potential drops using the variable resistor, measure them by the potentiometer and thus calibrate the voltmeter.

(b) Here the external circuit will consist of a cell, and a variable resistor in series with a standard resistor and the ammeter. The potential drop across the standard resistor is varied and measured using the calibrated potentiometer. From the known value of the resistance, one obtains different values of the current and thus the ammeter is calibrated.

3.40 290 kΩ (The ammeter's reading will be too small). This is an unusual use of a voltmeter meant only for high resistance measurement.

3.41 (a) $I = \mathrm{d}Q/\mathrm{d}t = A\,\mathrm{d}\sigma/\mathrm{d}t = \epsilon_0 A\,\mathrm{d}E/\mathrm{d}t$

(b) No; there is no conduction current across the plates However, if we agree to call $\epsilon_0 A\mathrm{d}E/\mathrm{d}t$ as 'current' across the capacitor, then the first rule is valid. Maxwell called this the **displacement current**

3.42 (a) About 10kΩ

(b) Because our body is sensitive to minute currents even as low as a few mA

(c) This is a misleading notion. There is no special attractive force that keeps a person 'stuck' with a high power line. What happens is that a current of the order of 0.05 A or even much less is enough to disorganise our nervous system. The result is that the affected person may lose temporarily his ability to exercise his nervous control to get himself 'free' from the high voltage point.

(d) The cause of death is not heating, though a person may receive burns if the currents are too large. The cause of death is the interference caused by external currents in our highly sensitive nervous system which is basically electrical in nature. External currents cause convulsive actions, and especially interfere with the nerve processes related to our heartbeating. Beyond a certain point, this interference is fatal

(e) About 0.1V.

Chapter 4

4.1 192 W

4.2 25%

4.3 5A

4.4 3A; 70Ω

4.5 7200 J. chemical energy stored in the battery

4.6 2.9 Ω (Hint: power dissipated as heat is 0.7 × 50 × 12 = 420W. Therefore $I^2R = 420$ i.e. $R = 420/144 = 2.9\Omega$).

4.7 25W; 30kJ

4.8 No, because the power dissipated would be 4.5 kW which exceeds the maximum power rating of the resistor

4.9 First heater

4.10 Mode (i) involves lesser power wastage, because for the same power transmitted, higher voltage corresponds to smaller current and therefore lesser power dissipated as heat in the cables.

4.11 (a) from copper to bismuth
(b) from copper to iron
(c) from lead to platinum.

Answers to Additional Exercises

4.12 (a) 3.0 W
(b) 0.40 W
(c) 2.6 W
(d) 2.6 W

4.13 Series resistor of 8.0 Ω
(a) 800 W
(b) 704 W
(c) 86400 J

4.14 (a) Power output $= I^2R = (\mathcal{E}^2R)/(R+r)^2$. This is maximum at $R = r$. Maximum power output $= \mathcal{E}^2/4r$.

(b) When the battery is shorted, power output = 0. In this case, the entire power is dissipated as heat inside the battery. Its magnitude is $\mathcal{E}^2/r$.

4.15 3.0 A

4.16 (a) Power output of the source $= \mathcal{E}I - I^2 r$, which is maximum at $I = \mathcal{E}/2r$.

(b) $I = (\mathcal{E} - \mathcal{E}')/r$ since external resistance R is neglible ($\mathcal{E}'$ stands for back emf). Power output of the electric motor equals power output of the source since $R \simeq 0$. The latter is maximum for $I = \mathcal{E}/2r$. This gives $\mathcal{E}' = \mathcal{E}/2$.

(c) The condition in 4.16(a) is more general than condition in 4.14(a), for a passive resistor, the former reduces to the latter.

4.17 (a) 252000 J

(b) 201600 J (Hint: Obtain first the power dissipated as heat The remaining power is the sum of the mechanical power yielded by the motor and the chemical power stored in the battery.)

4.18 Nichrome ribbon, because it has lesser resistance.

4.19 Hint: The temperature of the wire increases up to θ where the heat produced per second by the current equals heat lost (by radiation) per second, i.e.,

$$\frac{I^2 \rho l}{\pi r^2} = h \times 2\pi r l$$

(Ignoring heat loss from the ends)

where h is the heat lost per second per unit surface area of the wire. Clearly, h (and therefore the steady state temperature θ) is independent of l.

4.20 $r = 0.24$ mm. Hint: From the equation $I^2 \propto r^3$.

4.21 (a) No, the steady temperature acquired by a resistor depends not only on the power consumed but also on its characteristics (such as surface area, emittivity etc) which determine its power loss due to radiation.

(b) Tungsten bulb.

4.22 120° C

4.23 0 25 Ω

4.24 No, the phenomenon described is known as the Peltier effect. In ordinary Joule effect of current (for which the given formula applies) heat is produced by the current no matter what its direction is. (Notice, the formula involves I^2, i e. it depends only on the magnitude of I).

Chapter 5

5.2 $\pi \times 10^{-4}\text{T} \simeq 3.1 \times 10^{-4}\text{T}$

5.4 3.5×10^{-5}T

5.5 4×10^{-6} T vertical up

5.6 1.2×10^{-5}T towards south

5.7 (a) 1.9×10^{-4}T normal to the plane of the paper going out of it

(b) same magnitude of **B** but opposite in direction to that in (a)

5.8 0.6 Nm^{-1}

5.9 8.1×10^{-2}N; direction of force given by Fleming's lefthand rule

5.10 2×10^{-5}N; attractive force normal to A towards B

5.11 $8\pi \times 10^{-3}\text{T} \simeq 2.5 \times 10^{-2}\text{T}$

5.12 0.96 Nm

5.13 (a) 1.4 (b) 1

5.14 4.2 cm

5.15 18 MHz

5.16 (a) 3.1 Nm (b) No, the answer is unchanged because the formula $\boldsymbol{\tau} = NI\mathbf{A} \times \mathbf{B}$ is true for a planar loop of any shape.

Answers to Additional Exercises

5.17 $5\pi \times 10^{-4}\text{ T} = 1.6 \times 10^{-3}$ T towards west

5.18 Length about 50 cm, radius about 4 cm, number of turns about 400, current about 10 A. These particulars are not unique. Some adjustment within limits is possible.

5.19 (b) In a small region of length 2d about the mid-point between the coils,

$$B = \frac{\mu_0 I R^2 N}{2} \times \left[\left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-3/2} + \left\{\left(\frac{R}{2}-d\right) + R^2\right\}^{-3/2}\right]$$

$$\simeq \frac{\mu_0 I R^2 N}{2} \times \left(\frac{5R^2}{4}\right)^{-3/2} \times \left[\left(1+\frac{4d}{5R}\right)^{-3/2} + \left(1-\frac{4d}{5R}\right)^{-3/2}\right]$$

$$\simeq \frac{\mu_0 I R^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{3/2} \times \left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

where in the second and third steps above, terms containing d^2/R^2 and higher powers of d/R are neglected since $d/R \ll 1$. The terms linear in d/R cancel giving a uniform field B in a small region:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 I N}{R} \simeq 0.72 \frac{\mu_0 I N}{R}$$

5.20 Hint: B for a toroid is given by the same formula as for a solenoid: $B = \mu_0 n I$, where n in this case is given by $n = N/2\pi r$. The field is non-zero only inside the core surrounded by the windings. (a) zero (b) 3.0×10^{-2} T (c) zero Note, the field varies slightly across the cross-section of the toroid as r varies from the inner to outer radius. Answer (b) corresponds to the mean radius $r = 25.5$ cm.

5.21 (a) initial **v** is either parallel or antiparallel to **B**

(b) Yes, because magnetic force can change the direction of **v**, not its magnitude

(c) **B** should be in a vertically downward direction,

5.22 (a) Electron-positron pair production by a high energy gamma ray

(b) The charged particles ionize much more in a liquid than in a gas. They progressively lose energy and get slower after collisions with the environment. Since the radius of a track in a given **B** is proportional to the speed of the particle, it decreases progressively, giving rise to spiral paths.

(c) The edge L of the strip in the figure is positive, while the edge N is negative.

(d) Conduction is effectively via positive charge carriers as in conduction via holes (electron vacancies) in semiconductors.

5.23 (a) Circular trajectory of radius 1.0 mm normal to **B** (b) helical trajectory of radius 0 5 mm with velocity component $2.3 \times 10^7 \text{ms}^{-1}$ along **B**

5:24 B = 0.66T, Kinetic energy = 7.4 MeV

5.25 Deuterium ions or deuterons, the answer is not unique because only the ratio of charge to mass is determined Other possible answers are He^{++}, Li^{+++} etc.

5.26 (a) A horizontal magnetic field of magnitude 0.26 T normal to the conductor in such a direction that Fleming's left hand rule gives a magnetic force upward. (b) 1.176 N

5.27 1.2 N m^{-1}, repulsive. Note, obtaining total force on the wire as $1.2 \times 0.7 = 0.84$ N is only approximately correct because the formula $F = (\mu_0/2\pi r)\, I_1 I_2$ for force per unit length is strictly valid for infinitely long conductors.

5.28 (a) 2.1 N vertically downwards

(b) 2.1 N vertically downwards (true for any angle between current and direction and **B** since $l \sin\theta$ remains fixed, equal to 20 cm)

(c) 1.68 N vertically downwards.

5.29 Use $\tau = I\mathbf{A} \times \mathbf{B}$ and $\mathbf{F} = I\mathbf{l} \times \mathbf{B}$

(a) 1.8×10^{-2} N m along $-y$ direction

(b) same as in (a)

(c) $1\ 8 \times 10^{-2}$ N m along $-x$ direction

(d) $1\ 8 \times 10^{-2}$ N m at an angle of 240° with the $+x$ direction

(e) Zero (f) Zero

Force is zero in each case Case (e) corresponds to stable, and case (f) corresponds to unstable equilibrium.

5.30 (a) 0.384 N m; the coil experiences this torque whenever the x-axis lies in the plane of the coil.

(b) torque is zero whenever the coil lies in the $y-z$ plane. Equilibrium is stable in this plane when the sense of current circulation in the loop is such that the area vector (related to this sense via right-handed screw rule) is parallel to **B**; it is unstable when the area vector is antiparallel to **B**. (This rule can be equivalently stated in terms of magnetic moment of the loop: $\boldsymbol{\mu} = I\mathbf{A}$. When $\boldsymbol{\mu}$ is parallel to $\mathbf{B}$, equilibrium in stable; when $\boldsymbol{\mu}$ is antiparallel to $\mathbf{B}$, equilibrium is unstable.)

5.31 3.0×10^{-9}Nm per degree

5.32 Circular loop in a plane containing E – W direction, 1.54×10^{-3} N m (Hint: For a given perimeter, a circle encloses maximum area for any planar curve.)

5.33 4.0×10^{-6} N (attractive) [Hint: First calculate the force on the short conductor due to **B** of the long conductor, and then apply Newton's third law.]

5.34 11.3 A [Hint: Equilibrium condition is: $IBl\cos\theta = mg\sin\theta$ where θ is the inclination of the plane.]

5.35 (a) zero (b) zero (c) force on each electron is $evB = IB/(nA) = 5 \times 10^{-25}$N *Note:* Answer (c) denotes only the magnetic force. Actually there will be an additional balancing force due to the Hall emf generated across the width of the wire. See 5.36

5.36 $7.0 \times 10^{28}\text{m}^{-3}$ [Hint: Use Eq. (5.53).]

5.37 108 A

5.38 (a) (i) straight line (ii) parabola (iii) parabola

(b) (i) straight line (ii) circle in a plane normal to the field (iii) helix with its axis parallel to **B**

The speed is constant for each case under (b)

(c) (i) straight line (ii) same as case (ii) next (iii) resolve **v** along **B** and perpendicular to it, motion is circular, motion perpendicular to **B** is circular motion parallel to **B** (or **E**) is accelerated or retarded due to **E**

5.39 (c) Hint A straight conductor of finite length cannot by itself form a complete steady current circuit. Additional conductors are necessary to close the circuit. These will spoil the symmetry of the problem. The difficulty disappears if the conductor is infinitely long.

5.40 (a) No, because that would require $\boldsymbol{\tau}$ to be in the vertical direction. But $\boldsymbol{\tau} = I\,\mathbf{A} \times \mathbf{B}$, and since **A** of the horizontal loop is in the vertical direction, $\boldsymbol{\tau}$ must be in the plane of the loop.

(b) Orientation of stable equilibrium is one where the area vector **A** of the loop is in the direction of external magnetic field. In this orientation, the magnetic field produced by the loop is in the same direction as external field, both normal to the plane of the loop, thus giving rise to maximum flux of the total field.

(c) It assumes circular shape with its plane normal to the field to maximize flux, since for a given perimeter, a circle encloses greater area than any other shape

(d) Yes, it is, and this fact is crucial to the phenomenon of electromagnetic waves, as you will see in later chapter.

5.41 7.8×10^{-4} N, towards the long conductor if the current in the closer side of the loop is in the same direction as the current in the long conductor; repulsive otherwise. Note, the forces on the two smaller sides of the loop cancel each other.

Chapter 6

6.1 7.2×10^{-4}J; direction of torque is such as to end to bring the needle along **B**

6.2 $\omega = 8.0\,\text{s}^{-1}$

6.3 $0.36\,\text{JT}^{-1}$

6.4 (a) **m** parallel to **B**; $U = -mB = -4.8 \times 10^{-2}$J, stable

(b) **m** antiparallel to **B**; $U' = +mB = +4.8 \times 10^{-2}$J; unstable.

6.5 $m = 9\ 0 \times 10^{-3}\ \text{JT}^{-1}$, directed along normal to the plane of the loop away from the observer.

6.6 $69.3\ \text{JT}^{-1}$

6.7 $0.60\ \text{JT}^{-1}$ along the axis of the solenoid; the direction determined by the sense of flow of current.

6.8 7.5×10^{-2}J

6.9 (a) (i) 0.33J (ii) 0.66J

(b) (i) torque of magnitude 0.33 J in a direction that tends to align the magnetic moment vector along **B**. (ii) Zero.

6.10 (a) 1.28 Am^2 along the axis in the direction related to the sense of current via the right-handed screw rule.

(b) force is zero in uniform field, torque = 0 048 Nm in a direction that tends to align the axis of the solenoid (i.e. its magnetic moment vector) along **B**.

6.11 B = 0.35 sec 22° $\simeq$ 0.38 G.

6.12 The earth's field lies in a vertical plane 12° west of the geographic meridian making an angle of 60° (upwards) with the horizontal (magnetic south to magnetic north) direction. Magnitude = 0.32 G.

6.13 (i) 0.96 G along S-N direction.

(ii) 0.48 G along N-S direction

6.14 0.54 G in the direction of earth's field

6.15 At $14 \times 2^{-1/3} \simeq 11\ 1$ cm on the normal bisector.

6.16 (a) $(\mu_0 m)/(4\pi r^3) = 0.42 \times 10^{-4}$ which gives $r = 5.0$ cm

(b) $(\mu_0 m)/(2\pi r_1^3) = 0\ 42 \times 10^{-4}$ i.e. $r_1 = 2^{1/3}\ r \simeq 6\ 3$ cm

6.17 Use $I = mB/(4\pi^2\nu^2)$; $m = NIA$ to get $I = 1.2 \times 10^{-4}$ kgm^2

Answers to Additional Exercises

6.18 Parallel to and above the cable at a distance of 1.5 cm.

6.19 Below the cable:

$$\begin{aligned} R_h &= 0.39\cos 35^\circ - 0.2 \\ &= 0.1195\,\text{G}; \\ R_v &= 0.39\sin 35^\circ = 0.224\,\text{G}. \\ R &= \sqrt{R_h{}^2 + R_v{}^2} = 0.254\text{G}; \\ \theta &= \tan^{-1}\frac{R_v}{R_h} \simeq 62^\circ \end{aligned}$$

Above the cable:

$$\begin{aligned} R_h &= 0.39\cos 35^\circ + 0.2 \\ &= 0.5195\,\text{G} \\ R_v &= 0\ 224\,\text{G} \\ R &= 0.566\,\text{G},\ \theta \simeq 23^\circ \end{aligned}$$

6.20 (a) $B_h = (\mu_0 IN/2r)\cos 45^\circ = 0.39$G

(b) east to west (i.e the needle will reverse its original direction).

6.21 In a stable equilibrium, the loop is 34° west of the magnetic meridian, its sense of current flow being such that its magnetic moment vector is 56° east of magnetic north. If the current in the loop is reversed, this orientation is in unstable equilibrium. A slight perturbation will result in the rotation of the loop by 180° bringing it to stable equilibrium.

6.22 Magnitude of the other field

$$= \frac{1.2 \times 10^{-2} \times \sin 15^\circ}{\sin 45^\circ}$$
$$= 4.4 \times 10^{-3}\text{T}$$

6.23

$$R = \frac{meV}{e\,B}$$
$$= \frac{\sqrt{2m_e \times \text{kinetic energy}}}{e\,B}$$
$$= 11.3\,\text{m}$$

Up or down deflection $= R(1-\cos\theta)$, where $\sin\theta = 0.3/11.3$. We get deflection $\simeq$4 mm.

6.24 Hint: Potential energy of the configuration arises due to the potential energy of one dipole (say B) in the magnetic field due to the other (A). Use the result that

$$\mathbf{B}_A = -\frac{\mu_0}{4\pi}\frac{\mathbf{m}_A}{r^3}$$

(on the normal bisector)

$$\mathbf{B}_A = \frac{\mu_0}{2\pi}\frac{\mathbf{m}_A}{r^3}$$

(on the axis)

Equilibrium is stable when $\mathbf{m}_B$ is parallel to $\mathbf{B}_A$, and unstable when $\mathbf{m}_B$ is anti-parallel to $\mathbf{B}_A$. (a) AB_1 and AB_2

(b) (i) AB_3, AB_6 (stable)

(ii) AB_5, AB_4 (unstable)

(c) AB_6

6.25 Initially, total dipole moment

$$= 0.15 \times 1.5 \times 10^{-23} \times 2.0 \times 10^{24}$$
$$= 4.5\ \text{JT}^{-1}$$

Use Curie's law $m \propto B/T$ to get the final dipole movement

$$= 4.5 \times (0.98/0.84) \times (4.2/2.8)$$
$$= 7.9\ \text{JT}^{-1}$$

6.26 Use the formula $B = \mu_0 KnI$, $K = \mu/\mu_0$ (relative permeability) to get $B = 5.76\text{T}$.

6.27 Of the two, the relation $\boldsymbol{\mu}_l = -(e/2m)\mathbf{l}$ is in accordance with classical physics. It follows easily from the definitions of $\boldsymbol{\mu}_l$ and $\mathbf{l}$:

$$\mu_l = IA = (e/T)\pi r^2$$
$$l = mvr = m\frac{2\pi r^2}{\text{T}}$$

where r is the radius of the circular orbit which the electron of mass m and charge $(-e)$ completes in time T. Clearly $\mu_l/l = e/2m$.

Since charge of the electron is negative $(= -e)$, it is easily seen that $\boldsymbol{\mu}_l$ and $\mathbf{l}$ are antiparallel, both normal to the plane of the orbit. Therefore $\boldsymbol{\mu}_l = -e/2m\,\mathbf{l}$. Note μ_s/s in contrast is e/m, i.e. twice the classically expected value. This latter result (verified experimentally) is an outstanding consequence of modern quantum theory can not be obtained classically

6.28 (a) In either case, one gets two magnets, each with a north and south pole.

(b) Molten iron is above its Curie temperature (=770°C) and is therefore not ferromagnetic. An iron bar magnet when melted does not retain its magnetism.

(c) No force if the field is uniform. The iron nail experiences a nonuniform magnetic field due to the bar magnet. The induced magnetic moment in the nail therefore experiences both force and torque The net force is attractive because the induced (say) south pole in the nail is closer to the north pole of the magnet than the induced north pole.

(d) Not necessarily. True only if the source of the field has a net non-zero magnetic moment. This is not so for a toroid or even for a straight infinite conductor.

(e) Depends on what one means by three poles. Poles must always occur in pairs. But one can think of two bar magnets with (say) their north ends glued together as providing a three-pole field configuration.

(f) Try to bring different ends of the magnets closer. A *repulsive force* in some situations establishes that both are magnetized If it is always attractive, then one of them is not magnetized. To see which one, pick up one say A and lower one of its ends; first on one of the ends of the other (say B), and then on the middle of B. If you notice that in the middle of B, A experiences no force, then B is magnetized. If you do not notice any change from the end to the middle of B, then A is magnetized.

6.29 (a) Magnetic declination, angle of dip, horizontal component of earth's magnetic field.

(b) Greater in Britain, (it is about 70°), because Britain is closer to the magnetic north pole.

(c) Field lines of B due to the earth's magnetism would seem to come out of the ground

(d) A compass is free to move in a horizontal plane, while the earth's field is exactly vertical at the magnetic poles So the compass can point in any direction there

(e) Use the formula for field **B** on the normal bisector of a dipole of magnetic moment **m**.

$$\mathbf{B} = -\frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3}.$$

Take m = $8 \times 10^{22}\,\mathrm{JT}^{-1}$, $r = 6\,4 \times 10^6$m; one gets B = 0 3 G which checks with the order of magnitude of the observed field on the earth.

(f) Why not? The earth's field is only approximately a dipole field Local N–S poles may arise due to, for instance, magnetized mineral deposits.

6.30 (a) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(b) Because molten iron (which is the phase of the iron at the high temperatures of the core) is not ferromagnetic.

(c) One possibility is the radioactivity in the interior of the earth. But nobody really knows. You should consult a good modern text on geomagnetism for a proper view of the question.

(d) Earth's magnetic field gets weakly 'recorded' in certain rocks during solidification. Analysis of this rock magnetism offers clues to geomagnetic history.

(e) At large distances, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The latter is sensitive to extraterrestrial disturbances such as, for example, the solar wind.

(f) From the relation $R = m\nu/eB$, an extremely minute field bends charged particles in a circle of very large radius. Over a small distance, the deflection due to the circular orbit of such large R may not be noticeable, but over the gigantic interstellar-distances, the deflection can significantly affect the passage of e g. cosmic rays.

6.31 (a) *Wrong*. Magnetic field lines can never emanate from a point as shown. Over any closed surface, the net flux of **B** must always be zero; i.e , pictorially as many field lines should seem to enter the surface as the number of lines leaving it and going out. The field lines shown, in fact, represent electric field of a long positively charged wire. The correct magnetic field lines are lines circling the straight conductor, as described in chapter 5.

(b) *Wrong* Magnetic field lines (like electric field lines) can never cross each other, because otherwise the direction of the field at the point of intersection is ambiguous. There is further error in the figure. Magnetostatic field lines can never form closed loops around empty space. A closed loop of a static magnetic field line must enclose a region across which a current is passing. (By contrast, electrostatic field lines can never form closed loops, neither in empty space, nor even when the loop encloses charges).

(c) *Right*. Magnetic lines are completely confined within a toroid Nothing wrong here in field lines forming closed loops, since each loop encloses a region across which a current passes. Note, for clarity of figure, only a few field lines within the toroid have been shown. Actually, the entire region enclosed by the windings contains magnetic field

(d) *Wrong*. Field lines due to a solenoid at its ends and outside cannot be so completely straight and confined, such a thing violates Ampere's law. The lines should curve out at the ends, and meet eventually to form closed loops.

(e) *Right* These are field lines outside and inside of a bar magnet. Note carefully the direction of field lines inside. Not all field lines emanate out of a north pole (or converge into a south pole). Around both the N-pole, and the S-pole, the net flux of the field is zero.

(f) *Wrong*. These field lines cannot possibly represent a magnetic field. Look at the upper region. All the field lines seem to emanate out of the shaded plate. The net flux of field through a surface surrounding the shaded plate is not zero. This is impossible for a magnetic field. The given field lines, in fact, show the electrostatic field lines around a positively charged upper plate and a negatively charged lower plate. The difference between figures (e) and (f) should be carefully grasped.

(g) *Wrong* Magnetic field lines between two pole pieces cannot be precisely straight at the ends. Some fringing of the lines is inevitable. Otherwise, Ampere's law is violated. This is true also of electric field lines.

6.32 (a) *No*. The magnetic force is always normal to **B** (remember magnetic force = e **v** × **B**). It is misleading to call field lines of **B** as lines of force.

(b) If field lines were entirely confined between two ends of a straight solenoid, the flux through the cross-section at each end would be non-zero. But the flux of field **B** through any closed surface must always be zero. For a toroid, this difficulty is absent because it has no 'ends'.

(c) Gauss' law of magnetism states that the flux **B** through any closed surface is always zero $\oint_s \mathbf{B} \cdot \mathbf{ds} = 0$ for any closed S.

If monopoles existed, the r.h.s would equal the monopole (magnetic charge) q_m enclosed by S. (Analogous to Gauss' law of electrostatics). $\oint_s \mathbf{B} \cdot \mathbf{ds} = q_m$; where q_m is the (monopole) magnetic charge included by S.

(d)*No*. There is no force or torque on an element due to the field produced by that element itself. But there is a force (or torque) on an element of the same wire. (For the special case of a straight wire, this force is zero)

(e) *Yes*. The average of the charges in the system may be zero. Yet, the mean of the magnetic moments due to various current loops may not be zero. A neutron, for example, has zero charge but non-zero magnetic moment.

(f) This is not a simple question. Think of the nail as so many charges in motion. In the bar's magnetic field, each charge in motion in the nail experiences a magnetic force that alters its velocity but not speed. The total energy of the system (the nail) cannot change, since magnetic force does no work. But because of changes in individual velocity directions, the velocity of the centre of mass can increase, obviously at the expense of the nail's internal energy. Magnetic field provides force, while internal energy of the nail provides increase in kinetic energy of the nail as a whole. The above answer would be complete if magnetic moments arose only from the (orbital) motion of charges inside the nail. Actually, however, charges (electrons) have *intrinsic* magnetic moments also – a fact that can be understood only in the framework of quantum mechanics. In a non-uniform field, these magnetic dipoles experience force leading to an increase in the kinetic energy of the nail as a whole

6.33 (a) The tendency to disrupt the alignment of dipoles (with the magnetizing field) arising from random thermal motion is reduced at lower temperatures.

(b) The induced dipole moment in a diamagnetic sample is always opposite to the magnetizing field, no matter what the internal motion of the atoms is.

(c) Slightly less, since bismuth is diamagnetic

(d) *No*, as is evident from the magnetization curve. From the slope of magnetization curve, it is clear that μ is greater for lower fields.

(e) Proof of this important fact (of much practical use) is based on boundary conditions of magnetic fields (**B** and **H**) at the interface of two media. (When one of the media has $\mu \gg 1$, the field lines meet this medium nearly normally). Details are beyond the scope of this book.

(f) *Yes*. Apart from minor differences in strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetization will have the same order of magnetization. But of course, saturation requires impractically high magnetizing fields.

6.34 (b) Carbon steel piece, because heat lost per cycle is proportional to the

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area of the hysteresis loop.

(c)Magnetization of a ferromagnet is not a single-valued function of the magnetizing field. Its value for a particular field depends both on the field and also on history of magnetization (i.e how many cycles of magnetization it has gone through etc.). In other words, the value of magnetization is a record or 'memory' of its cycles of magnetization. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information

(d) Ceramics. (specially treated barium iron oxides) also called ferrites.

(e) Surround the region by soft iron rings. Magnetic field lines will be drawn into the rings, and the enclosed space will be free of magnetic field But this shielding is only approximate, unlike the perfect electric shielding of a cavity in a conductor placed in an external electric field.